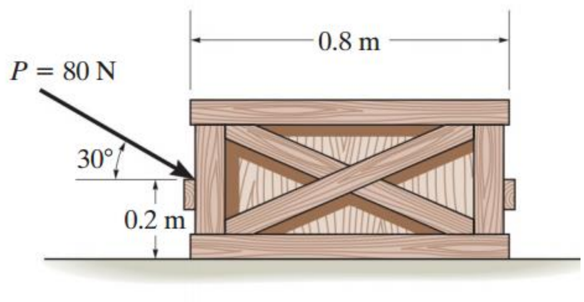
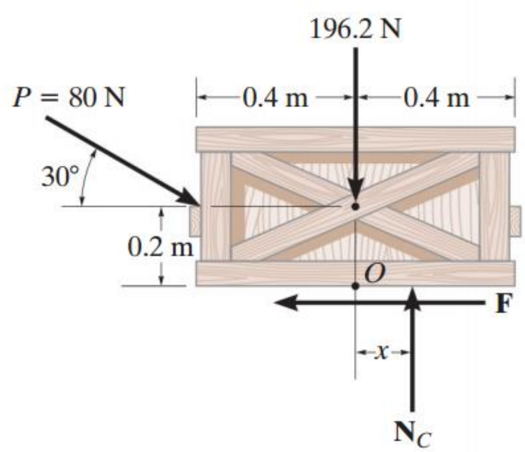
Example: The uniform crate shown in Fig. a has a mass of 20 kg. If a force P = 80 N is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is $\mu_s = 0.3$.





Solution:

The resultant normal force N_C must act a distance x from the crate's center line in order to counteract the tipping effect caused by P. There are three unknowns, F, N_C, and x, which can be determined strictly from the three equations of equilibrium.

Equations of Equilibrium.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 80 \cos 30° \text{ N} - F = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad -80 \sin 30° \text{ N} + N_C - 196.2 \text{ N} = 0$$

$$\circlearrowleft + \Sigma M_O = 0; \quad 80 \sin 30° \text{ N}(0.4 \text{ m}) - 80 \cos 30° \text{ N}(0.2 \text{ m}) + N_C(x) = 0$$

Solving,

$$F = 69.3 \text{ N}$$

$$N_C = 236.2 \text{ N}$$

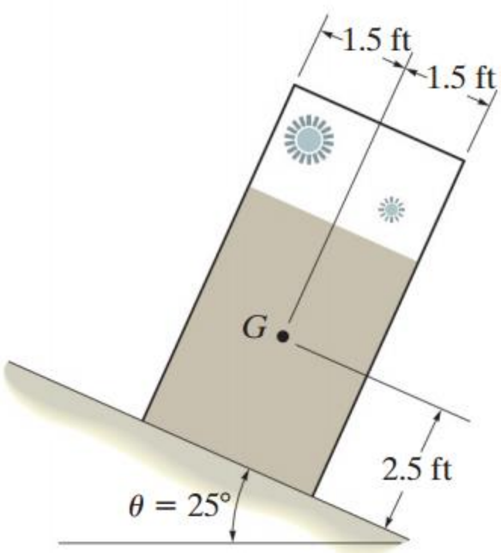
$$x = -0.00908 \text{ m} = -9.08 \text{ mm}$$

Since x is negative it indicates the *resultant* normal force acts (slightly) to the *left* of the crate's center line. No tipping will occur since $x < 0.4$ m. Also, the *maximum* frictional force which can be developed at the surface of contact is $F_{max} = \mu_s N_C = 0.3(236.2 \text{ N}) = 70.9$ N. Since $F = 69.3 \text{ N} < 70.9$ N, the crate will *not slip*, although it is very close to doing so.

