Chapter 5

Pressure

5.1 Pressure and Its Units

Pressure is defined as "the normal (perpendicular) **force** per unit **area** (Figure 5.1). The pressure at the bottom of the static (nonmoving) column of mercury exerted on the sealing plate is

$$p = \frac{F}{A} = \rho g h + p_0 \quad \dots 5.1$$

Where p = pressure at the bottom of the column of the fluid, F = force, A = area, ρ = density of fluid g = acceleration of gravity, h = height of the fluid column, and p_0 = pressure at the top of the column of fluid

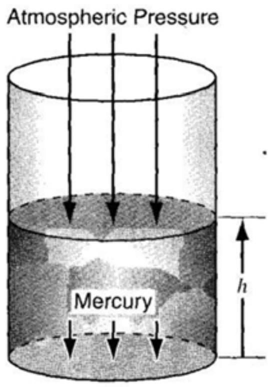


Figure 5.1 Pressure is the normal force per unit area. Arrows show the force exerted on the respective areas

For Example, suppose that the cylinder of fluid in Figure 5.1 is a column of mercury that has an area of 1 cm^2 and is 50 cm high. The density of the Hg is 13.55 g/cm^3. Thus, the force exerted by the mercury alone on the 1 cm^2 section of the bottom plate by the column of mercury is

$$F = \frac{13.55\ g}{cm^3}\left|\frac{980\ cm}{s^2}\right|\frac{50\ cm}{}\left|\frac{1\ cm^2}{}\right|\frac{1\ kg}{1000\ g}\left|\frac{1\ m}{100\ cm}\right|\frac{1(N)(s^2)}{1(kg)(m)}$$

$$= 6.64\ N$$

P=F/A …. F= P* A = ρ g h*A

The pressure on the section of the plate covered by the mercury is the force per unit area of the mercury plus the pressure of the atmosphere

$$p = \frac{6.64\ N}{1\ cm^2}\left|\left(\frac{100\ cm}{1\ m}\right)^2\right|\frac{(1\ m^2)(1\ Pa)}{(1\ N)}\left|\frac{1\ kPa}{1000\ Pa.}\right. + p_0 = 66.4\ kPa + p_0$$

If we had started with units in the AE system, the pressure would be computed as [the density of mercury is 845.5 lb_m/ft^3]

$$p = \frac{845.5\ lb_m}{1\ ft^3}\left|\frac{32.2\ ft}{s^2}\right|\frac{50\ cm}{}\left|\frac{1\ in.}{2.54\ cm}\right|\frac{1 ft}{12\ in.}\left|\frac{(s)^2(lb_f)}{32.174(ft)(lb_m)}\right| + p_0$$

$$= 1388\frac{lb_f}{ft^2} + p_0$$

5.2 Measurement of Pressure

Pressure, like temperature, can be expressed using either an **absolute** or a **relative scale**.

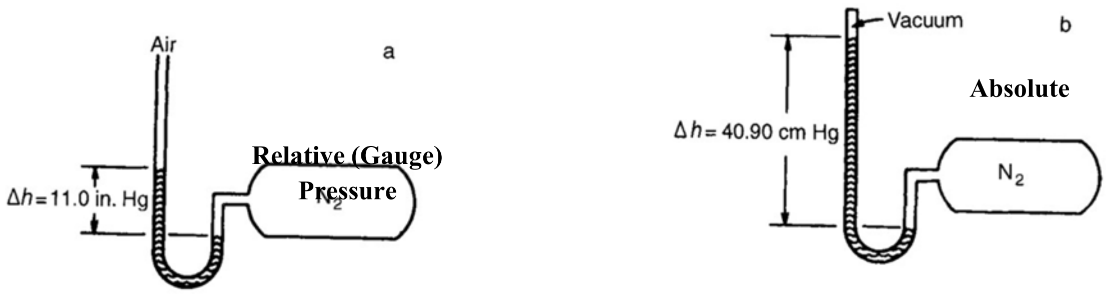


Figure 5.2 (a) **Open-end manometer** showing a pressure above atmospheric pressure. (b) **Manometer** measuring an **absolute pressure**.

The relationship between **relative** and **absolute pressure** is given by the following expression:

Gauge Pressure + Barometer Pressure (atmospheric)= Absolute Pressure …5.2

$$P_{vacuum} = p_{atmospheric} - p_{absolute}$$

☒ The **standard atmosphere** is defined as the pressure (in a standard gravitational field) equivalent to 1 atm or 760 mm Hg at 0°C or other equivalent.

