Lecture 7: Elastic Collisions

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Lecture
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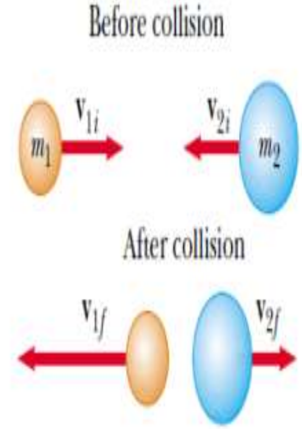
Elastic Collisions

Elastic Collisions

The two particles collide head-on and then leave the collision site with different velocities, v_{1f} *and* v_{2f}. If the collision is elastic, both the momentum and kinetic energy of the system are conserved. Therefore, considering velocities along the horizontal direction in Figure



$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad (1)$$

$$\frac{1}{2} m_1 {v_{1i}}^2 + \frac{1}{2} m_2 {v_{2i}}^2 = \frac{1}{2} m_1 {v_{1f}}^2 + \frac{1}{2} m_2 {v_{2f}}^2 \qquad (2)$$

Because all velocities in Figure are either to the left or the right, they can be represented by the corresponding speeds along with algebraic signs indicating direction. We shall indicate *v* as positive if a particle moves to the right and negative if it moves tothe left. In a typical problem involving elastic collisions, there are two unknown quantities, and Equations 1 and 2 can be solved simultaneously to find these. An alternative approach, however—one that involves a little mathematical manipulation of Equation often simplifies this process. To see how, let us cancel the factor in Equation 2 and rewrite it as

$$m_1({v_{1i}}^2 - {v_{1f}}^2) = m_2({v_{2f}}^2 - {v_{2i}}^2)$$

and then factor both sides:

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \qquad (3)$$

Next, let us separate the terms containing *m_1 and m_2 in Equation 1 to obtain*

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \qquad (4)$$

To obtain our final result, we divide Equation 3 by Equation 4 and obtain

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f}) \qquad (5)$$

The relative velocity of the two particles before the collision, $v_{1i} - v_{2i}$, equals the negative of their relative velocity after the collision,$-(v_{1f} - v_{2f})$.

Suppose that the masses and initial velocities of both particles are known. Equations 1 and 5 can be solved for the final velocities in terms of the initial velocities because there are two equations and two unknowns:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i} \qquad (6)$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i} \qquad (7)$$

It is important to use the appropriate signs for v_{1i} *and* v_{2i} in Equations *6* and *7.* For example, if particle *2* is moving to the left initially, then v_{2i} is negative*.* Let us consider some special cases. If *m1 ! m2,* then Equations *6* and *7* show us that $v_{1f} = v_{2i}$ *and* $v_{2f} = v_{1i}$.

That is, the particles exchange velocities if they have equal masses. This is approximately what one observes in head-on billiard ball collisions the cue ball stops, and the struck ball moves away from the collision with the same velocity that the cue ball had. If particle 2 is initially at rest, then $v_{2i} = 0$, and Equations *6* and *7* become

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} \qquad (8)$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} \qquad (9)$$

If m_1 *is much greater than* m_2 *and* $v_{2i} = 0$, *we see from Equations 8 and 9 that* $v_{1f} \approx v_{1i}$ *and* $v_{2f} \approx 2v_{1i}$. *That is, when a very heavy particle collides head-on with a very* light one that is initially at rest, the heavy particle continues its motion unaltered after the collision and the light particle rebounds with a speed equal to about twice the initial speed of the heavy particle. If m_2 *is much greater than m1 and particle 2 is initially at rest, then* $v_{1f} \approx v_{1i}$ *and* $v_{2f} = 0$.

EXP. An 1800-kg car stopped at a traffic light is struck from the rear by a 900-kg car, and the two become entangled, moving along the same path as that of the originally moving car. If the smaller car were moving at 20.0 m/s before the collision, what is the velocity of the entangled cars after the collision?

Solution:

We can guess that the final speed is less than 20.0 m/s, the initial speed of the smaller car. The total momentum of the system (the two cars) before the collision must equal the total momentum immediately after the collision because momentum of an isolated system is observed in any type of collision. The magnitude of the total momentum of the system before the collision is equal to that of the smaller car because the larger car is initially at rest

$$p_i = m_1 v_i = (900\ \mathrm{kg})(20.0\ \mathrm{m/s}) = 1.80 \times 10^4\ \mathrm{kg \cdot m/s}$$

