Ministry of Higher Education and Scientific Research
Almustaqbal University, College of Engineering
And Engineering Technologies
Computer Technology Engineering Department

Five week :

Series Ac Circuits (R L C)

Course Name : Fundamentals of Electricity
Stage : One
Academic Year : 2024
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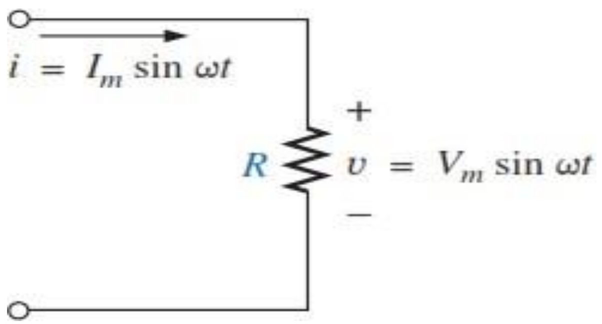
SERIES AND PARALLEL AC CIRCUITS

15.2 IMPEDANCE AND THE PHASOR DIAGRAM



Resistive Elements

For the purely resistive circuit, v and *i are in-phase,* and the magnitude

$$I_m = \frac{V_m}{R} \quad \text{or} \quad V_m = I_m R$$

In phasor form,

$$v = V_m \sin \omega t \Rightarrow \mathbf{V} = V \angle 0°$$

where $V = 0.707V_m$.

Applying Ohm's law and using phasor algebra, we have

$$\mathbf{I} = \frac{V \angle 0°}{R \angle \theta_R} = \frac{V}{R} \angle 0° - \theta_R$$

Since *i and v are in-phase, the angle associated with i also must be 0°. To* satisfy this condition, θ_R must equal 0°.

$$\mathbf{I} = \frac{V \angle 0°}{R \angle 0°} = \frac{V}{R} \angle 0° - 0° = \frac{V}{R} \angle 0°$$

so that in the time domain

$$i = \sqrt{2}\left(\frac{V}{R}\right) \sin \omega t$$

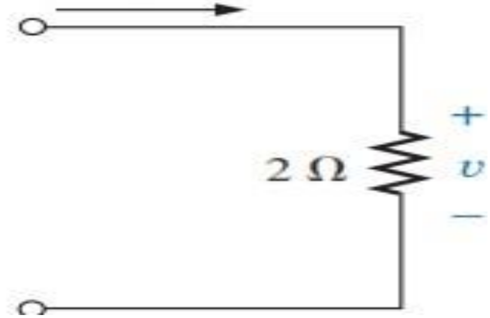
$$\boxed{\mathbf{Z}_R = R \angle 0°}$$

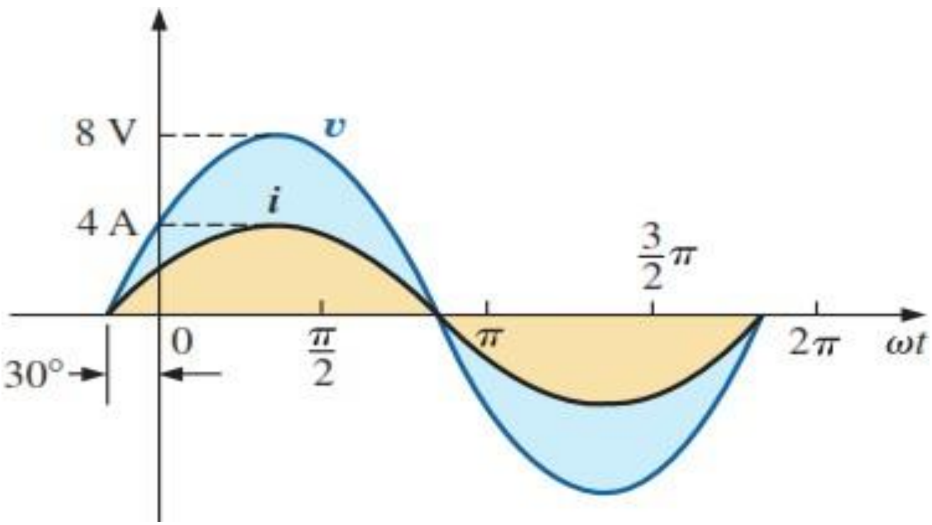
EXAMPLE 15.2 Using complex algebra, find the voltage v for the circuit in Fig. Sketch the waveforms of v and i.