The **standard atmosphere** is equal to

- 1.00 atmospheres (atm)
- 33.91 feet of water (ft H_2O)
- 14.7 pounds (force) per square inch absolute (psia)

- 29.92 inches of mercury (in. Hg)
- 760.0 millimeters of mercury (mm Hg)
- $1.013 * 10^5$ pascal (Pa) or newtons per square meter (N/m^2); or 101.3 kPa

For Example, convert 35 psia to inches of mercury and kPa.

$$\frac{35 \text{ psia}}{} \left| \frac{29.92 \text{ in. Hg}}{14.7 \text{ psia}} \right. = 71.24 \text{ in Hg}$$

And,

$$\frac{35 \text{ psia}}{} \left| \frac{101.3 \text{ kPa}}{14.7 \text{ psia}} \right. = 241 \text{ kPa}$$

<u>For Example</u>, What is the equivalent **pressure** to **1 kg/cm^2** (i.e., **kg$_f$/cm^2**) in pascal (g = 9.8 m/s^2)
[1 kg/cm^2] * [9.8 m/s^2] * [(100 cm/1 m)^2] = 9.8 * 10^4 N/m^2 (or Pa)

<u>Example 5.1</u>

What is the equivalent pressure to 60 Gpa (gigapascal) in

(a) atmospheres (b) psia (c) inches of Hg (d) mm of Hg

Solution

Basis: 60 GPa

(a) $$\frac{60\ \text{GPa}}{} \left| \frac{10^6\ \text{kPa}}{1\ \text{GPa}} \right| \frac{1\ \text{atm}}{101.3\ \text{kPa}} = 0.59 \times 10^6\ \text{atm}$$

(b) $$\frac{60\ \text{GPa}}{} \left| \frac{10^6\ \text{kPa}}{1\ \text{GPa}} \right| \frac{14.696\ \text{psia}}{101.3\ \text{kPa}} = 8.70 \times 10^6\ \text{psia}$$

(c) $$\frac{60\ \text{GPa}}{} \left| \frac{10^6\ \text{kPa}}{1\ \text{GPa}} \right| \frac{29.92\ \text{in. Hg}}{101.3\ \text{kPa}} = 1.77 \times 10^7\ \text{in. Hg}$$

(d) $$\frac{60\ \text{GPa}}{} \left| \frac{10^6\ \text{kPa}}{1\ \text{GPa}} \right| \frac{760\ \text{mm Hg}}{101.3\ \text{kPa}} = 4.50 \times 10^8\ \text{mm Hg}$$

<u>Example 5.2</u>

The pressure gauge on a tank of CO_2 used to fill soda-water bottles reads 51.0 psi. At the same time the barometer reads 28.0 in. Hg. What is the absolute pressure in the tank in psia? See Figure E5.2.

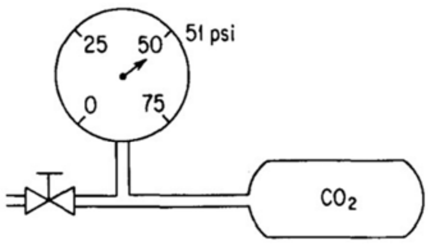


Figure E5.2

Solution

$$\text{Atmospheric pressure} = \frac{28.0\ \text{in. Hg}}{} \left| \frac{14.7\ \text{psia}}{29.92\ \text{in Hg}} \right. = 13.76\ \text{psia}$$

The absolute pressure in the tank is

51.0 psia + 13.76 psia = 64.8 psia

Example 5.3

Small animals such as mice can live (although not comfortably) at reduced air pressures down to 20 kPa absolute. In a test, a mercury manometer attached to a tank, as shown in Figure E5.3, reads 64.5 cm Hg and the barometer reads 100 kPa. Will the mice survive?

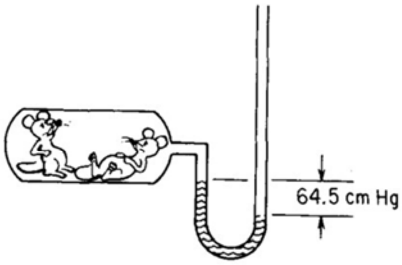


Figure E5.3

Solution

You are expected to realize from the figure that the tank is **below atmospheric pressure** because the **left leg** of the **manometer is higher** than the **right leg**, which is open to the atmosphere. Consequently, to get the **absolute pressure** you **subtract** the **64.5 cm Hg** from the **barometer reading**.