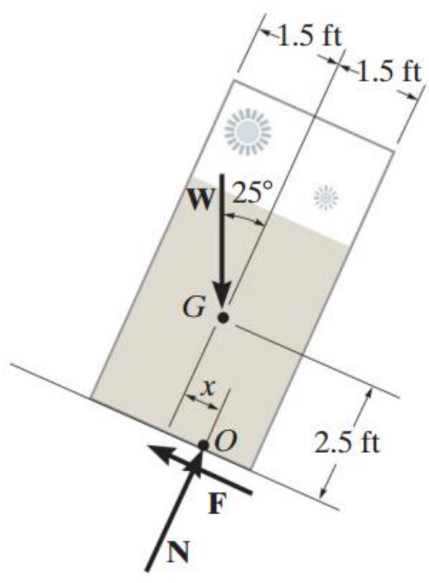
Example: It is observed that when the bed of the dump truck is raised to an angle of $\theta = 25^{\circ}$ the vending machines will begin to slide off the bed, Fig. a. Determine the static coefficient of friction between a vending machine and the surface of the truck bed.

(a)

Solution: An idealized model of a vending machine resting on the truckbed is shown in Fig. b. The dimensions have been measured and the center of gravity has been located. We will assume that the vending machine weighs W.



(b)



(c)

Free-Body Diagram. As shown in Fig. c, the dimension x is used to locate the position of the resultant normal force N. There are four unknowns, N, F, μ_s, and x.

Equations of Equilibrium.

$$+\searrow \Sigma F_x = 0; \qquad W \sin 25^\circ - F = 0 \qquad (1)$$

$$+\nearrow \Sigma F_y = 0; \qquad N - W \cos 25^\circ = 0 \qquad (2)$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad -W \sin 25^\circ (2.5 \text{ ft}) + W \cos 25^\circ (x) = 0 \qquad (3)$$

Since slipping impends at $\theta = 25^\circ$, using Eqs. 1 and 2, we have

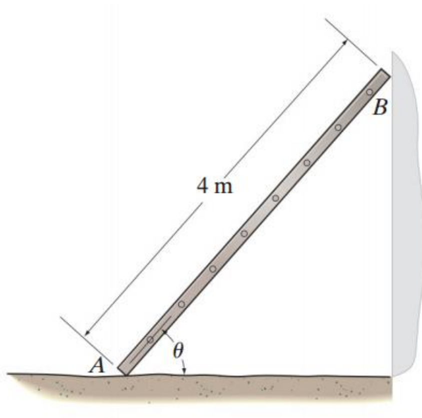
$$F_s = \mu_s N; \qquad W \sin 25^\circ = \mu_s (W \cos 25^\circ)$$

$$\mu_s = \tan 25^\circ = 0.466 \qquad \textit{Ans.}$$

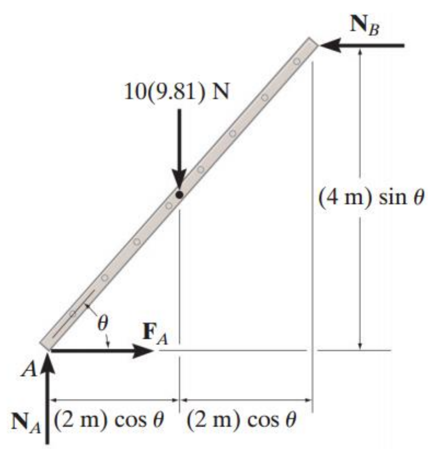
The angle of $\theta = 25^\circ$ is referred to as the ***angle of repose***, and by comparison, it is equal to the angle of static friction, $\theta = \phi_s$. Notice from the calculation that θ is independent of the weight of the vending machine, and so knowing θ provides a convenient method for determining the coefficient of static friction.

NOTE: From Eq. 3, we find x = 1.17 ft. Since 1.17 ft < 1.5 ft, indeed the vending machine will slip before it can tip

Example: The uniform 10-kg ladder in Fig. a rests against the smooth wall at B, and the end A rests on the rough horizontal plane for which the coefficient of static friction is $\mu_s = 0.3$. Determine the angle of inclination θ of the ladder and the normal reaction at B if the ladder is on the verge of slipping.



(a)



(b)

Solution: As shown on the free-body diagram, Fig. b, the frictional force $\mathbf{F_A}$ must act to the right since impending motion at A is to the left.

Equations of Equilibrium and Friction. Since the ladder is on the verge of slipping, then $F_A = \mu_s N_A = 0.3N_A$. By inspection, N_A can be obtained directly.

$+\uparrow \Sigma F_y = 0; \qquad N_A - 10(9.81)\text{ N} = 0 \qquad N_A = 98.1\text{ N}$

Using this result, $F_A = 0.3(98.1\text{ N}) = 29.43\text{ N}$. Now N_B can be found.