$$i = 4\sin(\omega t + 30°) \Rightarrow \text{phasor form } \mathbf{I} = 2.828 \text{ A} \angle 30°$$

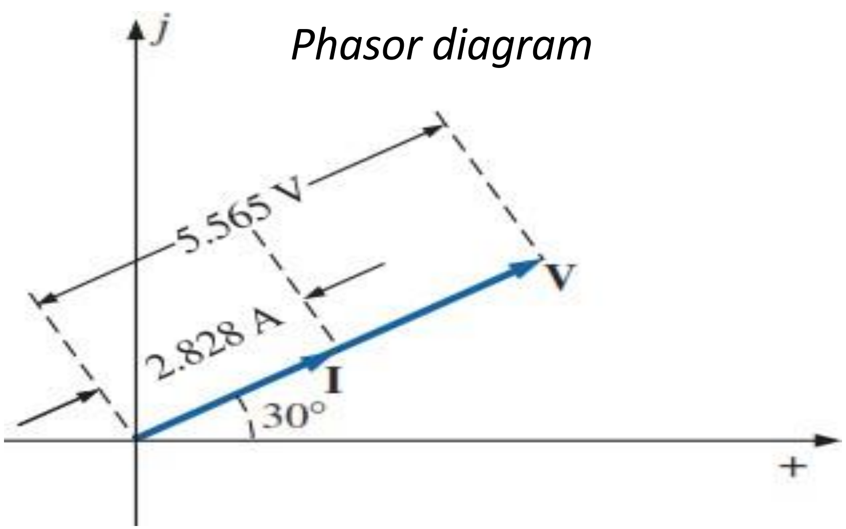
$$\mathbf{V} = \mathbf{I}\mathbf{Z}_R = (I \angle \theta)(R \angle 0°) = (2.828 \text{ A} \angle 30°)(2\ \Omega \angle 0°)$$
$$= 5.656 \text{ V} \angle 30°$$

$$v = \sqrt{2}\,(5.656)\sin(\omega t + 30°) = \mathbf{8.0 \sin(\omega t + 30°)}$$



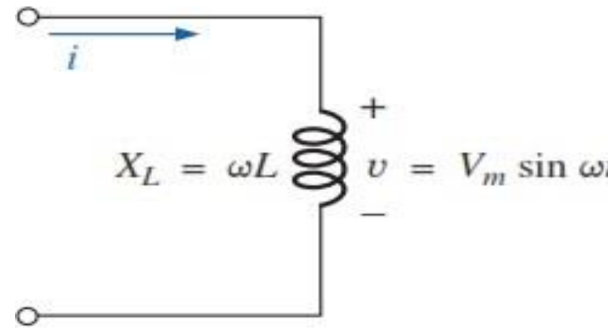


Phasor diagram



Inductive Reactance

For the pure inductor, the voltage leads the current by 90° and that the reactance of the coil X is determined by ωL.



$$v = V_m \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = V \angle 0°$$

By Ohm's law,

$$\mathbf{I} = \frac{V \angle 0°}{X_L \angle \theta_L} = \frac{V}{X_L} \angle 0° - \theta_L$$

Since v leads i by 90°, i must have an angle of -90° associated with it. To satisfy this condition, θ_L
must equal +90°. Substituting θ_L = 90° we obtain

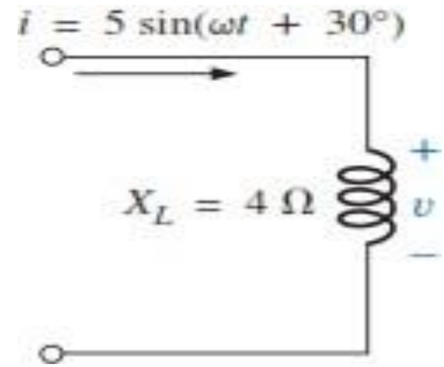
$$\mathbf{I} = \frac{V \angle 0°}{X_L \angle 90°} = \frac{V}{X_L} \angle 0° - 90° = \frac{V}{X_L} \angle -90°$$