The **absolute pressure** in the tank is

$$100 \text{ kPa} - \frac{64.5 \text{ cm Hg}}{} \left| \frac{101.3 \text{ kPa}}{76.0 \text{ cm Hg}} \right. = 100 - 86 = 14 \text{ kPa absolute}$$

The mice probably will **not survive**.

5.3 Differential Pressure Measurements

When the columns of fluids are at equilibrium (see Figure 5.3), the relationship among ρ_1, ρ_2, ρ_3, and the heights of the various columns of fluid is as follows:

$$\mathbf{P_1 + \rho_1 d_1 g = P_2 + \rho_2 d_2 g + \rho_3 d_3 \; g} \qquad \ldots 5.3$$

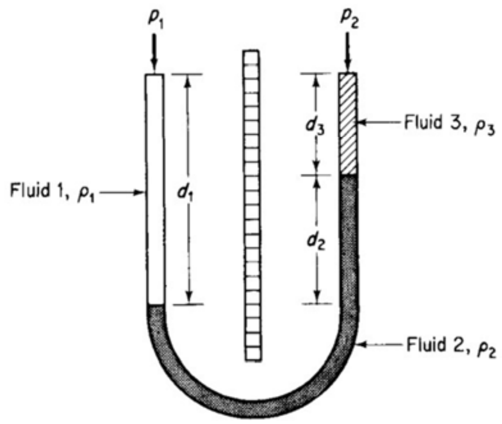


Figure 5.3 Manometer with three fluids.

Note

If fluids 1 and 3 are **gases**, and fluid 2 is **mercury**, the density of the **gas** is **so much less** than that of **mercury** that you can **ignore** the term involving the gas in Equation (5.3) for practical applications.

- ✶ Can you show for the case in which $\rho_1 = \rho_3 = \rho$ that the manometer expression reduces to the differential manometer equation:

$$P_1 - P_2 = (\rho_2 - \rho)\, g\, d_2 \qquad \dots 5.4$$

Example 5.4

In measuring the flow of fluid in a pipeline as shown in Figure E5.4, a differential manometer was used to determine the pressure difference across the orifice plate. The flow rate was to be calibrated with the observed pressure drop (difference). Calculate the **pressure drop p_1 - p_2** in pascals for the manometer reading in Figure E5.4.

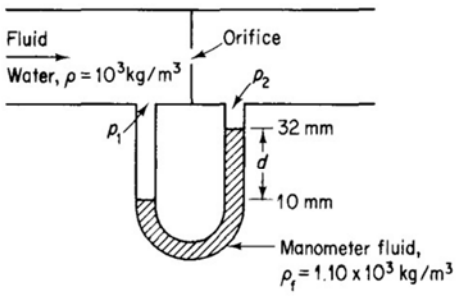


Figure E5.4

Solution

In this problem you **cannot ignore the water density** above the manometer fluid.

$$p_1 - p_2 = (\rho_f - \rho) g d$$

$$= \frac{(1.10 - 1.00)10^3\ \text{kg}}{\text{m}^3}\left|\frac{9.807\ \text{m}}{\text{s}^2}\right|\frac{(22)(10^{-3})\text{m}}{}\left|\frac{1(\text{N})(\text{s}^2)}{(\text{kg})(\text{m})}\right|\frac{1(\text{Pa})(\text{m}^2)}{1(\text{N})}$$

$$= 21.6\ \text{Pa}$$

Example 5.5

Air is flowing through a duct **under** a draft of 4.0 cm H_2O. The barometer indicates that the atmospheric pressure is 730 mm Hg. What is the absolute pressure of the air in inches of mercury? See Figure E5.5

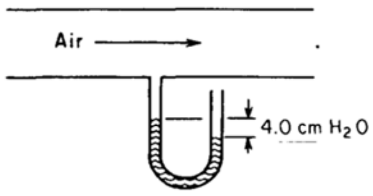


Figure E5.5

Solution

In this problem you **can ignore the gas density** above the manometer fluid and the air above the open end of the manometer.

$$\text{Atmospheric pressure} = \frac{730 \text{ mm Hg}}{} \left| \frac{29.92 \text{ in. Hg}}{760 \text{ mm Hg}} \right. = 28.7 \text{ in. Hg}$$