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 29.43\text{ N} - N_B = 0$

$$N_B = 29.43\text{ N} = 29.4\text{ N} \qquad \textit{Ans.}$$

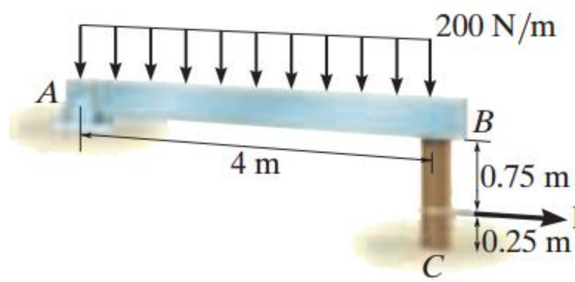
Finally, the angle θ can be determined by summing moments about point A.

$\circlearrowleft + \Sigma M_A = 0; \qquad (29.43\text{ N})(4\text{ m})\sin\theta - [10(9.81)\text{ N}](2\text{ m})\cos\theta = 0$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = 1.6667$$

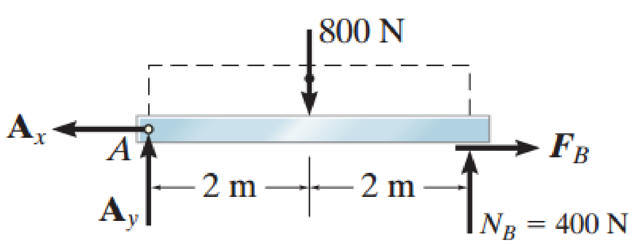
$$\theta = 59.04^\circ = 59.0^\circ \qquad \textit{Ans.}$$

Example: Beam AB is subjected to a uniform load of 200 N /m and is supported at B by post BC, Fig. a. If the coefficients of static friction at B and C are $\mu_B = 0.2$ and $\mu_C = 0.5$, determine the force **P** needed to pull the post out from under the beam. Neglect the weight of the members and the thickness of the beam.

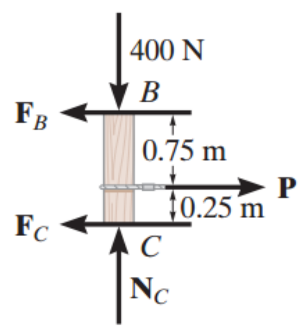


(a)

Solution:



(b)



(c)

Free-Body Diagrams. The free-body diagram of the beam is shown in Fig. *b*.

Applying $\sum M_A = 0$, we obtain $N_B = 400$ N. This result is shown on the free-body diagram of the post, Fig. *c*. Referring to this member, the *four* unknowns F_B, P, F_C, and N_C are determined from the *three* equations of equilibrium and *one* frictional equation applied either at *B* or *C*.

Equations of Equilibrium and Friction.

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad P - F_B - F_C = 0 \quad (1)$

$+\uparrow \Sigma F_y = 0;$ $\quad N_C - 400\text{ N} = 0 \quad (2)$

$\circlearrowleft + \Sigma M_C = 0;$ $\quad -P(0.25\text{ m}) + F_B(1\text{ m}) = 0 \quad (3)$

(Post Slips at *B* and Rotates about C.) This requires $F_C \le \mu_C N_C$ and

$F_B = \mu_B N_B;$ $\quad F_B = 0.2(400\text{ N}) = 80\text{ N}$

Using this result and solving Eqs. 1 through 3, we obtain

$$P = 320\text{ N}$$
$$F_C = 240\text{ N}$$
$$N_C = 400\text{ N}$$

Since $F_C = 240\text{ N} > \mu_C N_C = 0.5(400\text{ N}) = 200\text{ N}$, slipping at C occurs. Thus the other case of movement must be investigated.