so that in the time domain

$$i = \sqrt{2}\left(\frac{V}{X_L}\right) \sin(\omega t - 90°)$$

$$\mathbf{Z}_L = X_L \angle 90°$$

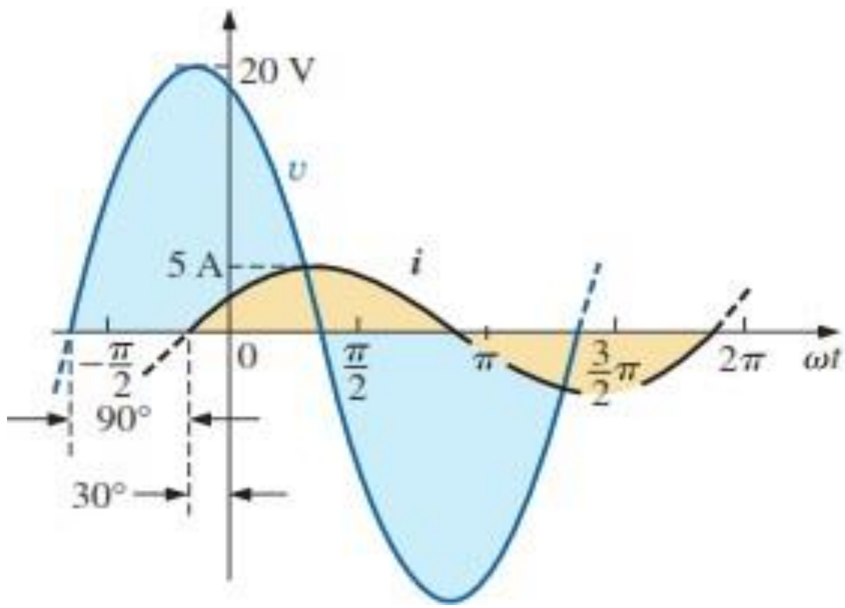
EXAMPLE 15.4 Using complex algebra, find the voltage v for the circuit in Fig. Sketch the v and i curves.



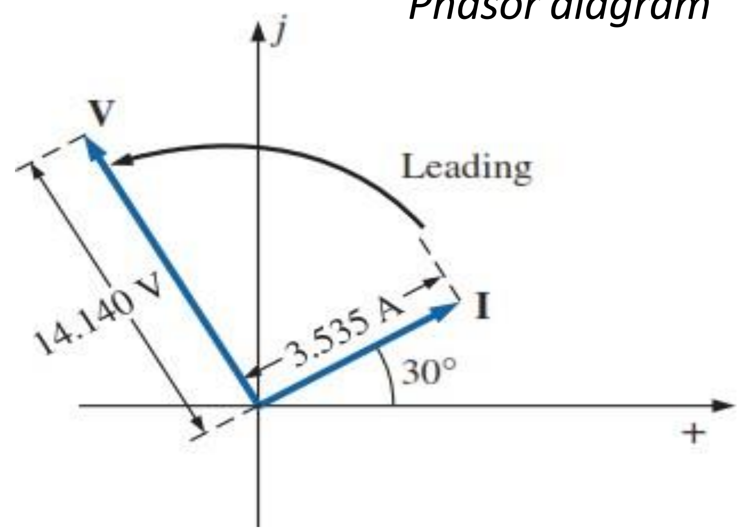
$$i = 5\sin(\omega t + 30°) \Rightarrow \text{phasor form } \mathbf{I} = 3.535\ \text{A} \angle 30°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_L = (I \angle \theta)(X_L \angle 90°) = (3.535\ \text{A} \angle 30°)(4\ \Omega \angle +90°)$$
$$= 14.140\ \text{V} \angle 120°$$

and $$v = \sqrt{2}(14.140)\sin(\omega t + 120°) = \mathbf{20\sin(\omega t + 120°)}$$

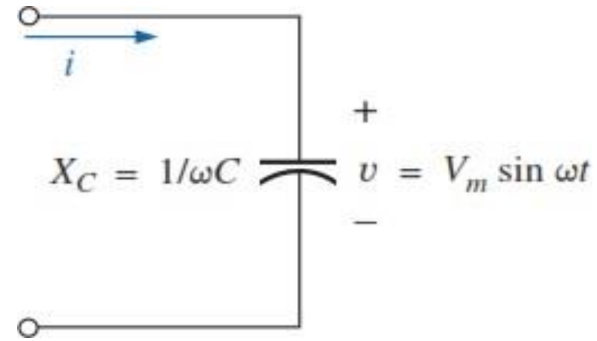


Phasor diagram



Capacitive Reactance

For the pure capacitor in Fig., the current leads the voltage by 90° and that the reactance of the capacitor Xc is determined by 1/ωC.



