ENGINEERING MECHANICS II

Al-Mustaqbal University
College of Engineering and Technologies
Prosthetics and Orthotics Engineering

MECHANICS-DYNAMIC TUTORIAL

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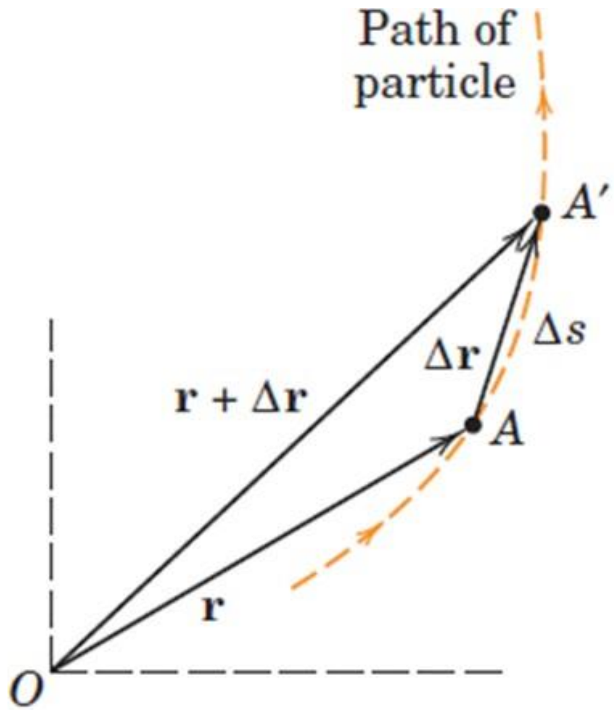
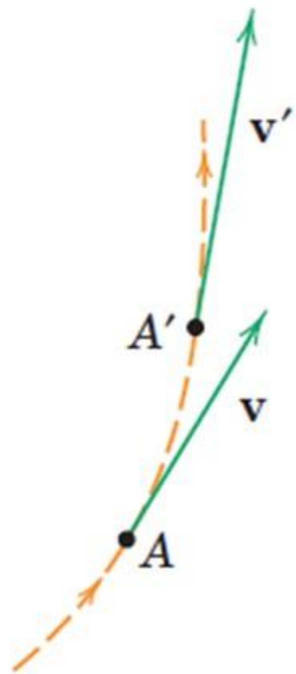
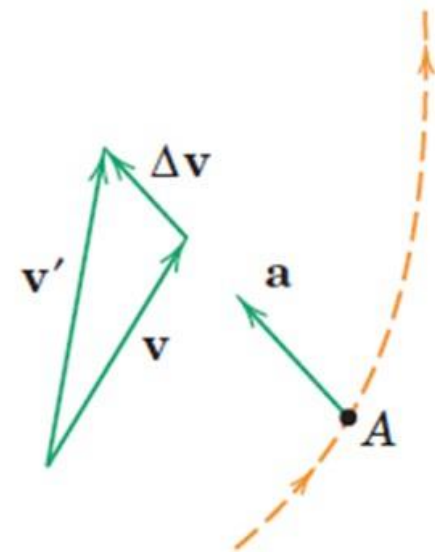
PLANE CURVILINEAR MOTION

- Plane Curvilinear Motion is the motion of a particle along a curved path which lies in a single plane.

- Consider the continuous motion of a particle along a plane curve as represented in Figure.


Path of
particle
A′
Δr
Δs
r + Δr
A
r
O


v′
A′
v
A


Δv
v′
a
v
A

Velocity

- At time t the particle is at position A.
- At time $t + \Delta t$ the particle is at A' .
- The displacement of the particle during time Δt is the vector Δr.
- The average velocity of the particle between A and A' is defined as

$$\mathrm{v}_{\mathrm{av}} = \frac{\Delta r}{\Delta t}$$

- The vector quantity of velocity write as:

$$\mathrm{v} = \frac{dr}{dt} = r$$

The scalar quantity of velocity write as:

$$v = |\mathrm{v}| = \frac{ds}{dt} = s$$

Acceleration:

The average acceleration of the particle between A and A' is defined as $\frac{\Delta v}{\Delta}$

The vector quantity of acceleration write as:

$$a = \frac{dv}{dt} = v$$

- **The direction of the acceleration of a particle in curvilinear motion is normal to the path points toward the center of curvature of the path.**

- **To find the angle between the velocity vector and acceleration vector:**

$$\cos\theta = \frac{\underline{\mathbf{v}} \cdot \underline{\mathbf{a}}}{\mathbf{va}}$$

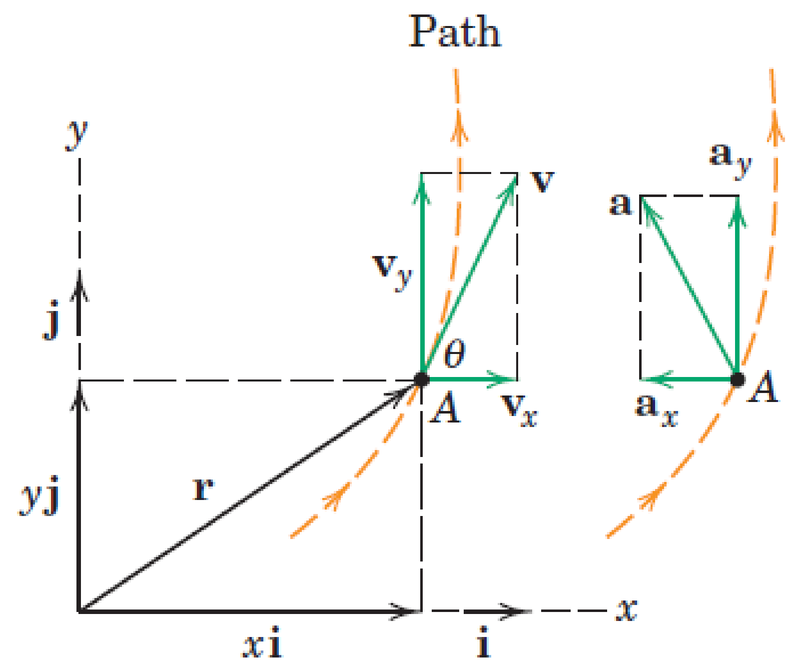
RECTANGULAR COORDINATES (X-Y)

