Al-Mustaqbal University

Department of Medical Instrumentation Techniques Engineering

Mathematics I

LECTURE 2

Methods Of Integration

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Methods Of Integration

Integral Formula (Standard Form)

1	$\int u^n du = \frac{u^{n+1}}{n+1} + c \quad n \neq -1$	$\int \frac{du}{u} = \ln(u) + c$
2	$\int e^u du = e^u + c \qquad e = 2.718$	$\int a^u du = \frac{a^u}{\ln(a)} + c$ **a is constant**
3	$\int \sin(u) du = -\cos(u) + c$	$\int \sinh(u) du = \cosh(u) + c$
4	$\int \cos(u) du = \sin(u) + c$	$\int \cosh(u) du = \sinh(u) + c$
5	$\int \sec^2(u) du = \tan(u) + c$	$\int \sec h^2(u) du = \tanh(u) + c$
6	$\int \csc^2(u) du = -\cot(u) + c$	$\int \csc h^2(u) du = -\coth(u) + c$
7	$\int \sec(u)\tan(u) du = \sec(u) + c$	$\int \sec h(u)\tanh(u) du = -\sec h(u) + c$
8	$\int \csc(u)\cot(u) du = -\csc(u) + c$	$\int \csc h(u)\coth(u) du = -\csc h(u) + c$
9	$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{\sqrt{u^2 + a^2}} = \sinh^{-1}(\frac{u}{a}) + c$

10	$\int \frac{-du}{\sqrt{a^2-u^2}} = \cos^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(\frac{u}{a}) + c$
11	$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}(\frac{u}{a}) + c & \lvert u\rvert \prec a \\ \frac{1}{a}\coth^{-1}(\frac{u}{a}) + c & \lvert u\rvert \succ a \end{cases}$
12	$\int \frac{-du}{a^2+u^2} = \frac{1}{a}\cot^{-1}(\frac{u}{a}) + c$	
13	$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sec}h^{-1}(\frac{u}{a}) + c$
14	$\int \frac{-du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\csc^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{u\sqrt{a^2+u^2}} = \frac{1}{a}\operatorname{csc}h^{-1}(\frac{u}{a}) + c$

Method [1]

Integration By Substitution

The goal of this method is to transform the integral into a standard from

To evaluate the integral $I = \int f[g(x)]\, g'(x)dx$ **carry out the following steps**

1- substitute $u = g(x)$ **the** $du = g'(x)dx$ **to obtain** $I = \int f(u)du$

2- Evaluate $I = \int f(u)du$ **by integrating w.r.t** u

3- Replace u **by** $g(x)$ **in the final result**

Evaluate $\int \sin(4x)dx$

let u = 4x $\rightarrow \frac{du}{dx} = 4 \qquad du = 4\,dx \qquad dx = \frac{du}{4}$

$$\int \sin(u)\,.\,\frac{du}{4} = \frac{1}{4}\int \sin(u)du$$

$$= \frac{1}{4}(-\cos u) + c$$

$$= -\frac{1}{4}\cos(4x) + c$$

Evaluate $I=\int \frac{dx}{\sqrt[3]{1-2x}}$

Solution :- $I=\int (1-2x)^{-\frac{1}{3}}dx$ **Let** $u=1-2x \Rightarrow du=-2dx \Rightarrow dx=\frac{du}{-2}$

$$I=\int (1-2x)^{-\frac{1}{3}}dx \Rightarrow I=\int u^{-\frac{1}{3}}\frac{du}{-2}=\frac{-1}{2}\int u^{-\frac{1}{3}}du=\frac{-1}{2}\frac{u^{\frac{2}{3}}}{\frac{2}{3}}+c=\frac{-3}{4}(1-2x)^{\frac{2}{3}}+c$$

Evaluate $I=\int \sin^2(5x)\cos(5x)dx$

Solution :- **Let** $u=\sin(5x) \Rightarrow du=5\cos(5x)dx \Rightarrow dx=\frac{du}{5\cos(5x)}$

$$I=\int \sin^2(5x)\cos(5x)dx \Rightarrow I=\int u^2\cos(5x)\frac{du}{5\cos(5x)}=\frac{1}{5}\int u^2du=\frac{1}{5}\frac{u^3}{3}+c=\frac{1}{15}[\sin(5x]^3+c$$

Evaluate $I=\int xe^{x^2+1}dx$

Solution :- **Let** $u=x^2+1 \Rightarrow du=2xdx \Rightarrow dx=\dfrac{du}{2x}$

$$I=\int xe^{x^2+1}dx \Rightarrow I=\int xe^{u}\frac{du}{2x}=\frac{1}{2}\int e^{u}du=\frac{1}{2}e^{u}+c=\frac{1}{2}e^{x^2+1}+c$$

Evaluate $I=\dfrac{1}{3}\displaystyle\int \frac{3\cos(3x)}{4+\sin(3x)}dx$

Solution :- **Let** $u=4+\sin(3x) \Rightarrow du=3\cos(3x)dx \Rightarrow dx=\dfrac{du}{3\cos(3x)}$

$$I=\frac{1}{3}\int \frac{3\cos(3x)}{4+\sin(3x)}dx$$

$$\Rightarrow I=\frac{1}{3}\int \frac{3\cos(3x)}{u}\frac{du}{3\cos(3x)}=\frac{1}{3}\int \frac{1}{u}du=\ln(u)+c=\frac{1}{3}\ln[4+\sin(3x)]+c$$

Evaluate $I=\dfrac{1}{3}\displaystyle\int \frac{3\cos(3x)}{4+\sin^2(3x)}dx$

Solution :- **Let** $u=\sin(3x) \Rightarrow du=3\cos(3x)dx \Rightarrow dx=\dfrac{du}{3\cos(3x)}$

$$\Rightarrow I=\frac{1}{3}\int \frac{3\cos(3x)}{2^2+u^2}\frac{du}{3\cos(3x)}=\frac{1}{3}\int \frac{1}{2^2+u^2}du=\frac{1}{3}\frac{1}{a}\tan^{-1}(\frac{u}{a})+c=\frac{1}{3}\frac{1}{2}\tan^{-1}(\frac{u}{2})+c$$