$$v = V_m \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = V \angle 0°$$

Applying Ohm's law and using phasor algebra, we find

$$\mathbf{I} = \frac{V \angle 0°}{X_C \angle \theta_C} = \frac{V}{X_C} \angle 0° - \theta_C$$

Since i leads v by 90°, i must have an angle of +90° associated with it. To satisfy this condition, θ_C must equal −90°. Substituting $\theta_C = -90°$ yields

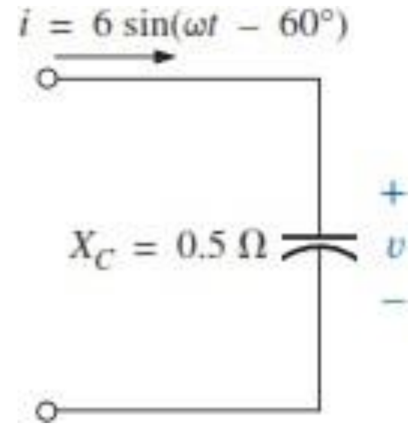
$$\mathbf{I} = \frac{V \angle 0°}{X_C \angle -90°} = \frac{V}{X_C} \angle 0° - (-90°) = \frac{V}{X_C} \angle 90°$$

so, in the time domain,

$$i = \sqrt{2}\left(\frac{V}{X_C}\right) \sin(\omega t + 90°)$$

$$\mathbf{Z}_C = X_C \angle -90°$$

EXAMPLE 15.6 Using complex algebra, find the voltage y for the circuit in Fig. Sketch the v and i curves.

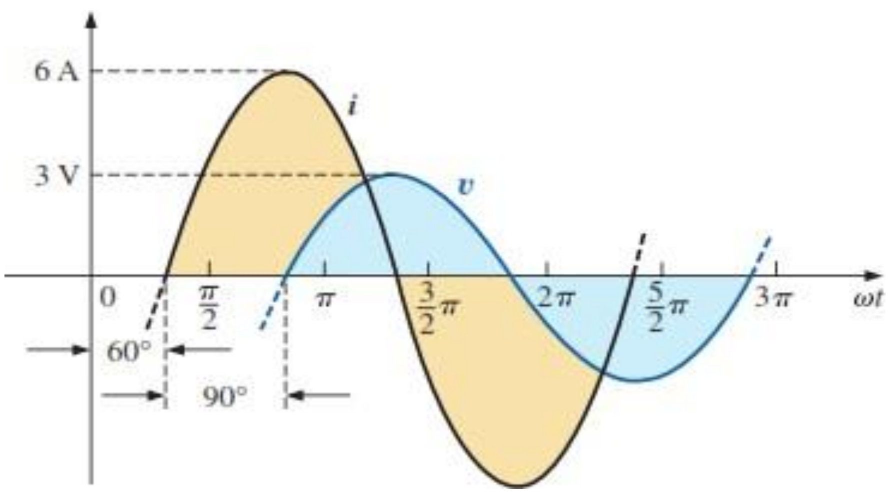


$$i = 6 \sin(\omega t - 60^\circ) \Rightarrow \text{phasor notation } \mathbf{I} = 4.242 \text{ A} \angle -60^\circ$$

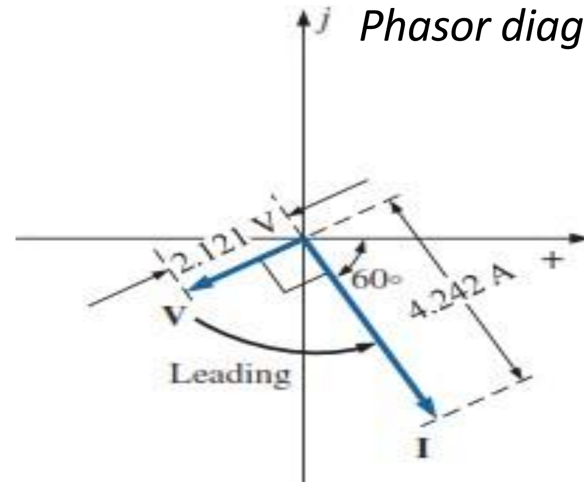
$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = (I \angle \theta)(X_C \angle -90^\circ) = (4.242 \text{ A} \angle -60^\circ)(0.5\ \Omega \angle -90^\circ)$$
$$= 2.121 \text{ V} \angle -150^\circ$$

and

$$v = \sqrt{2}(2.121)\sin(\omega t - 150^\circ) = \mathbf{3.0 \sin(\omega t - 150^\circ)}$$