Next, convert 4.0 cm H_2O to in. Hg:

$$\frac{4.0 \text{ cm } H_2O}{} \left| \frac{1 \text{ in.}}{2.54 \text{ cm}} \right| \frac{1 \text{ ft}}{12 \text{ in.}} \left| \frac{29.92 \text{ in. Hg}}{33.91 \text{ ft } H_2O} \right. = 0.12 \text{ in. Hg}$$

Since the reading is 4.0 cm H_2O draft (**under atmospheric**), the absolute reading in uniform units is

$$28.7 \text{ in. Hg} - 0.12 \text{ in. Hg} = 28.6 \text{ in. Hg absolute}$$

Questions

1. Figure SAT5.1Q2 shows four closed containers completely filled with water. Order the containers from the one exerting the highest pressure to the lowest on their respective base.

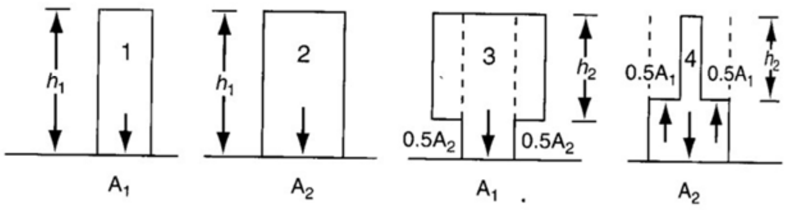


Figure SAT5.1Q2

2. Answer the following questions true or false:
   a. Atmospheric pressure is the pressure of the air surrounding us and changes from day to day
   b. The standard atmosphere is a constant reference atmosphere equal to 1.000 atm or the equivalent pressure in other units.
   c. Absolute pressure is measured relative to a vacuum.
   d. Gauge pressure is measured upward relative to atmospheric pressure.
   e. Vacuum and draft pressures are measured downward from atmospheric pressure.

f. You can convert from one type of pressure measurement to another using the standard atmosphere.
g. A manometer measures the pressure difference in terms of the height of fluid (s) in the manometer tube.

3. What is the equation to convert gauge pressure to absolute pressure?
4. What are the values and units of the standard atmosphere for six different methods of expressing pressure?
5. What is the equation to convert vacuum pressure to absolute pressure?

Answers:

1. 3 is the highest pressure; next are 1 and 2, which are the same; and 4 is last. The decisions are made by dividing the weight of water by the base area.
2. All are true
3. Gauge pressure + barometric pressure = absolute pressure
4. See lectures
5. Barometric pressure - vacuum pressure = absolute pressure

Problems

1. Convert a pressure of 800 mm Hg to the following units:

   a. psia b. kPa c. atm d. ft H_2O

2. Your textbook lists five types of pressures: atmospheric pressure, barometric pressure, gauge pressure, absolute pressure, and vacuum pressure.

   a. What kind of pressure is measured by the device in Figure SAT5.2P2A?

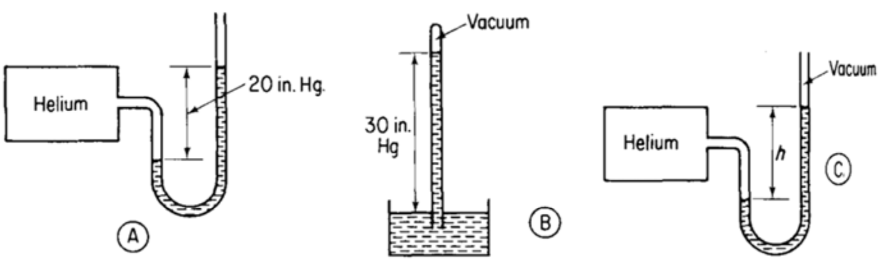


Figure SAT5.2P2A

   b. What kind of pressure is measured by the device in Figure SAT5.2P2B?

   c. What would be the reading in Figure SAT5.2P2C assuming that the pressure and temperature inside and outside the helium tank are the same as in parts (a) and (b)?

3. An evaporator shows a reading of 40 kPa vacuum. What is the absolute pressure in the evaporator in kPa?
4. A U-tube manometer filled with mercury is connected between two points in a pipeline. If the manometer reading is 26 mm of Hg, calculate the pressure difference in kPa between the

points when (a) water is flowing through the pipeline, and (b) also when air at atmospheric pressure and 20°C with a density of 1.20 kg/m^3 is flowing in the pipeline.