- For the particle path shown in figure along x- and y-axes:

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$



$v_x = \dot{x}$, $v_y = \dot{y}$ and $a_x = \dot{v}_x = \ddot{x}$, $a_y = \dot{v}_y = \ddot{y}$

- a_x is in the negative x-direction, so that x would be a negative a number.

- the direction of the velocity is always tangent to the path.

$$v^2 = {v_x}^2 + {v_y}^2 \qquad v = \sqrt{{v_x}^2 + {v_y}^2} \qquad \tan\theta = \frac{v_y}{v_x}$$

$$a^2 = {a_x}^2 + {a_y}^2 \qquad a = \sqrt{{a_x}^2 + {a_y}^2}$$

PROJECTILE MOTION

To solve projectile motion problem:

1. We neglect aerodynamic drag and the curvature and rotation of the earth.

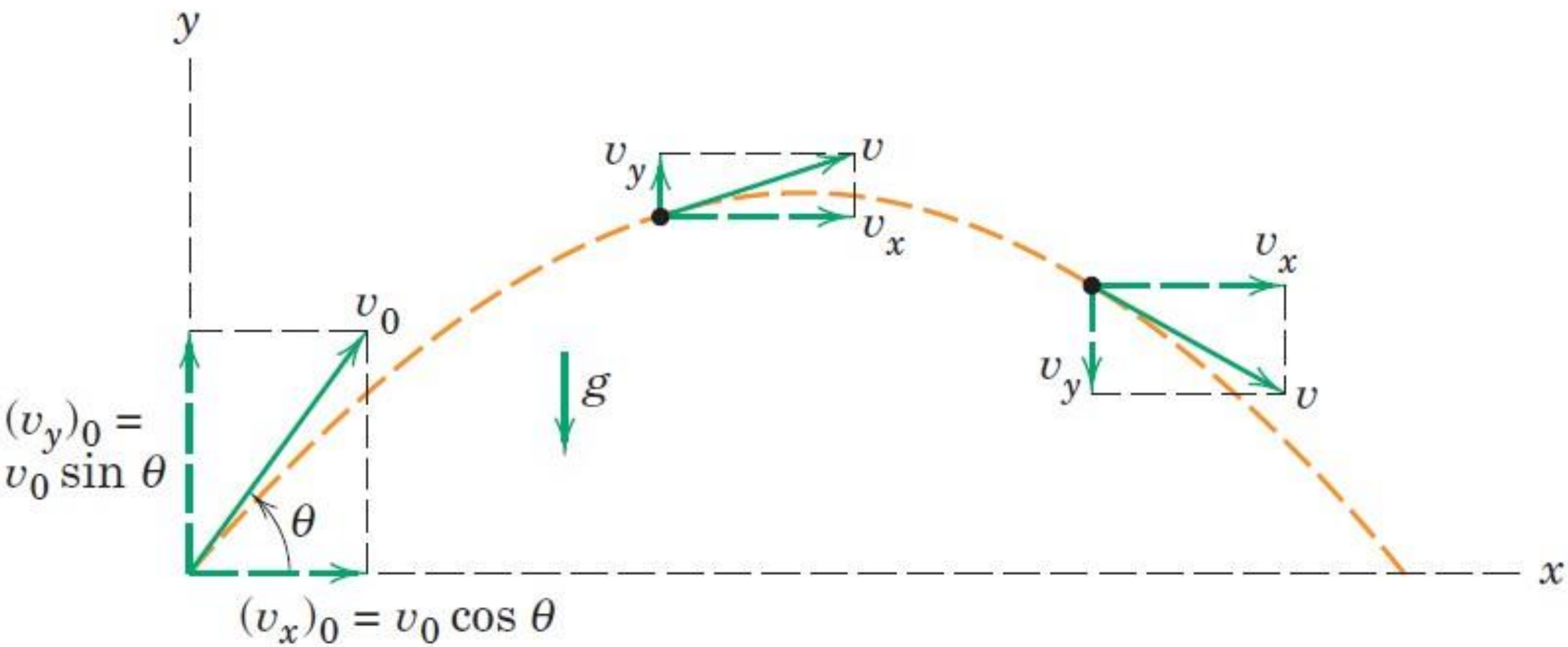
2. We assume that the altitude change is small enough so that the acceleration due to gravity can be considered constant.

$$a_x = 0 \qquad a_y = -g$$

$$v_x = (v_x)_0 \qquad v_y = (v_y)_0 - gt$$

$$x = x_0 + (v_x)_0 t \qquad y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

$$v_y{}^2 = (v_y)_0{}^2 - 2g(y - y_0)$$



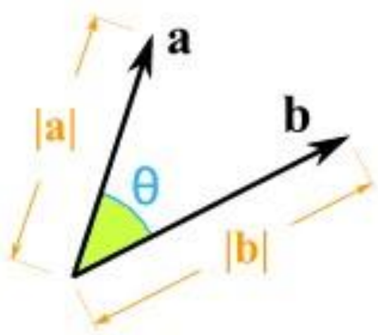
Calculating

The Dot Product is written using a central dot:

$$\mathbf{a} \cdot \mathbf{b}$$

This means the Dot Product of **a** and **b**

We can calculate the Dot Product of two vectors this way:

