Ministry of Higher Education and Scientific Research
Almustaqbal University, College of Engineering
And Engineering Technologies
Computer Technology Engineering Department

Six week :

Parallel Ac Circuits (R L C)

Course Name : Fundamentals of Electricity
Stage : One
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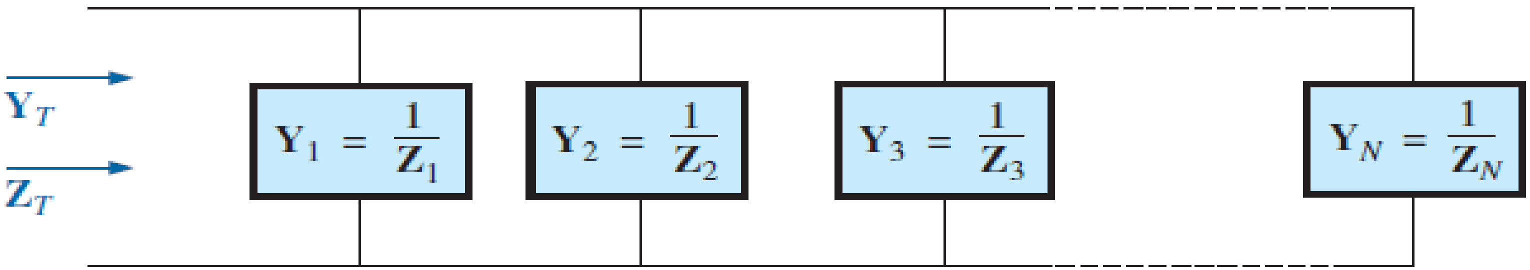
PARALLEL AC CIRCUITS

15.7 ADMITTANCE AND SUSCEPTANCE

In ac circuits, we define **admittance (Y) as being equal to 1/Z.** The unit of measure for admittance as defined by the SI system is ***siemens,*** which has the symbol **S**.

$$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \cdots + \mathbf{Y}_N$$



since $\mathbf{Z} = 1/\mathbf{Y}$,

$$\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \cdots + \frac{1}{\mathbf{Z}_N}$$

and

$$\mathbf{Z}_T = \frac{1}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \cdots + \frac{1}{\mathbf{Z}_N}}$$

Conductance is the reciprocal of resistance,

$$\mathbf{Y}_R = \frac{1}{\mathbf{Z}_R} = \frac{1}{R \angle 0^\circ} = G \angle 0^\circ$$

The reciprocal of reactance (1/X) is called **susceptance** and is a measure of how susceptible an element is to the passage of current through it. Susceptance is also measured in **Siemens** and is represented by the capital letter **B.**

For the inductor,

$$\mathbf{Y}_L = \frac{1}{\mathbf{Z}_L} = \frac{1}{X_L \angle 90^\circ} = \frac{1}{X_L} \angle -90^\circ$$

Defining $$B_L = \frac{1}{X_L}$$ (siemens, S)

we have $$\mathbf{Y}_L = B_L \angle -90^\circ$$

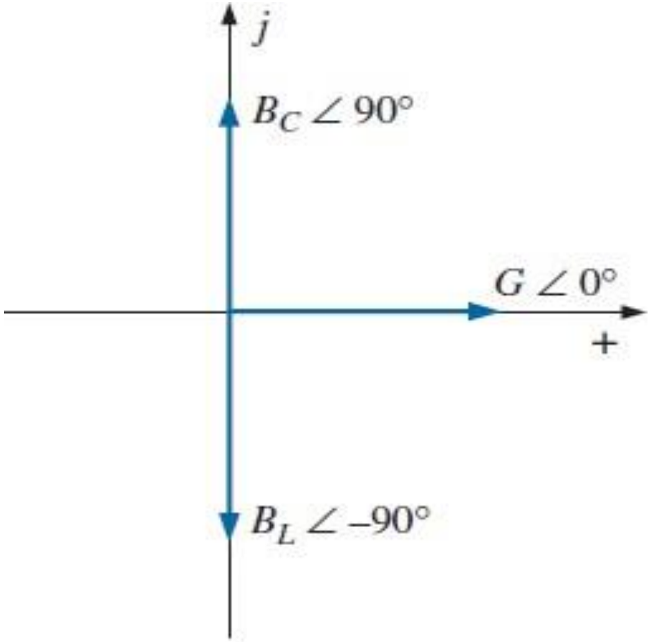
For the capacitor,

$$\mathbf{Y}_C = \frac{1}{\mathbf{Z}_C} = \frac{1}{X_C \angle -90^\circ} = \frac{1}{X_C} \angle 90^\circ$$

