

DERIVATIVES المشتقة

the definition of derivative of the function $f(x)$ and this denoted by y' or $\frac{dy}{dx}$ or $\frac{d}{dx}f(x)$ or $D_x f(x)$ or $f'(x)$ and given by the formula

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Example1: Find the derivative of the function $f(x) = x^2$ using the definition of derivative.

Sol: $f(x) = x^2$

$$f(x + \Delta x) = (x + \Delta x)^2$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(x^2 + 2x\Delta x + \Delta x^2) - x^2}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(2x\Delta x + \Delta x^2)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} (2x + \Delta x) = 2x + 0 = 2x$$

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Example2: Find the derivative of the function $f(x) = 3x$ using the definition of derivative.

Sol: $f(x) = 3x$

$$f(x + \Delta x) = 3(x + \Delta x)$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{3(x + \Delta x) - 3x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{3x + 3\Delta x - 3x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{3\Delta x}{\Delta x} = 3$$

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