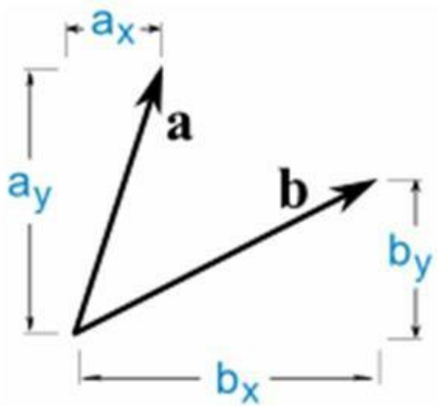


$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos(\theta)$$

Where:
|**a**| is the magnitude (length) of vector **a**
|**b**| is the magnitude (length) of vector **b**
θ is the angle between **a** and **b**

So we multiply the length of **a** times the length of **b**, then multiply by the cosine of the angle between **a** and **b**

OR we can calculate it this way:



$$\mathbf{a} \cdot \mathbf{b} = a_x \times b_x + a_y \times b_y$$

So we multiply the x's, multiply the y's, then add.

Example 1

Calculate the dot product of $\mathbf{a} = (1, 2, 3)$ and $\mathbf{b} = (4, -5, 6)$.

Solution: Using the component formula for the dot product of three-dimensional vectors,

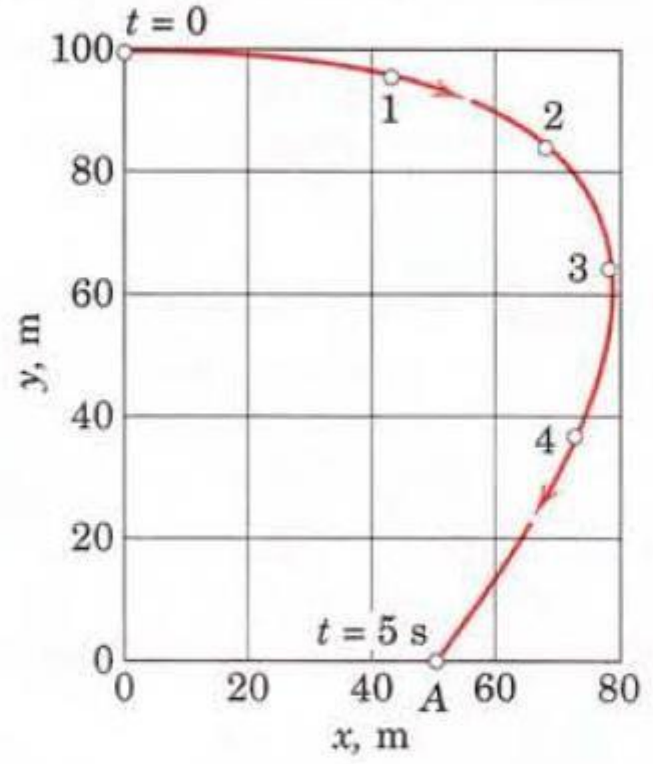
$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3,$$

we calculate the dot product to be

$$\mathbf{a} \cdot \mathbf{b} = 1(4) + 2(-5) + 3(6) = 4 - 10 + 18 = 12.$$

Sample Problem 2/5

The curvilinear motion of a particle is defined by $v_x = 50 - 16t$ and $y = 100 - 4t^2$, where v_x is in meters per second, y is in meters, and t is in seconds. It is also known that $x = 0$ when $t = 0$. Plot the path of the particle and determine its velocity and acceleration when the position $y = 0$ is reached.



Solution. The x-coordinate is obtained by integrating the expression for v_x, and the x-component of the acceleration is obtained by differentiating v_x. Thus,

$$\left[\int dx = \int v_x\, dt\right] \qquad \int_0^x dx = \int_0^t (50 - 16t)\, dt \qquad x = 50t - 8t^2 \text{ m}$$

$$[a_x = \dot{v}_x] \qquad a_x = \frac{d}{dt}(50 - 16t) \qquad a_x = -16 \text{ m/s}^2$$

The y-components of velocity and acceleration are

$$[v_y = \dot{y}] \qquad v_y = \frac{d}{dt}(100 - 4t^2) \qquad v_y = -8t \text{ m/s}$$

$$[a_y = \dot{v}_y] \qquad a_y = \frac{d}{dt}(-8t) \qquad a_y = -8 \text{ m/s}^2$$

We now calculate corresponding values of x and y for various values of t and plot x against y to obtain the path as shown.