Defining $$B_C = \frac{1}{X_C}$$ (siemens, S)

we have $$\mathbf{Y}_C = B_C \angle 90^\circ$$

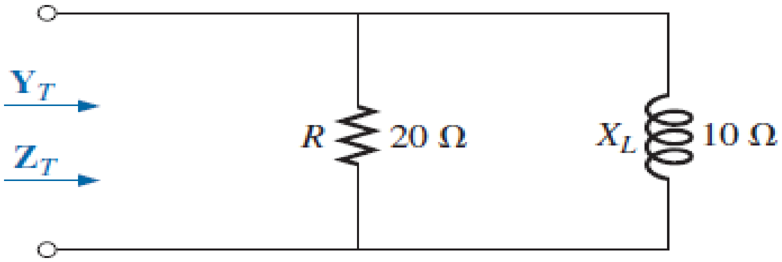
For any configuration (series, parallel, series-parallel, and so on), the angle associated with the total admittance is the angle by which the source current leads the applied voltage. For inductive networks, θ_T is negative, whereas for capacitive networks, θ_T is positive.



Admittance diagram.

EXAMPLE 15.13 For the network in Fig. :

a. Calculate the input impedance.
b. Draw the impedance diagram.
c. Find the admittance of each parallel branch.
d.Determine the input admittance and draw the admittance diagram.

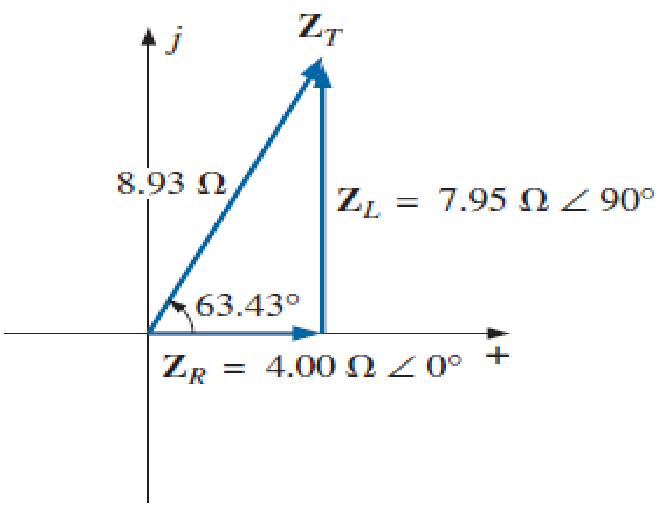


a. $\mathbf{Z}_T = \dfrac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} = \dfrac{(20\ \Omega\ \angle 0^\circ)(10\ \Omega\ \angle 90^\circ)}{20\ \Omega + j\,10\ \Omega}$

$$= \frac{200\ \Omega\ \angle 90^\circ}{22.361\ \angle 26.57^\circ} = \mathbf{8.93\ \Omega\ \angle 63.43^\circ}$$

$$= \mathbf{4.00\ \Omega + \mathit{j}\,7.95\ \Omega} = R_T + j\,X_{L_T}$$

b. The impedance diagram appears in Fig.



c. $\mathbf{Y}_R = G\angle 0^\circ = \dfrac{1}{R}\angle 0^\circ = \dfrac{1}{20\ \Omega}\angle 0^\circ = \mathbf{0.05\ S\ \angle 0^\circ}$

$$= \mathbf{0.05\ S + \mathit{j}\,0}$$

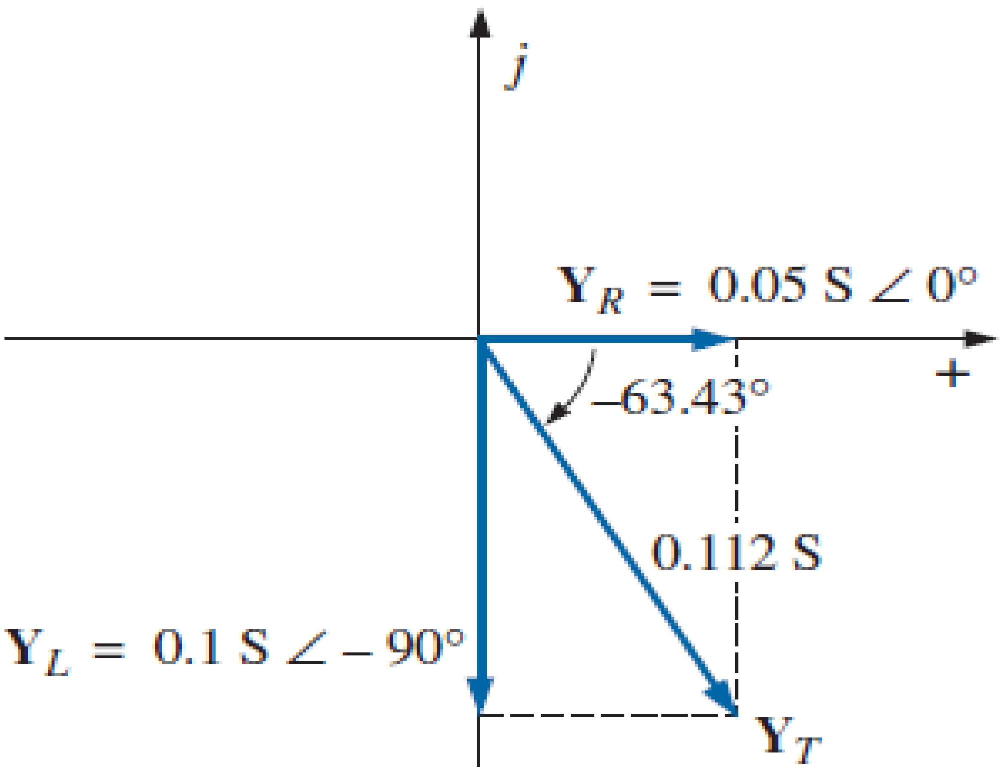
$$\mathbf{Y}_L = B_L\angle -90^\circ = \frac{1}{X_L}\angle -90^\circ = \frac{1}{10\ \Omega}\angle -90^\circ$$