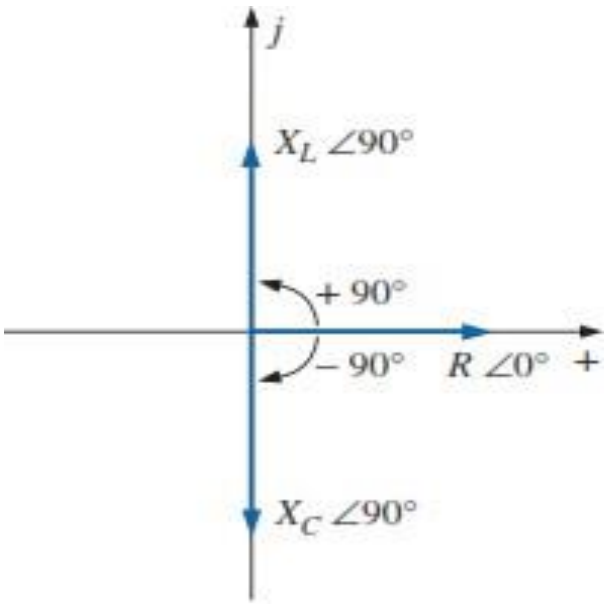


Phasor diagram



Impedance Diagram

For any network, the resistance will always appear on the positive real axis, the inductive reactance on the positive imaginary axis, and the capacitive reactance on the negative imaginary axis.

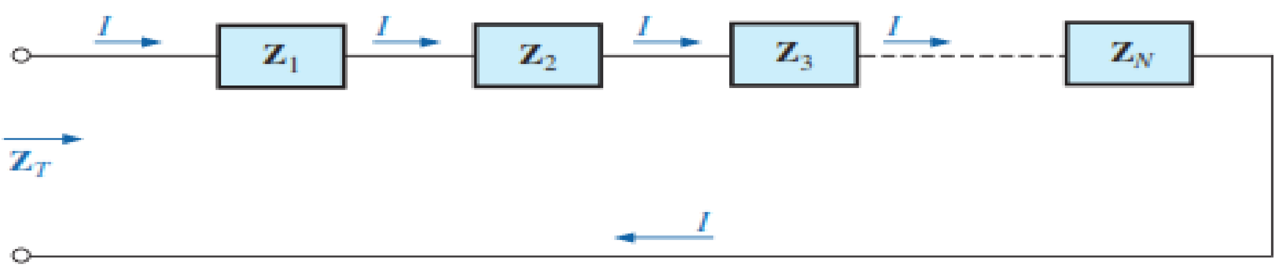


Impedance diagram.

For any configuration (series, parallel, series-parallel, and so on), the angle associated with the total impedance is the angle by which the applied voltage leads the source current. For inductive networks, θ_T will be positive, whereas for capacitive networks, θ_T will be negative.