When $y = 0$, $0 = 100 - 4t^2$, so $t = 5$ s. For this value of the time, we have

$$v_x = 50 - 16(5) = -30 \text{ m/s}$$

$$v_y = -8(5) = -40 \text{ m/s}$$

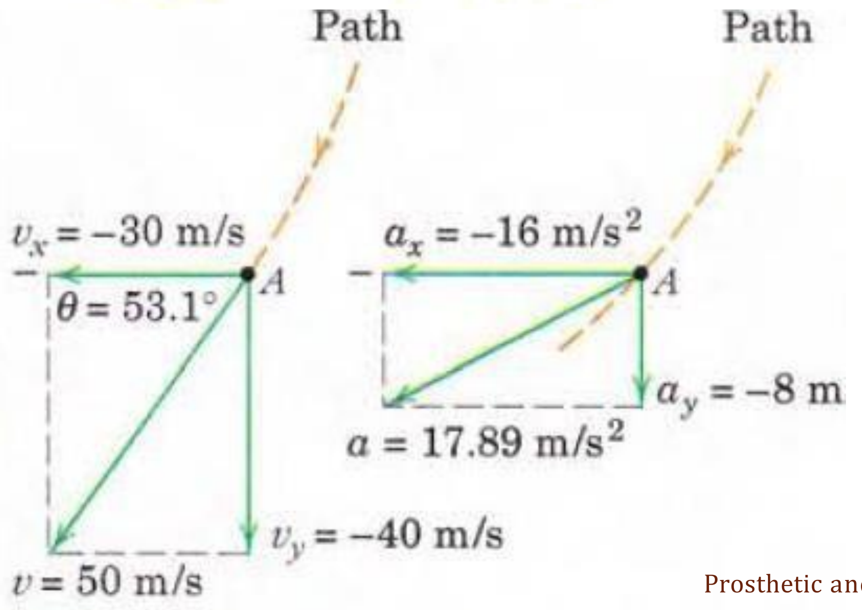
$$v = \sqrt{(-30)^2 + (-40)^2} = 50 \text{ m/s}$$

$$a = \sqrt{(-16)^2 + (-8)^2} = 17.89 \text{ m/s}^2$$

The velocity and acceleration components and their resultants are shown on the separate diagrams for point A, where $y = 0$. Thus, for this condition we may write

$$\mathbf{v} = -30\mathbf{i} - 40\mathbf{j} \text{ m/s}$$

$$\mathbf{a} = -16\mathbf{i} - 8\mathbf{j} \text{ m/s}^2$$



2/59 The position vector of a particle moving in the x-y plane at time $t = 3.60$ s is $2.76\mathbf{i} - 3.28\mathbf{j}$ m. At $t = 3.62$ s its position vector has become $2.79\mathbf{i} - 3.33\mathbf{j}$ m. Determine the magnitude v of its average velocity during this interval and the angle θ made by the average velocity with the x-axis.

Ans. $v = 2.92$ m/s, $\theta = -59.0°$

$\underline{2/59}$

$$\underline{V}_{av} = \frac{\Delta r}{\Delta t} = \frac{(2.79i - 3.33j) - (2.76i - 3.28j)}{(3.62 - 3.6)}$$

$$\underline{V}_{av} = \frac{0.03i - 0.05j}{0.02} = 1.5i - 2.5j \ \text{m/s}$$

$$v = |\underline{V}_{av}| = \sqrt{1.5^2 + 2.5^2} = 2.92 \ \text{m/s}$$

$$\tan\theta = \frac{V_y}{V_x} = \frac{-2.5}{1.5} \Rightarrow \theta = -59$$

2/61 The velocity of a particle moving in the x-y plane is given by $6.12\mathbf{i} + 3.24\mathbf{j}$ m/s at time $t = 3.65$ s. Its average acceleration during the next 0.02 s is $4\mathbf{i} + 6\mathbf{j}$ m/s^2. Determine the velocity $\mathbf{v}$ of the particle at $t = 3.67$ s and the angle θ between the average-acceleration vector and the velocity vector at $t = 3.67$ s.

Ans. $\mathbf{v} = 6.20\mathbf{i} + 3.36\mathbf{j}$ m/s, $\theta = 27.9°$

<u>2/61</u>

$$\underline{a} = \frac{\Delta V}{\Delta t}$$

$$4i + 6j = \frac{[\underline{V}_{3.67} - (6.12i + 3.24j)]}{0.02}$$

$$\underline{V} = \underline{V}_{3.67} = 6.2i + 3.36j \text{ m/s}$$

$$\cos\theta = \frac{\underline{V} \cdot \underline{a}}{Va}$$

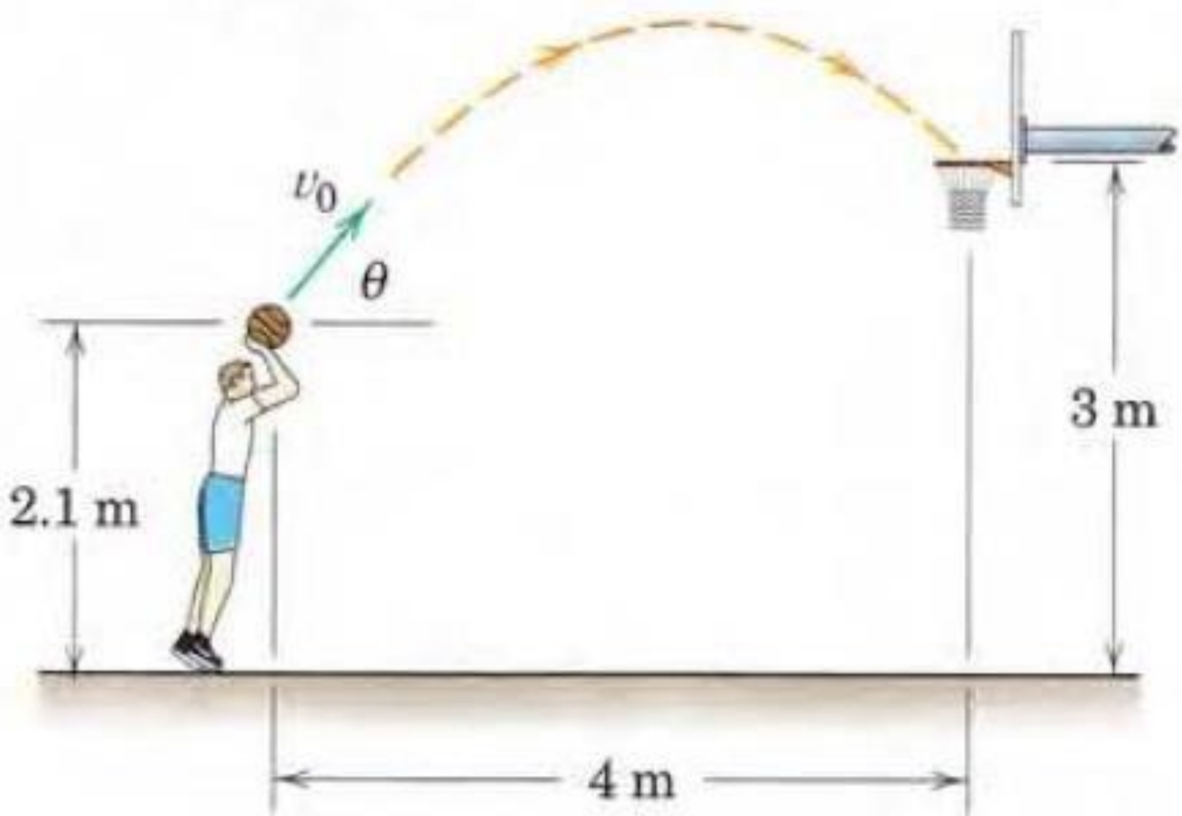
$$V = \sqrt{6.2^2 + 3.36^2} = 7.05 \ m/s$$