5. A Bourdon gauge and a mercury manometer are connected to a tank of gas, as shown in Figure SAT5.3P2. If the reading on the pressure gauge is 85 kPa, what is h in centimeters of Hg?

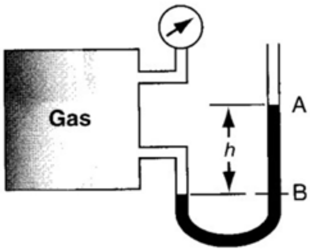


Figure SAT5.3P2

Answers:

1. (a) 15.5; (b) 106.6; (c) 1.052; (d) 35.6
2. (A) Gauge pressure; (B) barometric pressure, absolute pressure; (C) 50 in. Hg
3. In the absence of a barometric pressure value, assume 101.3 kPa. The absolute pressure is 61.3 kPa.
4. The Hg is static. (a) 3.21 kPa; (b) 3.47 kPa
5. 63.8 cm Hg

Supplementary Problems (Chapter Five):

Problem 1

A solvent storage tank, 15.0 m high contains liquid styrene (sp. gr. 0.909). A pressure gauge is fixed at the base of the tank to be used to determine the level of styrene.

a. Determine the gage pressure when the tank is full of styrene.
b. If the tank is to be used for storage of liquid hexane (sp. gr. 0.659), will the same pressure gage calibration be adequate ? What is the risk in using the same calibration to determine the level of hexane in the tank.
c. What will be the new pressure with hexane to indicate that the tank is full.

Solution

a. The liquid in full tank will exert a gage pressure at the bottom equal to 15.0 m of styrene. The tank has to operate with atmospheric pressure on it and in it, or it will break on expansion at high pressure or collapse at lower pressure.

$$p = h\,\rho\,g$$

$$= 15.0\text{ m}\;\frac{0.909\text{ g styrene/cm}^3}{1.0\text{ g }H_2O\text{/cm}^3}\left|\frac{1.0\text{ g }H_2O\text{/cm}^3}{}\right|\frac{10^3\text{ kg/m}^3}{1\text{ g/cm}^3}\left|\frac{9.80\text{ m/s}^2}{}\right|\frac{1\text{ Pa}}{1\text{ (kg)(m)}^{-1}\text{(s)}^{-2}}$$

$$= 134 \times 10^3\text{ Pa} = \mathbf{134\ kPa\ gage}$$

b. Hexane is a liquid of specific gravity lower than that of styrene; therefore a tank full of hexane would exert a proportionally lower pressure. If the same calibration is used the tank may overflow while the pressure gage was indicating only a partially full tank.

c. New $p = h \rho g$

$$= 15.0 \text{ m} \; \frac{0.659 \text{ g hexane/cm}^3}{1.0.0 \text{ g H}_2\text{O/cm}^3} \left| \frac{1.0 \text{ g H}_2\text{O/cm}^3}{} \right| \frac{10^3 \text{ kg/m}^3}{} \left| \frac{9.8 \text{ m/s}^2}{} \right| \frac{1 \text{ Pa}}{1(\text{kg})(\text{m})^{-1}(\text{s})^{-2}}$$

$$= 96900 \text{ Pa} = \mathbf{96.9\ kPa}$$

Problem 2

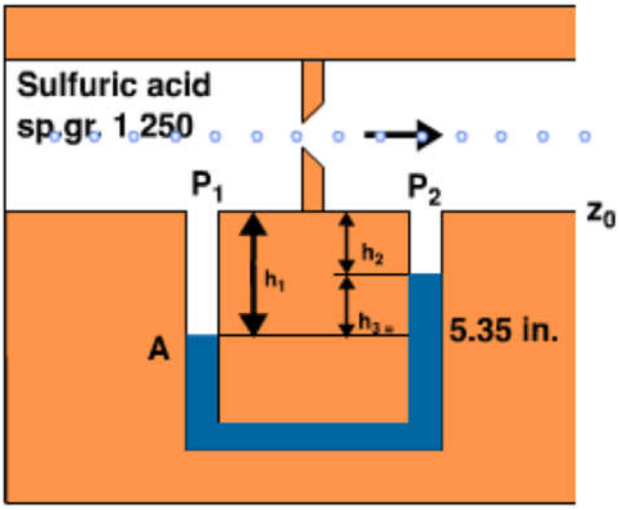
A U-tube manometer is used to determine the pressure drop across an orifice meter. The liquid flowing in the pipe line is a sulfuric acid solution having a specific gravity (60°/60°) of 1.250. The manometer liquid is mercury, with a specific gravity (60°/60°) of 13.56. The manometer reading is 5.35 inches, and all parts of the system are at a temperature of 60°F. What is the pressure drop across the orifice meter in psi.