$$= \mathbf{0.1\ S\ \angle -90^\circ = 0 - \mathit{j}\,0.1\ S}$$

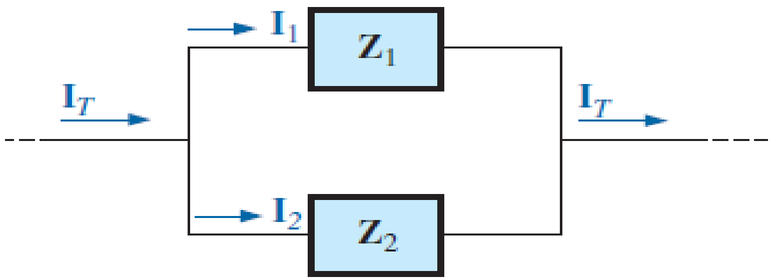
d. $\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L = (0.05\text{ S} + j\,0) + (0 - j\,0.1\text{ S})$

$= \mathbf{0.05\text{ S} - j\,0.1\text{ S}} = G - jB_L$

The admittance diagram appears in Fig.

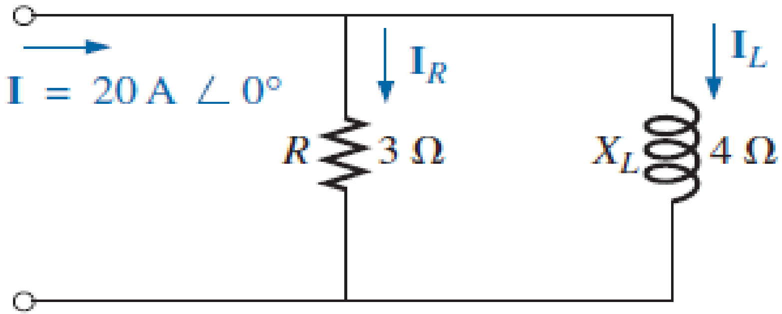


15.9 CURRENT DIVIDER RULE



The basic format for the **current divider** rule in ac circuits is exactly the same as that for dc circuits; that is, for two parallel branches with impedances **Z1 and Z2** as shown in Fig.

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} \quad \text{or} \quad \mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2}$$



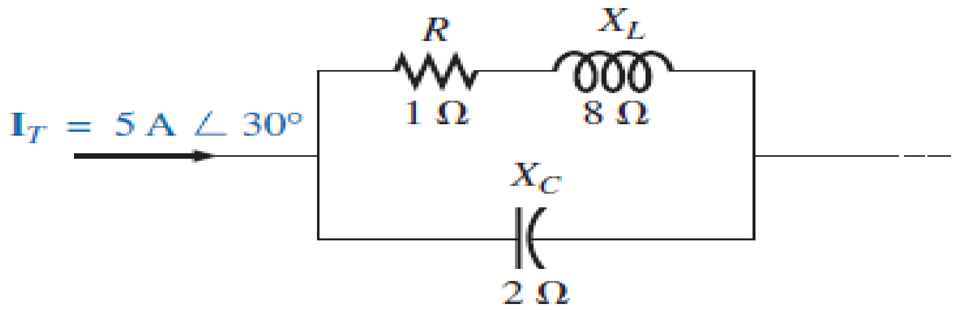
EXAMPLE 15.16 Using the current divider rule, find the current through each impedance in Fig.