$$a = \sqrt{4^2 + 6^2} = 7.21 \ m/s^2$$

$$\cos\theta = \frac{(6.2i + 3.36j) \cdot (4i + 6j)}{7.05\,(7.21)} = 0.884$$

$$\therefore \theta = \cos^{-1} 0.884 = 27.9$$

2/92 The basketball player likes to release his foul shots at an angle $\theta = 50°$ to the horizontal as shown. What initial speed v_0 will cause the ball to pass through the center of the rim?



2/92

$$x = x_o + V_{x_o} t$$

$$4 = 0 + (V_o \cos 50) t$$

$$t = \frac{4}{V_o \cos 50}$$

$$y = y_o + V_{y_o} t - \frac{1}{2} g t^2$$

$$3 = 2.1 + \cancel{V_o} \sin 50 \left(\frac{4}{\cancel{V_o} \cos 50} \right) - \frac{1}{2} (9.81) \left(\frac{4}{V_o \cos 50} \right)$$

$$V_o = 7.893 \text{ m/s}$$

2/63

Use coordinate origin at A

$$y = y_0 + V_{y_0} t - \frac{1}{2} g t^2$$

at B:

$$-4 = 0 + 0 - \frac{1}{2}(9.81) t_B^2 \Rightarrow t_B = 0.903 \text{ s}$$

$$x = x_0 + V_{x_0} t$$

$$6 = 0 + V_{x_0}(0.903) \Rightarrow V_{x_0} = 6.64 \text{ m/s}$$

at C:

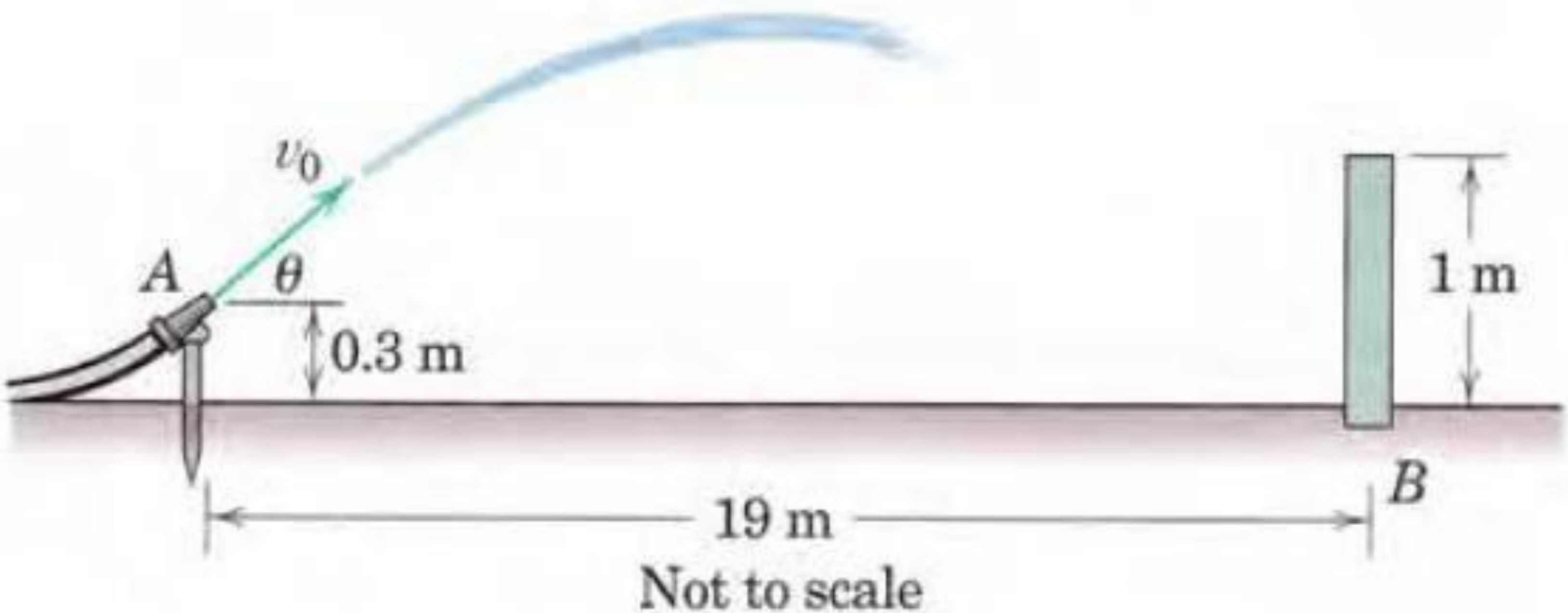
$$y = \cancel{y_0} + \cancel{V_{y_0}} t - \frac{1}{2} g t^2$$