After the collision, the magnitude of the momentum of the entangled cars is

$$p_f = (m_1 + m_2) v_f = (2\,700\ \mathrm{kg}) v_f$$

The final velocity of the entangled cars, we have

$$v_f = \frac{p_i}{m_1 + m_2} = \frac{1.80 \times 10^4\ \mathrm{kg \cdot m/s}}{2\,700\ \mathrm{kg}} = \boxed{6.67\ \mathrm{m/s}}$$

ELASTIC COLLISIONS

When a player kicks a soccer ball, the collision between the ball and the player's foot is much closer to elastic than the collisions we have studied so far. In this case, *elastic* means that the ball and the player's foot remain separate after the collision.

elastic collision

a collision in which the total momentum and the total kinetic energy are conserved

In an **elastic collision,** two objects collide and return to their original shapes with no loss of total kinetic energy. After the collision, the two objects move separately. In an elastic collision, both the total momentum and the total kinetic energy are conserved.

Most collisions are neither elastic nor perfectly inelastic

In the everyday world, most collisions are not perfectly inelastic. That is, colliding objects do not usually stick together and continue to move as one object. Most collisions are not elastic, either. Even *nearly* elastic collisions, such as those between billiard balls or between a football player's foot and the ball, result in some decrease in kinetic energy. For example, a football deforms when it is kicked. During this deformation, some of the kinetic energy is converted to internal elastic potential energy. In most collisions, some of the kinetic energy is also converted into sound, such as the click of billiard balls colliding. In fact, any collision that produces sound is not elastic; the sound signifies a decrease in kinetic energy.

Elastic and perfectly inelastic collisions are limiting cases; most collisions actually fall into a category between these two extremes. In this third category of collisions, called *inelastic collisions,* the colliding objects bounce and move separately after the collision, but the total kinetic energy decreases in the collision. *For the problems in this book, we will consider all collisions in which the objects do not stick together to be elastic collisions.* Therefore, we will assume that the total momentum and the total kinetic energy each will stay the same before and after a collision in all collisions that are not perfectly inelastic.

Kinetic energy is conserved in elastic collisions

Figure 12 shows an elastic head-on collision between two soccer balls of equal mass. Assume, as in earlier examples, that the balls are isolated on a frictionless surface and that they do not rotate. The first ball is moving to the right when it collides with the second ball, which is moving to the left. When considered as a whole, the entire system has momentum to the left.

After the elastic collision, the first ball moves to the left and the second ball moves to the right. The magnitude of the momentum of the first ball, which is now moving to the left, is greater than the magnitude of the momentum of the second ball, which is now moving to the right. The entire system still has momentum to the left, just as before the collision.

Another example of a nearly elastic collision is the collision between a golf ball and a club. After a golf club strikes a stationary golf ball, the golf ball moves at a very high speed in the same direction as the golf club. The golf club continues to move in the same direction, but its velocity decreases so that the momentum lost by the golf club is equal to and opposite the momentum gained by the golf ball. *The total momentum is always constant throughout the collision. In addition, if the collision is perfectly elastic, the value of the total kinetic energy after the collision is equal to the value before the collision.*

MOMENTUM AND KINETIC ENERGY ARE CONSERVED IN AN ELASTIC COLLISION

$$m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$$

$$\frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2 = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2$$

Remember that v is positive if an object moves to the right and negative if it moves to the left.

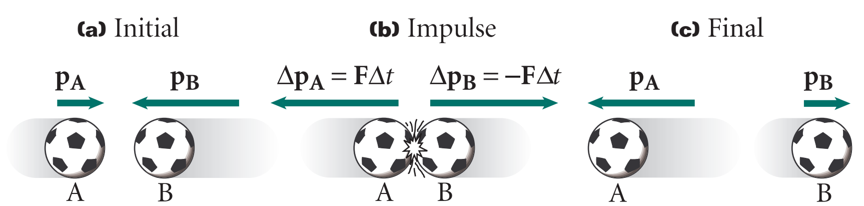


Figure 12
In an elastic collision like this one **(b),** both objects return to their original shapes and move separately after the collision **(c).**

Quick Lab

Elastic and Inelastic Collisions

MATERIALS LIST

- 2 or 3 small balls of different types

SAFETY CAUTION

Perform this lab in an open space, preferably outdoors, away from furniture and other people.

Drop one of the balls from shoulder height onto a hard-surfaced floor or sidewalk. Observe the motion of the ball before and after it collides with the ground. Next, throw the ball down from the same height. Perform several trials, giving the ball a different velocity each time. Repeat with the other balls.

During each trial, observe the height to which the ball bounces. Rate the collisions from most nearly elastic to most inelastic. Describe what evidence you have for or against conservation of kinetic energy and conservation of momentum for each collision. Based on your observations, do you think the equation for elastic collisions is useful to make predictions?