$$\mathbf{I}_R = \frac{\mathbf{Z}_L\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(4\ \Omega \angle 90°)(20\text{ A} \angle 0°)}{3\ \Omega \angle 0° + 4\ \Omega \angle 90°} = \frac{80\text{ A} \angle 90°}{5 \angle 53.13°}$$
$$= 16\text{ A} \angle 36.87°$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_R\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(3\ \Omega \angle 0°)(20\text{ A} \angle 0°)}{5\ \Omega \angle 53.13°} = \frac{60\text{ A} \angle 0°}{5 \angle 53.13°}$$
$$= 12\text{ A} \angle -53.13°$$

Draw the Current Phasor Diagram

EXAMPLE 15.17 Using the current divider rule, find the current through each parallel branch in Fig.



$$\mathbf{I}_{R\text{-}L} = \frac{\mathbf{Z}_C \mathbf{I}_T}{\mathbf{Z}_C + \mathbf{Z}_{R\text{-}L}} = \frac{(2\ \Omega \angle -90^\circ)(5\ \text{A} \angle 30^\circ)}{-j\,2\ \Omega + 1\ \Omega + j\,8\ \Omega} = \frac{10\ \text{A} \angle -60^\circ}{1 + j6}$$

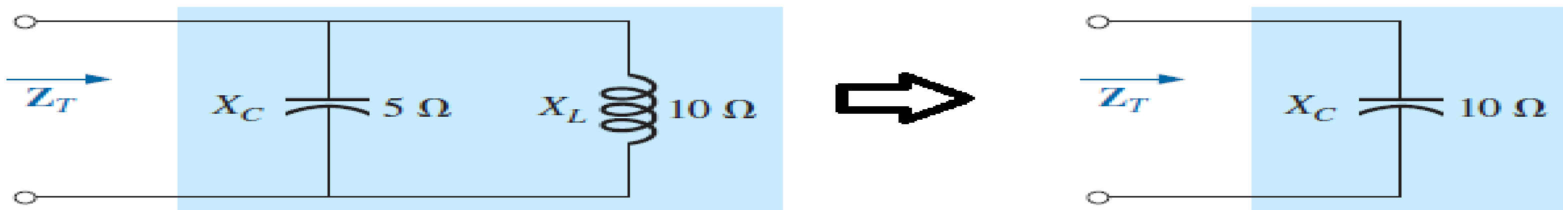
$$= \frac{10\ \text{A} \angle -60^\circ}{6.083 \angle 80.54^\circ} \cong \mathbf{1.64\ A \angle -140.54^\circ}$$

$$\mathbf{I}_C = \frac{\mathbf{Z}_{R\text{-}L} \mathbf{I}_T}{\mathbf{Z}_{R\text{-}L} + \mathbf{Z}_C} = \frac{(1\ \Omega + j\,8\ \Omega)(5\ \text{A} \angle 30^\circ)}{6.08\ \Omega \angle 80.54^\circ}$$

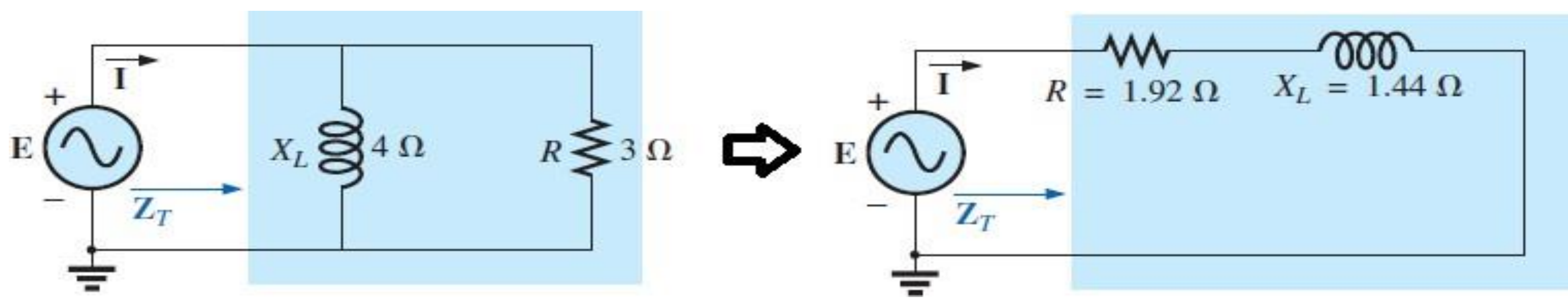
$$= \frac{(8.06 \angle 82.87^\circ)(5\ \text{A} \angle 30^\circ)}{6.08 \angle 80.54^\circ} = \frac{40.30\ \text{A} \angle 112.87^\circ}{6.083 \angle 80.54^\circ}$$