$$-8 = -\frac{1}{2}(9.81) t_c^2 \Rightarrow t_c = 1.277 \text{ S}$$

$$x = \cancel{x_0} + V_0 t$$

$$S + 6 = 6.64(1.277) \Rightarrow S = 2.49 \text{ m}$$

2/78 The water nozzle ejects water at a speed $v_0 = 14$ m/s at the angle $\theta = 40°$. Determine where, relative to the wall base point B, the water lands. Neglect the effects of the thickness of the wall.



2/78

$$V_{x_0} = V_0 \cos\theta$$

$$V_{x_0} = 14 \cos 40 = 10.724 \text{ m/s}$$

$$V_{y_0} = V_0 \sin\theta$$

$$= 14 \sin 40 = 9 \text{ m/s}$$

at B:

$$x = x_o + V_{x_o} t$$

$$19 = 0 + 10.724\, t \Rightarrow t = 1.771\ s$$

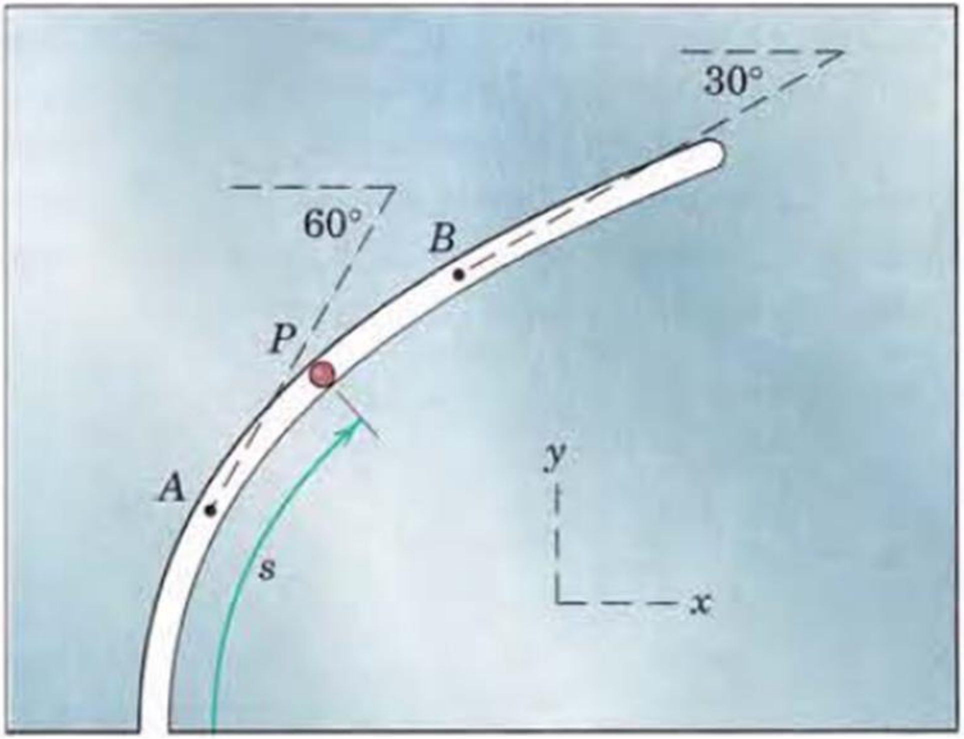
$$y = y_o + V_{y_o} t - \frac{1}{2} g t^2$$

$$y = 0.3 + 9(1.771) - \frac{1}{2}(9.81)(1.771)^2$$

$$y = 0.854\ m$$

So water strikes the wall at 0.854 m above B.

2/64 The particle P moves along the curved slot, a portion of which is shown. Its distance in meters measured along the slot is given by $s = t^2/4$, where t is in seconds. The particle is at A when $t = 2.00$ s and at B when $t = 2.20$ s. Determine the magnitude a_{av} of the average acceleration of P between A and B. Also express the acceleration as a vector $\mathbf{a}_{av}$ using unit vectors $\mathbf{i}$ and $\mathbf{j}$.