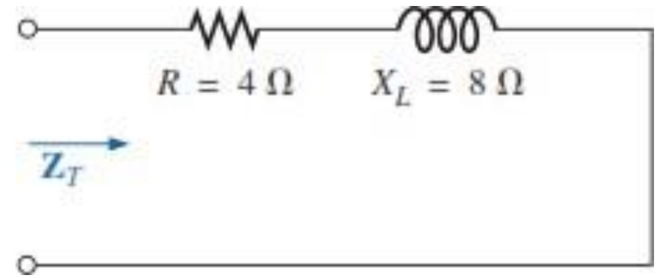
15.3 SERIES CONFIGURATION

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N$$



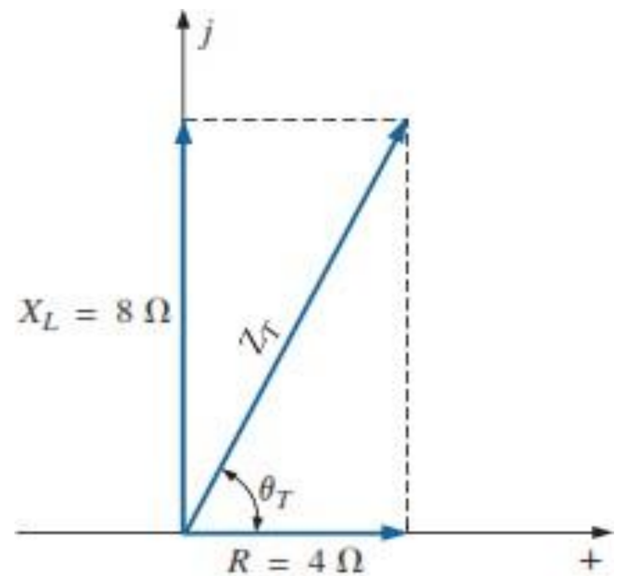
EXAMPLE 15.7 Draw the impedance diagram for the circuit in Fig., and find the total impedance.

As indicated by Fig., the input impedance can be found graphically from the impedance diagram by properly scaling the real and imaginary axes and finding the length of the resultant vector Z_T and angle θ_T. Or, by using vector algebra, we obtain



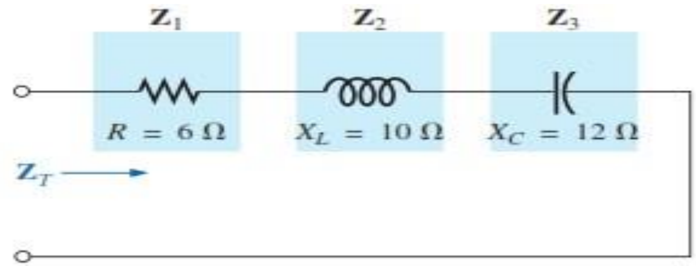
$$\begin{aligned} \mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 \\ &= R \angle 0^\circ + X_L \angle 90^\circ \\ &= R + jX_L = 4\ \Omega + j\,8\ \Omega \end{aligned}$$

$$\mathbf{Z}_T = 8.94\ \Omega \angle 63.43^\circ$$



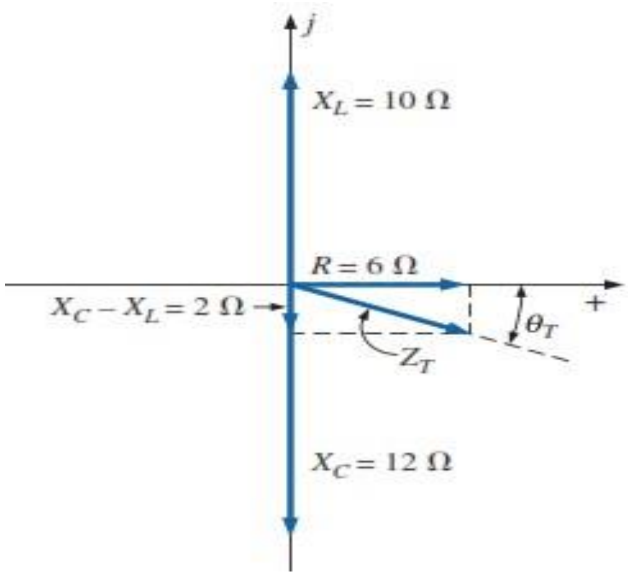
Impedance diagram.

EXAMPLE 15.8 Determine the input impedance to the series network in Fig. Draw the impedance diagram.



$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3$$
$$= R\angle 0^\circ + X_L\angle 90^\circ + X_C\angle -90^\circ$$
$$= R + jX_L - jX_C$$
$$= R + j(X_L - X_C) = 6\ \Omega + j(10\ \Omega - 12\ \Omega) = 6\ \Omega - j2\ \Omega$$
$$\mathbf{Z}_T = \mathbf{6.32\ \Omega \angle -18.43^\circ}$$

Note that in this example, the series inductive and capacitive reactances are in direct opposition.
But if the inductive reactance were equal to the capacitive reactance, the input impedance would be purely resistive.