$$= \mathbf{6.63\ A \angle 32.33^\circ}$$

15.12 EQUIVALENT CIRCUITS

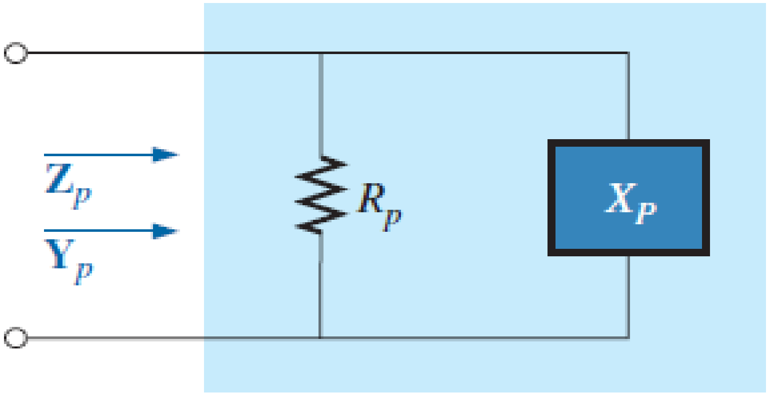


$$Z_T = \frac{Z_C Z_L}{Z_C + Z_L} = \frac{(5\ \Omega \angle -90^\circ)(10\ \Omega \angle 90^\circ)}{5\ \Omega \angle -90^\circ + 10\ \Omega \angle 90^\circ} = \frac{50 \angle 0^\circ}{5 \angle 90^\circ}$$
$$= 10\ \Omega \angle -90^\circ$$



$$Z_T = \frac{Z_L Z_R}{Z_L + Z_R} = \frac{(4\ \Omega \angle 90^\circ)(3\ \Omega \angle 0^\circ)}{4\ \Omega \angle 90^\circ + 3\ \Omega \angle 0^\circ}$$
$$= \frac{12 \angle 90^\circ}{5 \angle 53.13^\circ} = 2.40\ \Omega \angle 36.87^\circ \quad = 1.92\ \Omega + j\,1.44\ \Omega$$

$$\mathbf{Y}_p = \frac{1}{R_p} + \frac{1}{\pm jX_p} = \frac{1}{R_p} \mp j\frac{1}{X_p}$$



and

$$\mathbf{Z}_p = \frac{1}{\mathbf{Y}_p} = \frac{1}{(1/R_p) \mp j(1/X_p)}$$

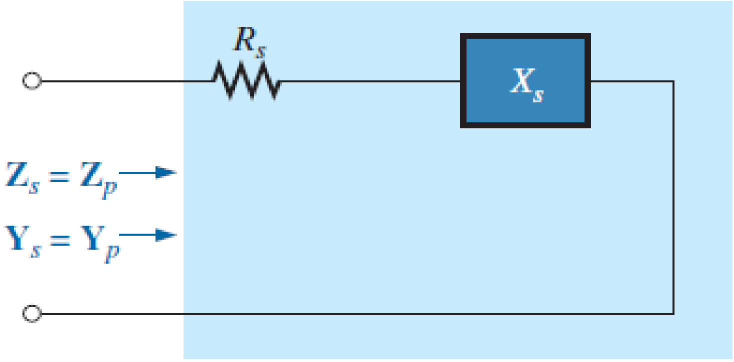
$$= \frac{1/R_p}{(1/R_p)^2 + (1/X_p)^2} \pm j\frac{1/X_p}{(1/R_p)^2 + (1/X_p)^2}$$

Multiplying the numerator and denominator of each term by $R_p^2 X_p^2$

results in

$$\mathbf{Z}_p = \frac{R_p X_p^2}{X_p^2 + R_p^2} \pm j\frac{R_p^2 X_p}{X_p^2 + R_p^2}$$

$$= R_s \pm j X_s$$



and

$$R_s = \frac{R_p X_p^2}{X_p^2 + R_p^2}$$

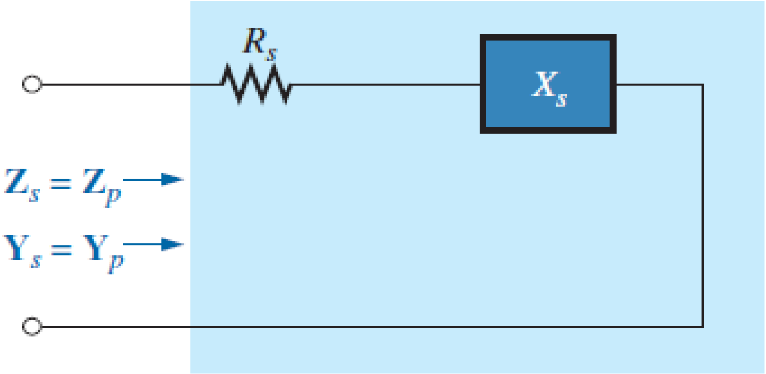
$$X_s = \frac{R_p^2 X_p}{X_p^2 + R_p^2}$$

$$\mathbf{Z}_s = R_s \pm jX_s$$

$$\mathbf{Y}_s = \frac{1}{\mathbf{Z}_s} = \frac{1}{R_s \pm j X_s} = \frac{R_s}{R_s^2 + X_s^2} \mp j \frac{X_s}{R_s^2 + X_s^2}$$

