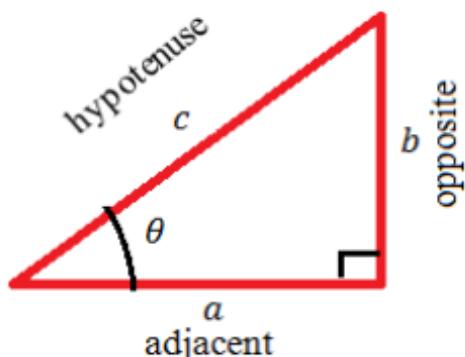


## Trigonometric functions

There are six basic trigonometric functions used in Trigonometry. These functions are trigonometric ratios. The six basic trigonometric functions are sine function, cosine function, secant function, co-secant function, tangent function, and co-tangent function. The trigonometric functions and identities are the ratio of sides of a right-angled triangle. The sides of a right triangle are the perpendicular side, hypotenuse, and base, which are used to calculate the sine, cosine, tangent, secant, cosecant, and cotangent values using trigonometric formulas.

A right triangle is a triangle with a right angle ( $90^\circ$ )



For every angle  $\theta$  in the triangle, there is the side of the triangle adjacent to it, the side opposite of it and the hypotenuse such that  $a^2 + b^2 = c^2$ .

For angle  $\theta$ , the trigonometric functions are defined as follows:

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{b}{c}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{a}{c}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{opp}}{\text{adj}} = \frac{b}{a}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{\text{adj}}{\text{opp}} = \frac{a}{b}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hyp}}{\text{adj}} = \frac{c}{a}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{\text{hyp}}{\text{opp}} = \frac{c}{b}$$

### Trigonometric Functions Values

The trigonometric functions have a domain  $\theta$ , which is in degrees or radians.

Some of the principal values of  $\theta$  for the different trigonometric functions are presented below in a table.

$\theta$ degrees	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$180^\circ$	$270^\circ$	$360^\circ$
$\theta$ radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$	0	$\infty$	0
$\csc \theta$	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\infty$	-1	$\infty$
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$	-1	$\infty$	1
$\cot \theta$	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$\infty$	0	$\infty$

### Trigonometric functions of negative angles

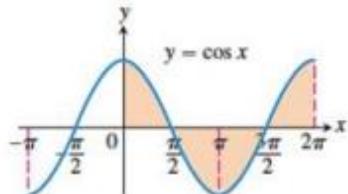
$$\sin(-\theta) = -\sin \theta, \quad \cos(-\theta) = \cos \theta \quad \text{and} \quad \tan(-\theta) = -\tan \theta$$

### Some useful relationships among trigonometric functions

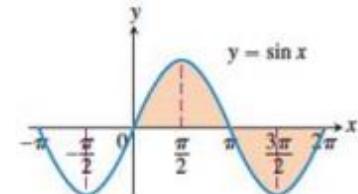
1.  $\sin^2 x + \cos^2 x = 1, \quad \sec^2 x - \tan^2 x = 1, \quad \csc^2 x - \cot^2 x = 1$
2.  $\sin 2x = 2 \sin x \cos x, \quad \cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
3.  $\sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2}$

## Lecture (6)

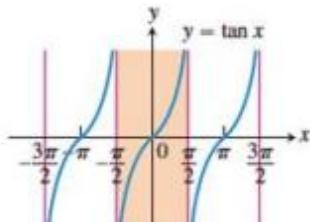
### Graphs of Trigonometric Functions



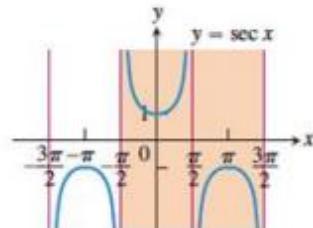
Domain:  $-\infty < x < \infty$   
Range:  $-1 \leq y \leq 1$   
Period:  $2\pi$



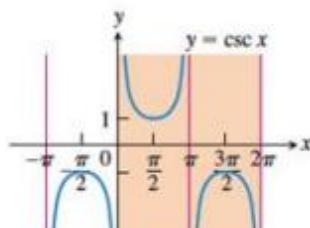
Domain:  $-\infty < x < \infty$   
Range:  $-1 \leq y \leq 1$   
Period:  $2\pi$



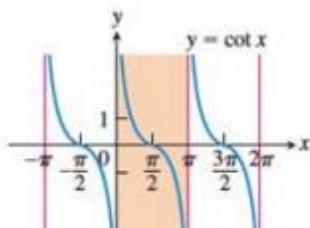
Domain:  $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots$   
Range:  $-\infty < y < \infty$   
Period:  $\pi$



Domain:  $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots$   
Range:  $y \leq -1$  and  $y \geq 1$   
Period:  $2\pi$



Domain:  $x \neq 0, \pm\pi, \pm2\pi, \dots$   
Range:  $y \leq -1$  and  $y \geq 1$   
Period:  $2\pi$



Domain:  $x \neq 0, \pm\pi, \pm2\pi, \dots$   
Range:  $-\infty < y < \infty$   
Period:  $\pi$