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Implicit Differentiation: المشتقة الضمنية

In some cases, it is difficult to solve $y = f(x)$, so to find $\frac{d}{dx}$ for such cases, implicit differentiation will be use.

Example1: Find $\frac{d}{dx}$ of the function $y^2 + x^2 = 1$

Sol: $2y\frac{d}{dx} + 2x = 0$

$$2y\frac{d}{dx} = -2x$$

$$\frac{d}{dx} = \frac{-2x}{2y}.$$

Example2: Find the Implicit Differentiation of the function

$2y = x^2 + 3xy^2$ at x=1, y=2

(Or) Find the slope of the tangent line at the point(1,2)

Sol: $2\frac{d}{dx} = 2x + \left(3x * 2y * \frac{d}{dx}\right) + (y^2 * 3)$

$$2\frac{d}{dx} = 2x + \left(6xy\frac{d}{dx}\right) + (3y^2)$$

$$2\frac{d}{dx} - 6xy\frac{d}{dx} = 2x + (3y^2)$$

$$(2 - 6xy)\frac{d}{dx} = 2x + (3y^2)$$

$$\frac{d}{dx} = \frac{2x + 3y^2}{2 - 6xy} = \frac{2 * (1) + 3 * (2)^2}{2 - (6 * (1) * (2))} = \frac{14}{-10}$$

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Example3: Find $\frac{d}{dx}$ for $y^3 + y^2 - 5y - x^2 = -4$

$$3y^2\frac{d}{dx} + 2y\frac{d}{dx} - 5\frac{d}{dx} - 2x = 0$$

$$\frac{d}{dx}(3y^2 + 2y - 5) = 2x$$

$$\frac{d}{dx} = \frac{2x}{(3y^2 + 2y - 5)}$$

H.W:

1) $x^2 - y^2 = 10$ **2)** $x^2 + y^2 - 4y = 20$ **3)** $3x^2 - xy = 4$

4) $x^2y + 3xy^3 - x = 3$ **5)** $\left(x^2 + 3y^2\right)^{35} = x$ **6)** $\sin\left(x^2y^2\right) = x$

7) For $x^3y + xy^3 = 10$, evaluate dy/dx at the point (2,2).

(Or) Find the slope of the tangent line at the point (2,2)

8) Given $x^{2/3} - y^{2/3} - y = 1$, find the equation of the tangent line at the point (1,1).

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The Chain Rule قاعدة السلسلة

If $y = f(u)$; $u = g(x)$, and the derivatives $\frac{dy}{du}$ and both $\frac{du}{dx}$ exist then the composite function defined by $f(g(x))$ has a derivative given by:

Ex1: Let $y = \sqrt{u^2+1}$, $u = \frac{1}{x} + x^2$, find $\frac{dy}{dx}$.

Sol: $\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx}$

$\frac{dy}{du} = \frac{2u}{2\sqrt{u^2+1}} = \frac{u}{\sqrt{u^2+1}}$; $\frac{du}{dx} = -\frac{1}{x^2} + 2x$

$$\therefore \frac{dy}{dx} = \frac{2u}{2\sqrt{u^2+1}} * \left(2x - \frac{1}{x^2}\right) = \frac{\left(\frac{1}{x} + x^2\right)}{\sqrt{\left(\frac{1}{x} + x^2\right)^2 + 1}} * (2x - \frac{1}{x^2})$$

Ex2: If $y = (3x^2 - 7x + 1)^2$, use the chain rule to find $\frac{dy}{dx}$.

Sol.: we may express y as a composite function of x by letting:

$y = u^2$ and $u = 3x^2 - 7x + 1$

So , $\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx} = 5u^2 * (6x - 7) = 5(3x^2 - 7x + 1)^4(6x - 7)$

H.W:

1- find $\frac{dy}{dx}$ at $x = -1$ if $y = u^3 + 5u - 4$ and $u = x^2 + x$.

2- find $\frac{dy}{dx}$ at $x = 2$ if $y = 2u^2 + 3u$and $u = 2x^2 + 3x - 1$.

3- find $\frac{dy}{dx}$ if $y = 2u^2$ and $u = \frac{x-1}{x}$.

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Derivative of Parametric Equations) مشتقة الدوال المحددة:

If $y = f(t)$, and $x = g(t)$, and the Derivative $\frac{dy}{dt}, \frac{dx}{dt}$ both exist, then:.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Example1: Find $\frac{dy}{dx}$ of the function $y = t^2 - 1$, $x = 2t + 3$

Sol:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$dy/dt = 2t \quad \text{and} \quad dx/dt = 2$$

$$\frac{dy}{dx} = \frac{2t}{2} = t = \frac{x-3}{2}.$$

Example2: Find $\frac{d^2y}{dx^2}$ if $y = t - t^3$ and $x = t - t^2$

Sol:

$$dy/dt = 1 - 3t^2 \quad \text{and} \quad dx/dt = 1 - 2t$$

$$\frac{dy}{dx} = \frac{1-3t^2}{1-2t} = t = \frac{x-3}{2}$$

$$\frac{d^2y}{dx^2} = \frac{(1-2t)*(-6t) - (1-3t^2)*(-2)}{(1-2t)^2}$$