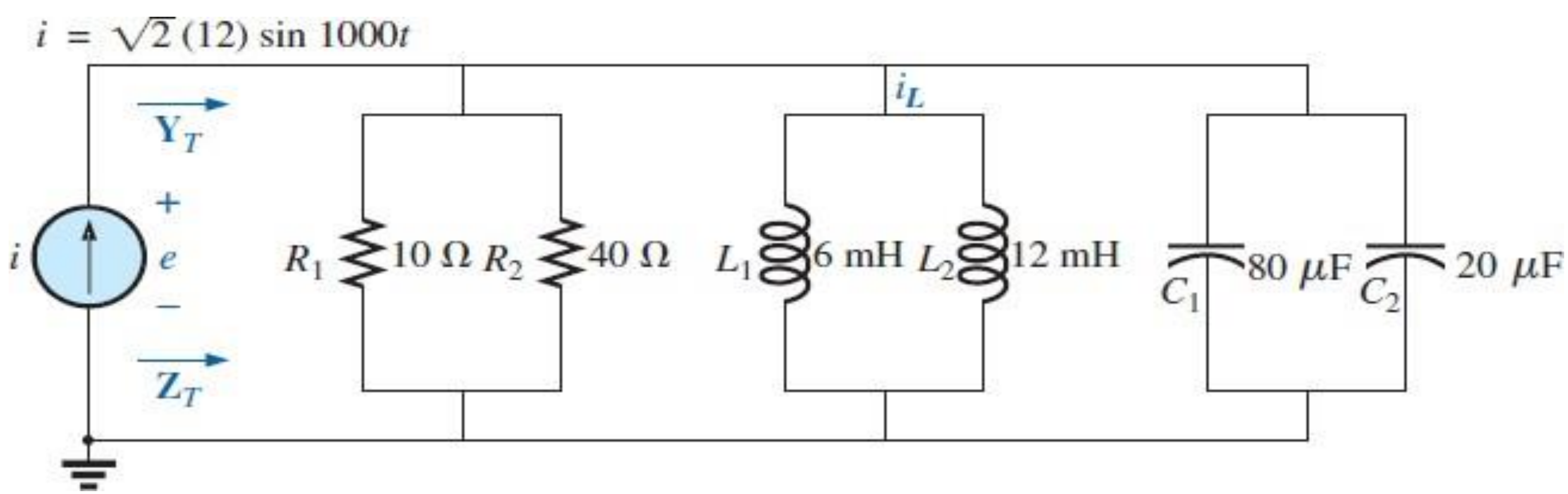
$$= G_p \mp j B_p = \frac{1}{R_p} \mp j \frac{1}{X_p}$$

or $R_p = \dfrac{R_s^2 + X_s^2}{R_s}$ $\quad X_p = \dfrac{R_s^2 + X_s^2}{X_s}$



EXAMPLE 15.19 For the network in Fig. :

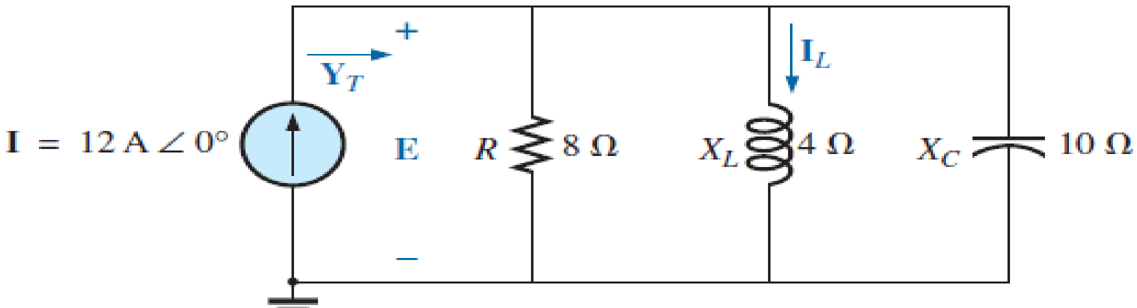
a. Determine $\mathbf{Y}_T$ ***and*** $\mathbf{Z}_T$.
b. Sketch the admittance diagram.
c. Find **E and** $\mathbf{I}_L$.
d. Compute the power factor of the network and the power delivered to the network.
e. Determine the equivalent series circuit.
f. Using the equivalent circuit developed in part (e), calculate **E,** and compare it with the result of part (c).
g. Determine the power delivered to the network, and compare it with the solution of part (d).
h.Determine the equivalent parallel network from the equivalent series circuit, and calculate the total admittance $\mathbf{Y}_T$. Compare the result with the solution of part (a).



