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DERIVATIVES المشتقة

the definition of derivative of the function $f(x)$ and this denoted by y' or $\frac{dy}{dx}$ or $\frac{d}{dx}f(x)$ or $D_x f(x)$ or $f'(x)$ and given by the formula

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Example1: Find the derivative of the function $f(x) = x^2$ using the definition of derivative.

Sol: $f(x) = x^2$

$$f(x + \Delta x) = (x + \Delta x)^2$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(x^2 + 2x\Delta x + \Delta x^2) - x^2}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(2x\Delta x + \Delta x^2)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} (2x + \Delta x) = 2x + 0 = 2x$$

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Example2: Find the derivative of the function $f(x) = 3x$ using the definition of derivative.

Sol: $f(x) = 3x$

$$f(x + \Delta x) = 3(x + \Delta x)$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{3(x + \Delta x) - 3x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{3x + 3\Delta x - 3x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{3\Delta x}{\Delta x} = 3$$

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Laws of Derivatives:

1- $\frac{d}{dx} c = 0$ where c is constant.

➡ $\frac{d}{dx} 4 = 0$

➡ $\frac{d}{dx} \sqrt{2} = 0$

2- $\frac{d}{dx} x^n = nx^{n-1}$

➡ $\frac{d}{dx} x^4 = 4x^3$

➡ $\frac{d}{dx} x^{-3} = -3x^{-4}$

➡ $\frac{d}{dx} \sqrt{x} = \frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2} x^{-\frac{1}{2}}$

3- If U and V are two functions of x then:

a) $\frac{d}{dx} (c * U) = c \frac{d}{dx} (U)$

➡ $\frac{d}{dx} 3x^3 = 3 \frac{d}{dx} x^3 = 3(3x^2) = 9x^2$

b) $\frac{d}{dx} (U \pm V) = \frac{d}{dx} (U) \pm \frac{d}{dx} (V)$

➡ $\frac{d}{dx} (x^2 + x^3) = \frac{d}{dx} (x^2) + \frac{d}{dx} (x^3) = 2x + 3x^2$

c) $\frac{d}{dx} (U * V) = U \frac{d}{dx} (V) + V \frac{d}{dx} (U)$

➡ $\frac{d}{dx} (x^2 * x^3) = x^2 \frac{d}{dx} (x^3) + x^3 \frac{d}{dx} (x^2) = x^2 * (3x^2) + x^3 * (2x)$

d) $\frac{d}{dx} U^n = nU^{n-1} * \frac{d}{dx} U$

➡ $\frac{d}{dx} (x^5)^3 = 3(x^5)^2 * \frac{d}{dx} x^5 = 3(x^5)^2 * 5x^4$

e) $\frac{d}{dx} \left(\frac{U}{V} \right) = \frac{V \frac{dU}{dx} - U \frac{dV}{dx}}{V^2}$

➡ $\frac{d}{dx} \left(\frac{x^2}{x^3} \right) = \frac{x^3 * (2x) - x^2 * 3x^2}{(x^3)^2}$

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Example: If $y = x^3 + 4x^2 - 5x + 4$ find $f'(2)$.

Sol: $f'(x) = 3x^2 + 8x - 5$

$$f'(2) = 3 * (2)^2 + 8 * (2) - 5$$

$$f'(2) = 23$$

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Example: If $y_1 = 3x^2 + 2x + 1$, $y_2 = 5x + 8$ find $f'(y_1 + y_2)_{(2)}$

Sol: $f'(y_1 + y_2) = f'(y_1) + f'(y_2)$

$$= f'(3x^2 + 2x + 1) + f'(5x + 8)$$

$$= (6x + 2) + (5)$$

$$= 6x + 7$$

$$f'(y_1 + y_2)_{(2)} = 6 * (2) + 7$$

$$f'(y_1 + y_2)_{(2)} = 19.$$

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H.W:

1- If $y = 3x^4 + 4x^3 - 2x + 6$ find $f'(3)$.

2- If $y_1 = 5x^4 + 2x^3 - 1$, $y_2 = 5x^3 - x + 2$ find $f'(y_1 + y_2)_{(3)}$

3- If $y = \frac{2x^2 - 6x}{4x}$ find $f'(1)$.

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المشتقة الثانية والمشتقات من الرتب العليا (Second and Higher Order Derivative)

$$f''(x) = \frac{d}{dx} \left(\frac{d}{dx} y \right)$$

Example: If $y = x^4 + 2x^3 - 3x + 1$ find $f''(1)$.

Sol: $f'(x) = 4x^3 + 2 * 3x^2 - 3 = 4x^3 + 6x^2 - 3$

$$f''(x) = 4 * 3x^2 + 6 * 2x = 12x^2 + 12x$$

$$f''(x) = 12(1)^2 + 12 = 24.$$

Example: If $y = 2x^4 + 5x^3 - 3x^2 + 6$ find $f''(-1)$.

Sol: $f'(x) = 8x^3 + 15x^2 - 6x$

$$f''(x) = 24x^2 + 30x - 6$$

$$f''(x) = 24(-1)^2 + 30(-1) - 6$$

$$= 24 - 30 - 6 = -12$$

H.W:

1- If $y = 3x^4 + 4x^3 - 2x + 6$ find $f''(4)$.

2- If $y_1 = 5x^4 + 2x^3 - 1$, $y_2 = 5x^3 - x + 2$ find $f''(y_1 - y_2)_{(1)}$

3- If $y = 5x^3 + 4x - 7$ find $f''(2)$.