(Post Slips at *C* and Rotates about *B*.) Here $F_B \le \mu_B N_B$ and

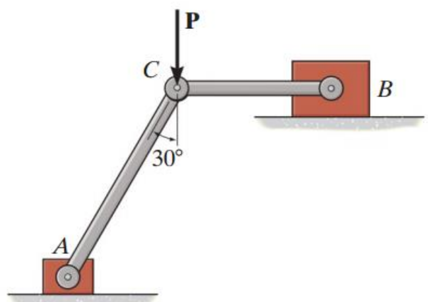
$F_C = \mu_C N_C;$ $\quad F_C = 0.5 N_C \quad (4)$

Solving Eqs. 1 through 4 yields

$$P = 267\text{ N} \qquad \textit{Ans.}$$
$$N_C = 400\text{ N}$$
$$F_C = 200\text{ N}$$
$$F_B = 66.7\text{ N}$$

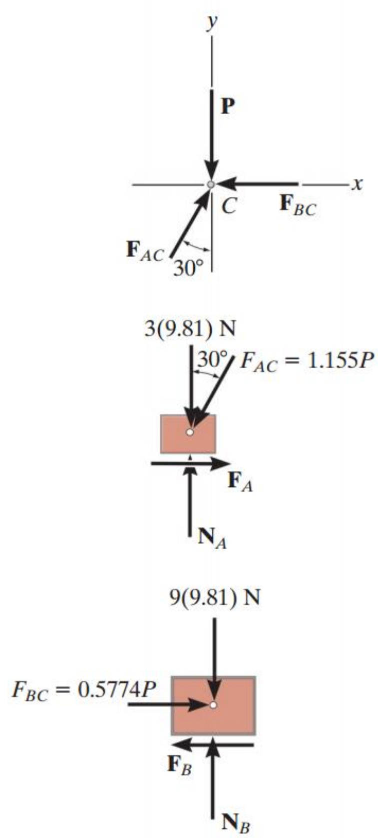
Obviously, this case occurs first since it requires a *smaller* value for P.

Example: Blocks A and B have a mass of 3 kg and 9 kg, respectively, and are connected to the weightless links shown in Fig. a. Determine the largest vertical force P that can be applied at the pin C without causing any movement. The coefficient of static friction between the blocks and the contacting surfaces is $\mu_s = 0.3$.



(a)

Solution:



(b)

Free-Body Diagram. The links are two-force members and so the free-body diagrams of pin *C* and blocks *A* and *B* are shown in Fig. *b*. Since the horizontal component of $\mathbf{F}_{AC}$ tends to move block *A* to the left, F_A must act to the right. Similarly, $\mathbf{F}_B$ must act to the left to oppose the tendency of motion of block *B* to the right, caused by $\mathbf{F}_{BC}$. There are ***seven*** unknowns and ***six*** available force equilibrium equations, two for the pin and two for each block, so that *only one* frictional equation is needed.

Equations of Equilibrium and Friction. The force in links AC and BC can be related to P by considering the equilibrium of pin C.

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC} \cos 30^\circ - P = 0; \qquad F_{AC} = 1.155P$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1.155P \sin 30^\circ - F_{BC} = 0; \qquad F_{BC} = 0.5774P$$

Using the result for F_{AC}, for block A,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A - 1.155P \sin 30^\circ = 0; \qquad F_A = 0.5774P \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad N_A - 1.155P \cos 30^\circ - 3(9.81 \text{ N}) = 0;$$

$$N_A = P + 29.43 \text{ N} \qquad (2)$$

Using the result for F_{BC}, for block B,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad (0.5774P) - F_B = 0; \qquad F_B = 0.5774P \qquad (3)$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B - 9(9.81) \text{ N} = 0; \qquad N_B = 88.29 \text{ N}$$

Movement of the system may be caused by the initial slipping of *either* block A or block B. If we assume that block A slips first, then

$$F_A = \mu_s N_A = 0.3N_A \qquad (4)$$

Substituting Eqs. 1 and 2 into Eq. 4,

$$0.5774P = 0.3(P + 29.43)$$

$$P = 31.8 \text{ N} \qquad \textit{Ans.}$$

Substituting this result into Eq. 3, we obtain $F_B = 18.4$ N. Since the maximum static frictional force at B is $(F_B)_{max} = \mu_s N_B = 0.3(88.29 \text{ N}) = 26.5 \text{ N} > F_B$, block B will not slip. Thus, the above assumption is correct. Notice that if the inequality were not satisfied, we would have to assume slipping of block B and then solve for P.