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مشتقة الدوال المثلثية (Derivative of Trigonometric Function)

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1. $\frac{dy}{dx} \sin u = \cos u * \frac{du}{dx}$
2. $\frac{dy}{dx} \cos u = -\sin u * \frac{du}{dx}$
3. $\frac{dy}{dx} \tan u = \sec^2 u * \frac{du}{dx}$
4. $\frac{dy}{dx} \cot u = -\csc^2 u * \frac{du}{dx}$
5. $\frac{dy}{dx} \sec u = \sec u * \tan u * \frac{du}{dx}$
6. $\frac{dy}{dx} \csc u = -\csc u * \cot u * \frac{du}{dx}$

Examples:

- $y = x^2 \sin x$

sol\ $\frac{dy}{dx} = x^2 * \cos x + \sin x * 2x$

- $y = \frac{\sin x}{x}$

sol\ $\frac{dy}{dx} = \frac{x \cos x - \sin x}{x^2}$

- $y = \cos^2 3x$

sol\ $\frac{dy}{dx} = 3 \cos 3x * (-\sin 3x) * 3$

- $y = \sin(1 + \tan 2x)$

sol\ $\frac{dy}{dx} = \cos(1 + \tan 2x) \sec^2 2x * 2$

- $x = \sin y - \sqrt{y}$ (implicit differentiation)

Sol\ $x = \sin y - y^{\frac{1}{2}}$

$$1 = \cos y * \frac{dy}{dx} - \frac{1}{2} y^{-\frac{1}{2}} * \frac{dy}{dx}$$

$$1 = (\cos y - \frac{1}{2} y^{-\frac{1}{2}}) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{(\cos y - \frac{1}{2} y^{-\frac{1}{2}})}$$

Example: if $y = \sec x$ prove that $y'' + y = 2y^3$

Sol\

$$y = \sec x$$

$$y' = \sec x \tan x$$

$$y'' = \sec x \sec^2 x + \tan x \sec x \tan x$$

$$= \sec^3 x + \sec x \tan^2 x$$

$$= \sec^3 x + \sec x (\sec^2 x - 1)$$

$$= \sec^3 x + \sec^3 x - \sec x$$

$$= 2\sec^3 x - \sec x$$

$$= 2y^3 - y$$

$$y'' + y = 2y^3.$$

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Examples: For each equation find $\frac{dy}{dx}$

1. $y = \tan(x) - \sin(x)$

2. $y = x^2 \sec(x)$

3. $y = \frac{\cos(x)}{1 + \sin(x)}$

4. $y = e^x (\sin(x) + x)$

5. $y = x^3 \cos(x) \sin(x)$

6. $y = \frac{\csc(x)}{\sqrt{x}}$

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Example : find $\frac{dy}{dx}$ if $y = \frac{\cos x}{(1 + \sin x)}$

Sol\

$$\frac{dy}{dx} = \frac{(1 + \sin x) * \frac{d}{dx}(\cos x) - \cos x * \frac{d}{dx}((1 + \sin x))}{(1 + \sin x)^2}$$

$$\frac{dy}{dx} = \frac{(1 + \sin x) * (-\sin x) - \cos x \cos x}{(1 + \sin x)^2}$$

$$\frac{dy}{dx} = \frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2}$$

$$\frac{dy}{dx} = \frac{-\sin x - (\sin^2 x + \cos^2 x)}{(1 + \sin x)^2}$$

$$\frac{dy}{dx} = \frac{-\sin x - (1)}{(1 + \sin x)^2}$$

$$\frac{dy}{dx} = \frac{-(1 + \sin x)}{(1 + \sin x)^2}$$

$$\frac{dy}{dx} = \frac{-(1)}{1 + \sin x}$$