a. Combining common elements and finding the reactance of the inductor and capacitor, we obtain

$$R_T = 10\ \Omega \parallel 40\ \Omega = 8\ \Omega$$
$$L_T = 6\text{ mH} \parallel 12\text{ mH} = 4\text{ mH}$$
$$C_T = 80\ \mu\text{F} + 20\ \mu\text{F} = 100\ \mu\text{F}$$

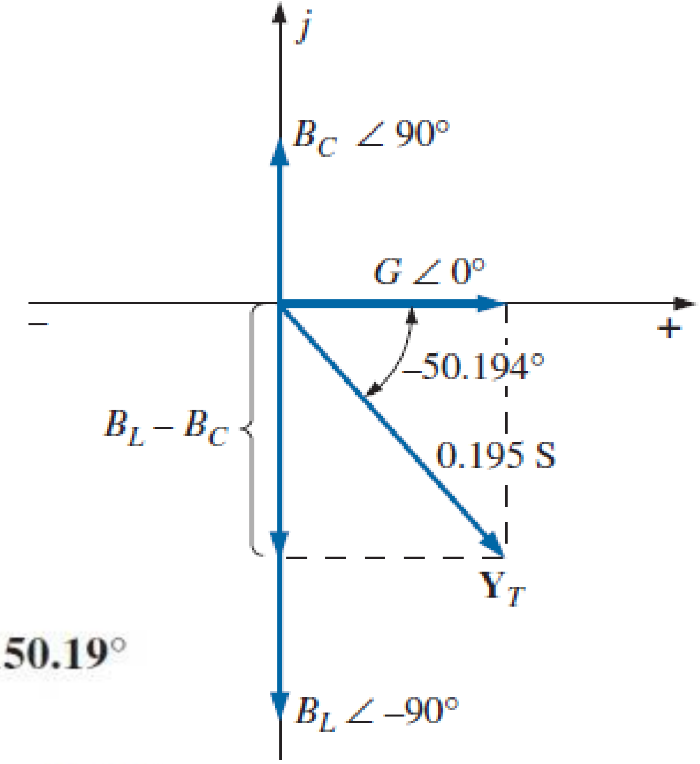
$$X_L = \omega L = (1000\text{ rad/s})(4\text{ mH}) = 4\ \Omega$$
$$X_C = \frac{1}{\omega C} = \frac{1}{(1000\text{ rad/s})(100\ \mu\text{F})} = 10\ \Omega$$



$$\begin{aligned}
\mathbf{Y}_T &= \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C \\
&= G\angle 0^\circ + B_L\angle -90^\circ + B_C\angle +90^\circ \\
&= \frac{1}{8\ \Omega}\angle 0^\circ + \frac{1}{4\ \Omega}\angle -90^\circ + \frac{1}{10\ \Omega}\angle +90^\circ \\
&= 0.125\text{ S}\angle 0^\circ + 0.25\text{ S}\angle -90^\circ + 0.1\text{ S}\angle +90^\circ \\
&= 0.125\text{ S} - j\,0.25\text{ S} + j\,0.1\text{ S} \\
&= 0.125\text{ S} - j\,0.15\text{ S} = \mathbf{0.195\ S\angle -50.194^\circ}
\end{aligned}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.195\text{ S}}\angle -50.194^\circ = \mathbf{5.13\ \Omega\angle 50.19^\circ}$$

b- Admittance diagram for the parallel R-L-C network



c. $$\mathbf{E} = \mathbf{I}\mathbf{Z}_T = \frac{\mathbf{I}}{\mathbf{Y}_T} = \frac{12 \text{ A} \angle 0°}{0.195 \text{ S} \angle -50.194°} = \mathbf{61.54 \text{ V} \angle 50.19°}$$

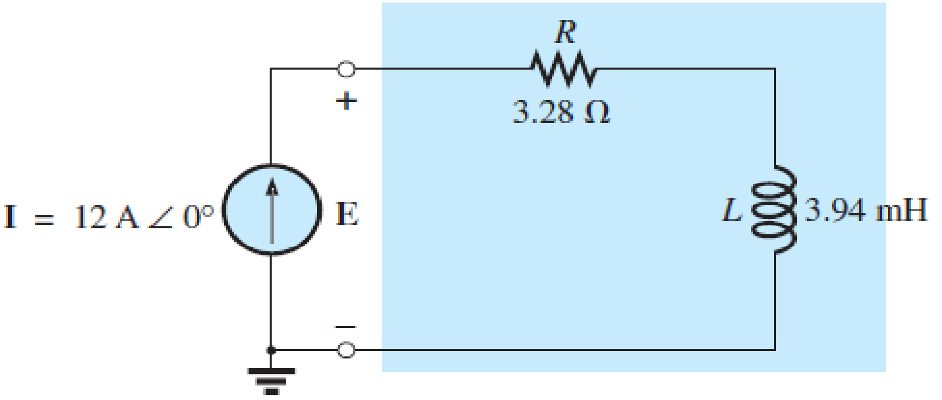
$$\mathbf{I}_L = \frac{\mathbf{V}_L}{\mathbf{Z}_L} = \frac{\mathbf{E}}{\mathbf{Z}_L} = \frac{61.538 \text{ V} \angle 50.194°}{4 \ \Omega \angle 90°} = \mathbf{15.39 \text{ A} \angle -39.81°}$$