2/64

$$V = \dot{S} = \frac{t}{2}$$

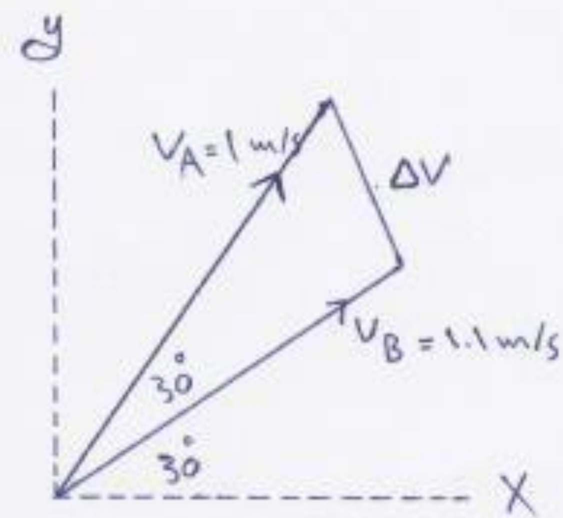
$$\therefore V_A = \frac{2}{2} = 1 \text{ m/s}$$

$$\therefore V_B = \frac{2.2}{2} = 1.1 \text{ m/s}$$

$$\Delta V_x = V_{Bx} - V_{Ax}$$

$$= 1.1 \cos 30 - 1 \cos 60$$

$$= 0.453 \text{ m/s}$$



$$\Delta V_y = V_{By} - V_{Ay}$$

$$= 1.1 \sin 30 - 1 \sin 60$$

$$= -0.316 \ m/s$$

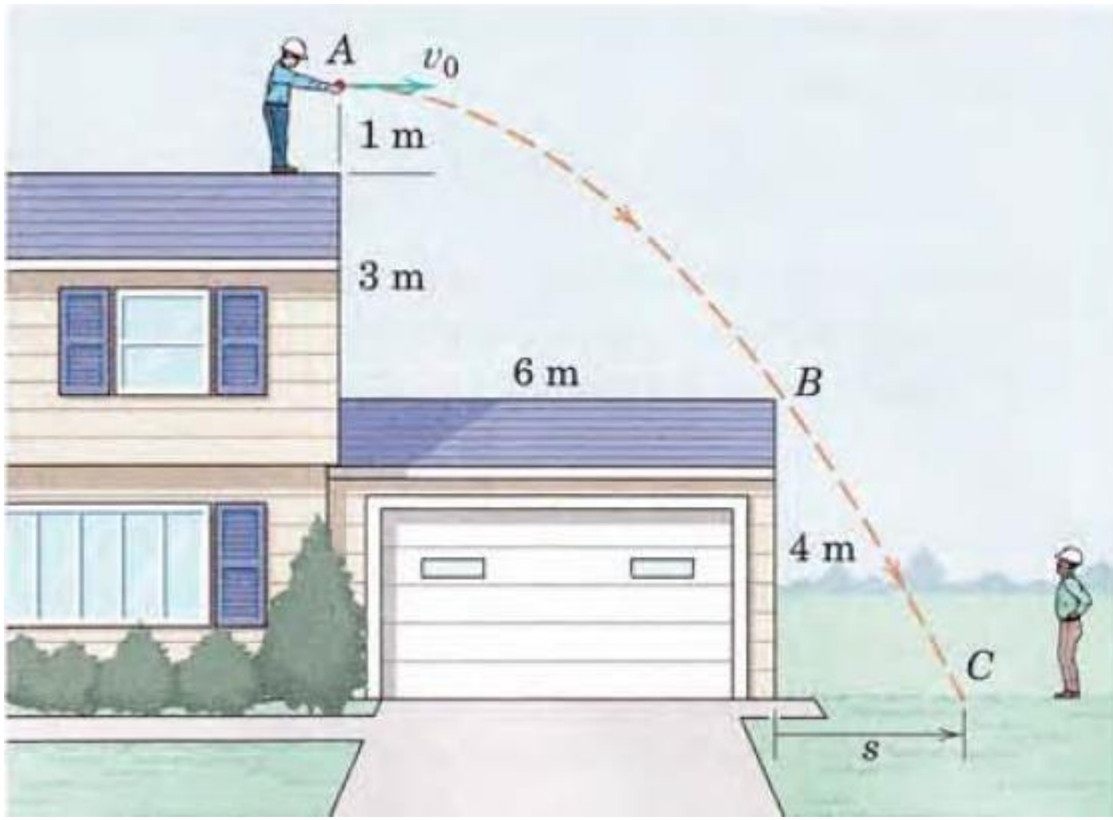
$$\Delta V = \sqrt{0.453^2 + 0.316^2} = 0.552 \ m/s$$

$$a_{av} = \frac{\Delta V}{\Delta t} = \frac{0.552}{0.2} = 2.76 \ m/s^2$$

$$\underline{a}_{av} = \frac{\Delta V}{\Delta t} = \frac{0.453i - 0.316j}{0.2} = 2.26i - 1.58j \ m/s^2$$

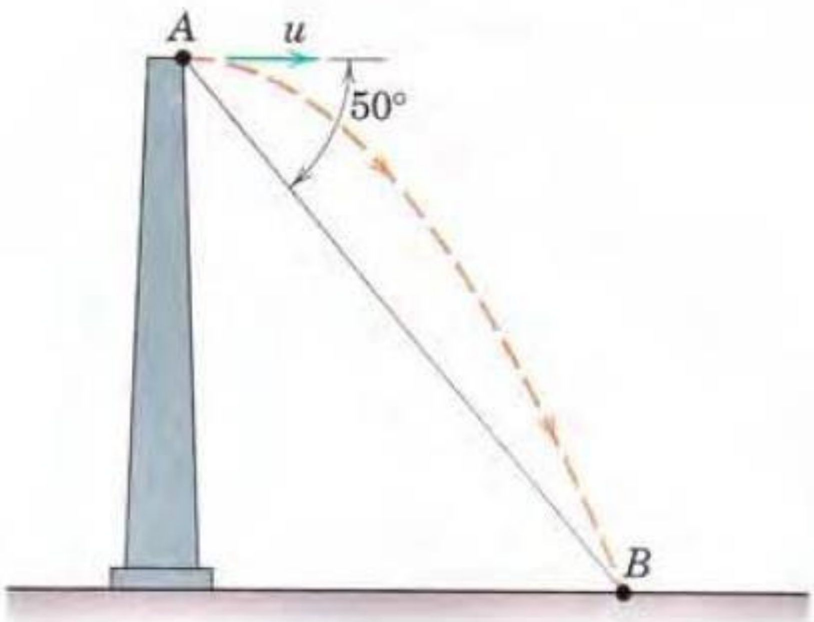
2/63 A roofer tosses a small tool toward a coworker on the ground. What is the minimum horizontal velocity v_0 necessary so that the tool clears point B? Locate the point of impact by specifying the distance s shown in the figure.