Quick Lab

TEACHER'S NOTES

The purpose of this lab is to show that in any collision, the elasticity of the materials involved affects the changes in kinetic energy. Test the balls before the lab in order to ensure a noticeable difference in elasticity. An interesting contrast can be observed by comparing new tennis balls with older ones.

Quick Lab As Homework

Teaching Tip — GENERAL

Point out to students that they should recognize the first equation in the box. This equation, which expresses the principle of conservation of momentum, holds for both types of collisions. The conservation of kinetic energy, on the other hand, which is expressed by the second equation in the box, is valid only for elastic collisions.

Classroom Practice

Elastic Collisions
Two billiard balls, each with a mass of 0.35 kg, strike each other head-on. One ball is initially moving left at 4.1 m/s and ends up moving right at 3.5 m/s. The second ball is initially moving to the right at 3.5 m/s. Assume that neither ball rotates before or after the collision and that both balls are moving on a frictionless surface. Predict the final velocity of the second ball.

Answer
4.1 m/s to the left

Two nonrotating balls on a frictionless surface collide elastically head on. The first ball has a mass of 15 g and an initial velocity of 3.5 m/s to the right, while the second ball has a mass of 22 g and an initial velocity of 4.0 m/s to the left. The final velocity of the 15 g ball is 5.4 m/s to the left. What is the final velocity of the 22 g ball?

Answer
2.0 m/s to the right

SAMPLE PROBLEM G

Elastic Collisions

PROBLEM

A 0.015 kg marble moving to the right at 0.225 m/s makes an elastic head-on collision with a 0.030 kg shooter marble moving to the left at 0.180 m/s. After the collision, the smaller marble moves to the left at 0.315 m/s. Assume that neither marble rotates before or after the collision and that both marbles are moving on a frictionless surface. What is the velocity of the 0.030 kg marble after the collision?

SOLUTION

1. DEFINE

Given: $m_1 = 0.015$ kg $\quad m_2 = 0.030$ kg

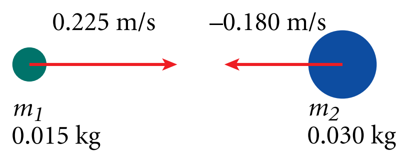
$\mathbf{v_{1,i}} = 0.225$ m/s to the right, $v_{1,i} = +0.225$ m/s

$\mathbf{v_{2,i}} = 0.180$ m/s to the left, $v_{2,i} = -0.180$ m/s

$\mathbf{v_{1,f}} = 0.315$ m/s to the left, $v_{1,f} = -0.315$ m/s

Unknown: $\mathbf{v_{2,f}} = ?$

Diagram:



2. PLAN

Choose an equation or situation: Use the equation for the conservation of momentum to find the final velocity of m_2, the 0.030 kg marble.

$$m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$$

Rearrange the equation to isolate the final velocity of m_2.

$$m_2\mathbf{v_{2,f}} = m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} - m_1\mathbf{v_{1,f}}$$

$$\mathbf{v_{2,f}} = \frac{m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} - m_1\mathbf{v_{1,f}}}{m_2}$$

3. CALCULATE

Substitute the values into the equation and solve: The rearranged conservation-of-momentum equation will allow you to isolate and solve for the final velocity.

$$v_{2,f} = \frac{(0.015\text{ kg})(0.225\text{ m/s}) + (0.030\text{ kg})(-0.180\text{ m/s}) - (0.015\text{ kg})(-0.315\text{ m/s})}{0.030\text{ kg}}$$

$$v_{2,f} = \frac{(3.4 \times 10^{-3}\text{ kg}\bullet\text{m/s}) + (-5.4 \times 10^{-3}\text{ kg}\bullet\text{m/s}) - (-4.7 \times 10^{-3}\text{ kg}\bullet\text{m/s})}{0.030\text{ kg}}$$

$$v_{2,f} = \frac{2.7 \times 10^{-3}\text{ kg}\bullet\text{m/s}}{3.0 \times 10^{-2}\text{ kg}}$$

$$\boxed{\mathbf{v_{2,f}} = 9.0 \times 10^{-2}\text{ m/s to the right}}$$

4. EVALUATE Confirm your answer by making sure kinetic energy is also conserved using these values.

Conservation of kinetic energy

$$\tfrac{1}{2}m_1v_{1,i}^2 + \tfrac{1}{2}m_2v_{2,i}^2 = \tfrac{1}{2}m_1v_{1,f}^2 + \tfrac{1}{2}m_2v_{2,f}^2$$

$$KE_i = \tfrac{1}{2}(0.015\text{ kg})(0.225\text{ m/s})^2 + \tfrac{1}{2}(0.030\text{ kg})(-0.180\text{ m/s})^2 = 8.7\times10^{-4}\text{ kg}\bullet\text{m}^2/\text{s}^2 = 8.7\times10^{-4}\text{ J}$$

$$KE_f = \tfrac{1}{2}(0.015\text{ kg})(0.315\text{ m/s})^2 + \tfrac{1}{2}(0.030\text{ kg})(0.090\text{ m/s})^2 = 8.7\times10^{-4}\text{ kg}\bullet\text{m}^2/\text{s}^2 = 8.7\times10^{-4}\text{ J}$$

Kinetic energy is conserved.