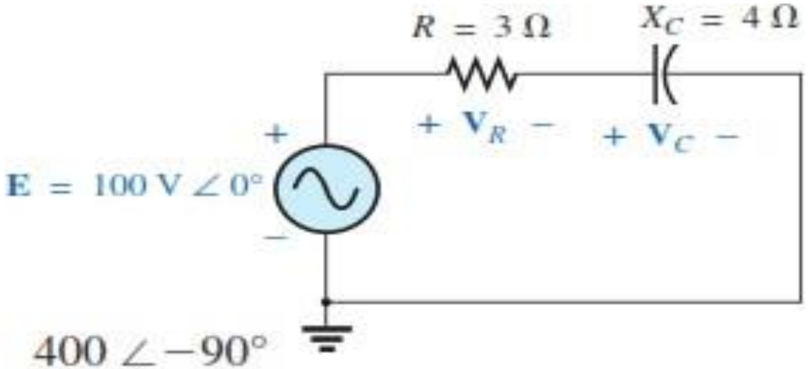


15.4 VOLTAGE DIVIDER RULE

The basic format for the **voltage divider** rule in ac circuits is exactly the same as that for dc circuits:

$$\mathbf{V}_x = \frac{\mathbf{Z}_x\mathbf{E}}{\mathbf{Z}_T}$$

EXAMPLE 15.9 Using the voltage divider rule, find the voltage across each element of the circuit in Fig.



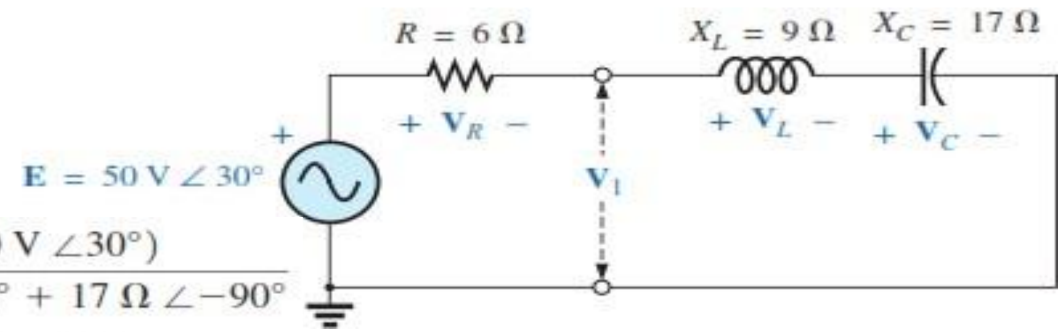
$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(4\ \Omega \angle -90^\circ)(100\ \text{V} \angle 0^\circ)}{4\ \Omega \angle -90^\circ + 3\ \Omega \angle 0^\circ} = \frac{400 \angle -90^\circ}{3 - j4}$$

$$= \frac{400 \angle -90^\circ}{5 \angle -53.13^\circ} = \mathbf{80\ V \angle -36.87^\circ}$$

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(3\ \Omega \angle 0^\circ)(100\ \text{V} \angle 0^\circ)}{5\ \Omega \angle -53.13^\circ} = \frac{300 \angle 0^\circ}{5 \angle -53.13^\circ}$$

$$= \mathbf{60\ V \angle +53.13^\circ}$$

EXAMPLE 15.10 Using the voltage divider rule, find the unknown voltages V_R, V_L, V_C and V_1 for the circuit shown.



$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C} = \frac{(6\ \Omega\ \angle 0^\circ)(50\ \text{V}\ \angle 30^\circ)}{6\ \Omega\ \angle 0^\circ + 9\ \Omega\ \angle 90^\circ + 17\ \Omega\ \angle -90^\circ}$$

$$= \frac{300\ \angle 30^\circ}{6 + j\,9 - j\,17} = \frac{300\ \angle 30^\circ}{6 - j\,8}$$

$$= \frac{300\ \angle 30^\circ}{10\ \angle -53.13^\circ} = \mathbf{30\ V\ \angle 83.13^\circ}$$

$$\mathbf{V}_L = \frac{\mathbf{Z}_L\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ} = \frac{450\ \text{V}\ \angle 120^\circ}{10\ \angle -53.13^\circ}$$

$$= \mathbf{45\ V \angle 173.13^\circ}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(17\ \Omega\ \angle -90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ} = \frac{850\ \text{V}\ \angle -60^\circ}{10\ \angle -53^\circ}$$

$$= \mathbf{85\ V\ \angle\ -6.87^\circ}$$

$$\mathbf{V}_1 = \frac{(\mathbf{Z}_L + \mathbf{Z}_C)\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90^\circ + 17\ \Omega\ \angle -90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ}$$

$$= \frac{(8\ \angle -90^\circ)(50\ \angle 30^\circ)}{10\ \angle -53.13^\circ} = \frac{400\ \angle -60^\circ}{10\ \angle -53.13^\circ} = \mathbf{40\ V\ \angle -6.87^\circ}$$

Thank you very much