Ans. $v_0 = 6.64$ m/s, $s = 2.49$ m



2/81 A rock is thrown horizontally from a tower at A and hits the ground 3.5 s later at B. The line of sight from A to B makes an angle of 50° with the horizontal. Compute the magnitude of the initial velocity $\mathbf{u}$ of the rock.

Ans. $u = 14.41$ m/s



2/81

$$y = \cancel{y_0} + \cancel{V_{y_0}} t - \frac{1}{2} g t^2$$

$$\therefore -y = -\frac{1}{2} g t^2$$

$$y = \frac{1}{2} g t^2$$

$$x = \cancel{x_0} + Ut$$

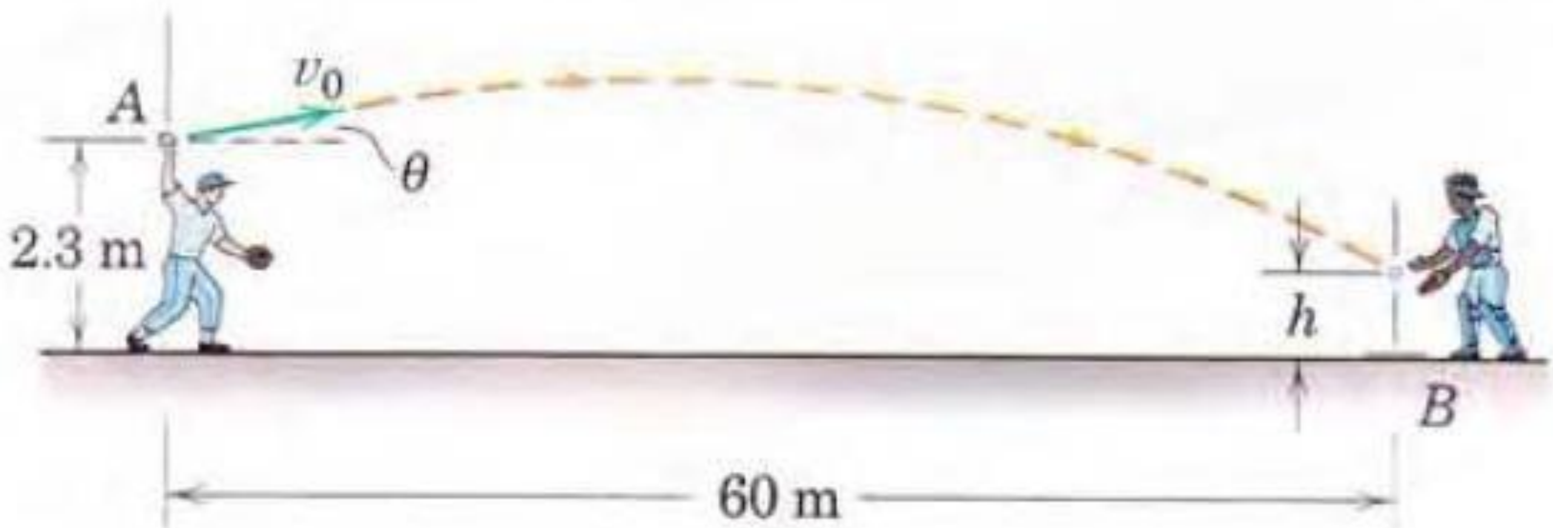
$$\therefore x = Ut$$

$$\tan\theta = \frac{y}{x}$$

$$\tan 50 = \frac{g t^2}{2 U t}$$

$$U = \frac{g t}{2 \tan 50} = \frac{9.81(3.5)}{2 \tan 50} = 14.41 \text{ m/s}$$

2/82 An outfielder experiments with two different trajectories for throwing to home plate from the position shown: (*a*) $v_0 = 42$ m/s with $\theta = 8°$ and (*b*) $v_0 = 36$ m/s with $\theta = 12°$. For each set of initial conditions, determine the time t required for the baseball to reach home plate and the altitude h as the ball crosses the plate.



2/82

(a) $V_0 = 42 \text{ m/s}$, $\theta = 8^\circ$

$$X = X_0 + V_{x_0} t$$

at B:

$$60 = 0 + (42 \cos 8) t \Rightarrow t = 1.442 \text{ s}$$

$$y = y_0 + V_{y_0} t - \frac{1}{2} g t^2$$

$$h = 2.3 + (42 \sin 8)(1.442) - \frac{1}{2}(9.81)(1.442)^2$$

$$h = 0.529 \text{ m}$$

(b) $V_0 = 36\ m/s$, $\theta = 12^\circ$

$$X = X_0 + V_{x_0}\, t$$

$$60 = 0 + (36 \cos 12)\, t \Rightarrow t = 1.704\ s$$

$$y = y_0 + V_{y_0}\, t - \frac{1}{2} g t^2$$

$$h = 2.3 + (36 \sin 12)(1.704) - \frac{1}{2}(9.81)(1.704)^2$$

$$h = 0.811\ m$$