PRACTICE G

Elastic Collisions

1. A 0.015 kg marble sliding to the right at 22.5 cm/s on a frictionless surface makes an elastic head-on collision with a 0.015 kg marble moving to the left at 18.0 cm/s. After the collision, the first marble moves to the left at 18.0 cm/s.

a. Find the velocity of the second marble after the collision.

b. Verify your answer by calculating the total kinetic energy before and after the collision.

2. A 16.0 kg canoe moving to the left at 12.5 m/s makes an elastic head-on collision with a 14.0 kg raft moving to the right at 16.0 m/s. After the collision, the raft moves to the left at 14.4 m/s. Disregard any effects of the water.

a. Find the velocity of the canoe after the collision.

b. Verify your answer by calculating the total kinetic energy before and after the collision.

3. A 4.0 kg bowling ball sliding to the right at 8.0 m/s has an elastic head-on collision with another 4.0 kg bowling ball initially at rest. The first ball stops after the collision.

a. Find the velocity of the second ball after the collision.

b. Verify your answer by calculating the total kinetic energy before and after the collision.

4. A 25.0 kg bumper car moving to the right at 5.00 m/s overtakes and collides elastically with a 35.0 kg bumper car moving to the right. After the collision, the 25.0 kg bumper car slows to 1.50 m/s to the right, and the 35.0 kg car moves at 4.50 m/s to the right.

a. Find the velocity of the 35 kg bumper car before the collision.

b. Verify your answer by calculating the total kinetic energy before and after the collision.

SECTION 3

PROBLEM GUIDE G

Use this guide to assign problems.
SE = Student Edition Textbook
PW = Problem Workbook
PB = Problem Bank on the One-Stop Planner (OSP)

Solving for:

v_f	**SE** Sample, 1–3; Ch. Rvw. 32–34, 46* **PW** Sample, 6–7 **PB** 7–10
v_i	**SE** 4 **PW** Sample, 1–3 **PB** 3–6
m	**PW** 4–5 **PB** Sample, 1–2

***Challenging Problem**
Consult the printed Solutions Manual or the OSP for detailed solutions.

ANSWERS

Practice G

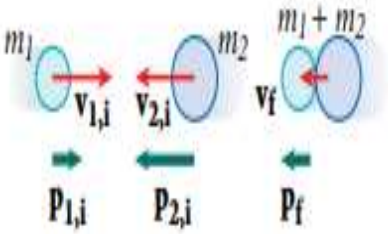
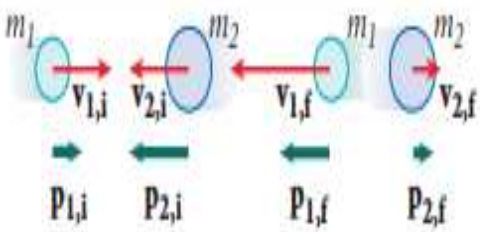
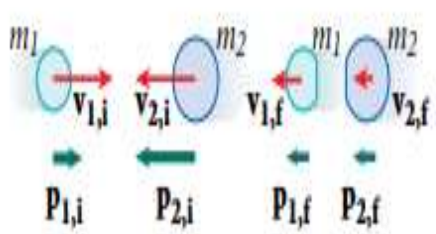
1. a. 22.5 cm/s to the right
b. $KE_i = 6.2\times10^{-4}\text{ J} = KE_f$

2. a. 14.1 m/s to the right
b. $KE_i = 3.04\times10^3$ J, $KE_f = 3.04\times10^3$ J, so $KE_i = KE_f$

3. a. 8.0 m/s to the right
b. $KE_i = 1.3\times10^2\text{ J} = KE_f$

4. a. 2.0 m/s to the right
b. $KE_i = 382\text{ J} = KE_f$

Table 2 Types of Collisions

Type of collision	Diagram	What happens	Conserved quantity
perfectly inelastic		The two objects stick together after the collision so that their final velocities are the same.	momentum
elastic		The two objects bounce after the collision so that they move separately.	momentum kinetic energy
inelastic		The two objects deform during the collision so that the total kinetic energy decreases, but the objects move separately after the collision.	momentum