Solution

First we calculate density of acid and mercury.

$$\rho_{acid} = \frac{1.250}{} \left| \frac{62.4 \text{ lb/ft}^3}{} \right| \frac{1 \text{ ft}^3}{1.728 \times 10^3 \text{ in}^3} = 0.0451 \text{ lb/in}^3$$

$$\rho_{Hg} = \frac{13.56}{} \left| \frac{62.4 \text{ lb/ft}^3}{} \right| \frac{1 \text{ ft}^3}{1.728 \times 10^3 \text{ in}^3} = 0.490 \text{ lb/in}^3$$


Sulfuric acid
sp.gr. 1.250
P_1
P_2
z_0
h_1
h_2
h_3 =
5.35 in.
A

left column *right column*

$$\text{At } z_o \quad p_1 + \rho_a h_1 g = p_2 + \rho_a h_2 g + \rho_{Hg} h_3 g$$

$$p_1 - p_2 + \rho_a (h_1 - h_2) g = \rho_{Hg} h_3 g$$

$$p_1 - p_2 + \rho_a h_3 g = \rho_{Hg} h_3 g$$

$$p_1 - p_2 = (\rho_{Hg} - \rho_a) h_3 g$$

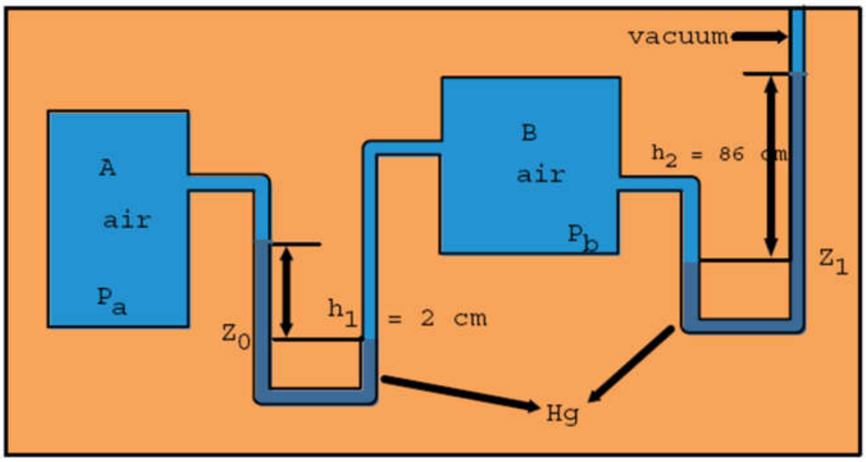
$$p_1 - p_2 = \frac{(0.490 - 0.0451)\, lb_f}{in^2} \Big| (5.35)\, in \Big| \frac{32.2\, ft/s^2}{32.174\, (ft)(lb_m)/(s^2)(lb_f)} = \mathbf{2.38\, lb_f/in^2\, (psi)}$$

Problem 3

The pressure difference between two air tanks A and B is measured by a U - tube manometer, with mercury as the manometer liquid. The barometric pressure is 700 mm Hg.

a. What is the absolute pressure in the tank A ?
b. What is the gauge pressure in the tank A ?

Solution



a. At Z_0 $\quad p_a + h_1 \rho_{Hg} g = p_b$ (neglecting the effect of air in the U - tube) (1)

at Z_1 $\quad p_b = h_2 \rho_{Hg} g$ (2)

Eliminate p_b from the equations

$$p_a + h_1 \rho_{Hg} g = h_2 \rho_{Hg} g$$

$$p_a = (h_2 - h_1) \rho_{Hg} g$$

= 840 mm Hg absolute

The pressure measured by this manometer system is the absolute pressure because the reference (pressure above the mercury) in the vertical tube is a vacuum.

b. $p_a = 840 - 700 = 140$ mm Hg

<u>Chapter 6</u>

Introduction to Material Balances