d. $$F_p = \cos\theta = \frac{G}{Y_T} = \frac{0.125 \text{ S}}{0.195 \text{ S}} = \mathbf{0.641 \text{ lagging}} \ (\mathbf{E} \text{ leads } \mathbf{I})$$

$$P = EI\cos\theta = (61.538 \text{ V})(12 \text{ A}) \cos 50.194°$$
$$= \mathbf{472.75 \text{ W}}$$

e. $$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.195\ \text{S}\ \angle -50.194^\circ} = 5.128\ \Omega\ \angle +50.194^\circ$$
$$= 3.28\ \Omega + j\,3.94\ \Omega$$
$$= R + j\,X_L$$

$$X_L = 3.94\ \Omega = \omega L$$
$$L = \frac{3.94\ \Omega}{\omega} = \frac{3.94\ \Omega}{1000\ \text{rad/s}} = \mathbf{3.94\ mH}$$

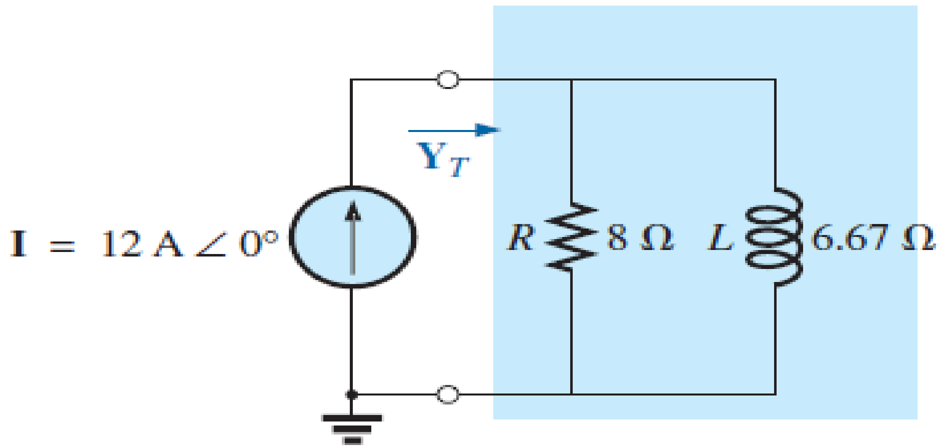


f. $$\mathbf{E} = \mathbf{I}\mathbf{Z}_T = (12\ \text{A}\ \angle 0^\circ)(5.128\ \Omega\ \angle 50.194^\circ)$$
$$= \mathbf{61.54\ V\ \angle 50.194^\circ} \quad \text{(as above)}$$

g. $$P = I^2R = (12\ \text{A})^2(3.28\ \Omega) = \mathbf{472.32\ W} \qquad \text{(as above)}$$

h. $$R_p = \frac{R_s^2 + X_s^2}{R_s} = \frac{(3.28\ \Omega)^2 + (3.94\ \Omega)^2}{3.28\ \Omega} = \mathbf{8\ \Omega}$$
$$X_p = \frac{R_s^2 + X_s^2}{X_s} = \frac{(3.28\ \Omega)^2 + (3.94\ \Omega)^2}{3.94\ \Omega} = \mathbf{6.67\ \Omega}$$

The parallel equivalent circuit



$$\mathbf{Y}_T = G\angle 0^\circ + B_L\angle -90^\circ = \frac{1}{8\ \Omega}\angle 0^\circ + \frac{1}{6.675\ \Omega}\angle -90^\circ$$
$$= 0.125\ \text{S}\angle 0^\circ + 0.15\ \text{S}\angle -90^\circ$$
$$= 0.125\ \text{S} - j\,0.15\ \text{S} = \mathbf{0.195\ S\angle -50.194^\circ} \quad \text{(as above)}$$

PROBLEMS

SECTION 15.2 Impedance and the Phasor Diagram: 1, 2

SECTION 15.3 Series Configuration: 4, 5, 6, 7, 9, 10

SECTION 15.4 Voltage Divider Rule: 15, 17, 20

SECTION 15.7 Admittance and Susceptance: 25, 26

SECTION 15.9 Current Divider Rule: 34

SECTION 15.12 Equivalent Circuits: 40, 42, 44

Thank you very much