Lecture (4)

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(Derivative of Parametric Equations) مشتقة الدوال المحددة:

If $y = f(t)$, and $x = g(t)$, and the Derivative $\frac{dy}{dt}, \frac{dx}{dt}$ both exist, then:.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Example1: Find $\frac{dy}{dx}$ of the function $y = t^2 - 1$, $x = 2t + 3$

Sol:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$dy/dt = 2t \quad \text{and} \quad dx/dt = 2$$

$$\frac{dy}{dx} = \frac{2t}{2} = t = \frac{x-3}{2}.$$

Example2: Find $\frac{d^2y}{dx^2}$ if $y = t - t^3$ and $x = t - t^2$

Sol:

$$dy/dt = 1 - 3t^2 \quad \text{and} \quad dx/dt = 1 - 2t$$

$$\frac{dy}{dx} = \frac{1 - 3t^2}{1 - 2t} = t = \frac{x - 3}{2}$$

$$\frac{d^2y}{dx^2} = \frac{(1 - 2t) * (-6t) - (1 - 3t^2) * (-2)}{(1 - 2t)^2}$$

Lecture (4)

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(Derivative of Trigonometric Function) مشتقة الدوال المثلثية:

1. $\frac{dy}{dx}\sin u = \cos u * \frac{du}{dx}$
2. $\frac{dy}{dx}\cos u = -\sin u * \frac{du}{dx}$
3. $\frac{dy}{dx}\tan u = \sec^2 u * \frac{du}{dx}$
4. $\frac{dy}{dx}\cot u = -\csc^2 u * \frac{du}{dx}$
5. $\frac{dy}{dx}\sec u = \sec u.\tan u * \frac{du}{dx}$
6. $\frac{dy}{dx}\csc u = -\csc u.\cot u * \frac{du}{dx}$

Examples:

- $y = x^2 \sin x$

sol\ $\frac{dy}{dx} = x^2 * \cos x + \sin x * 2x$

- $y = \frac{\sin x}{x}$

sol\ $\frac{dy}{dx} = \frac{x\cos x - \sin x}{x^2}$

- $y = \cos^2 3x$

sol\ $\frac{dy}{dx} = 3\cos 3x * (-\sin 3x) * 3$

- $y = \sin(1 + \tan 2x)$

sol\ $\frac{dy}{dx} = \cos(1 + \tan 2x)\sec^2 2x * 2$

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- $x = \sin y - \sqrt{y}$ (implicit differentiation)

sol\ $x = \sin y - y^{\frac{1}{2}}$

$$1 = \cos y * \frac{dy}{dx} - \frac{1}{2} y^{-\frac{1}{2}} * \frac{dy}{dx}$$

$$1 = (\cos y - \frac{1}{2} y^{-\frac{1}{2}}) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{(\cos y - \frac{1}{2} y^{-\frac{1}{2}})}$$

Example: if $y = \sec x$ prove that $y'' + y = 2y^3$

Sol\

$$y = \sec x$$
$$y' = \sec x \tan x$$
$$y'' = \sec x \sec^2 x + \tan x \sec x \tan x$$
$$= \sec^3 x + \sec x \tan^2 x$$
$$= \sec^3 x + \sec x (\sec^2 x - 1)$$
$$= \sec^3 x + \sec^3 x - \sec x$$
$$= 2\sec^3 x - \sec x$$
$$= 2y^3 - y$$
$$y'' + y = 2y^3.$$

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