Tutorial Lecture 12

Q1) Design Shannon-Fano for the following text

[DDABCCAADDDEEEDDDEEE] Find code efficiency and redundancy.

Solution:

$$P(A) = \frac{3}{20} = 0.15, P(B) = \frac{1}{20} = 0.05, P(C) = \frac{2}{20} = 0.1,$$

$$P(D) = \frac{8}{20} = 0.4, \quad P(E) = \frac{6}{20} = 0.3$$

Symbol	probability	Code				li
D	0.4	0				1
E	0.3	1	0			2
A	0.15	1	1	0		3
C	0.1	1	1	1	0	4
B	0.05	1	1	1	1	4

$$H(X) = -\sum_{i=1}^{n} p(x_i)\, log_2 p(x_i)$$

$$= -\left[\frac{0.4ln0.4 + 0.3ln0.3 + 0.15ln0.15 + 0.1ln0.1 + 0.05ln0.05}{ln2}\right]$$

$$= 2.008\ bit/symbol$$

$$L_C = 0.4 * 1 + 0.3 * 2 + 0.15 * 3 + 0.1 * 4 + 0.05 * 4 = 2.05\ bits/symbol$$

$$\eta = \frac{H(X)}{L_C} \times 100 = \frac{2.008}{2.05} \times 100 = 97.95\%$$

$$redunduncy = 100 - efficiency\% = 100 - 97.95\% = 2.05\%$$

Q2) Develop variable length code by Shannon-Fano Code for the message.

Symbol	B	A	D	C	F	E
P(x)	0.1	0.24	0.5	0.04	0.11	0.01

Find code table, and code efficiency.

Solution:

Variable	P(x)	Codeword(Ci)	l_i
D	0.50	0	1
A	0.24	10	2
F	0.11	110	3
B	0.10	1110	4
C	0.04	11110	5
E	0.01	11111	5

$H(x) = -\sum_{i=1}^{6} P(x)\ Log_2 \mathrm{P(xi)} = 1.93$ bits/symbol

$$L_C = \sum_{i=1}^{6} l_i p(x_i) = 1.96 bits/symbol$$

$$\eta = \frac{H(X)}{L_C} \times 100\% = 98.47\%$$