Al-Mustaqbal University

Department of Medical Instrumentation Techniques Engineering

Mathematics I

LECTURE 3

Integration By Parts

Lect. Dr. Ammar Nomi AL jenabi

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Integration By Parts

Integration by parts is the most useful integration technique for evaluating the integral of a product of functions. After separating a single function into a product of two functions, we can easily evaluate the function's integral by applying the integration by parts formula:

$$\int \boldsymbol{u\,dv = uv - \int v\,du}$$

In this formula, (du) represents the derivative of (u) , while (v) represents the integral of (dv). *The integral of the product of u and v is then evaluated in terms of the integral of the product of du and v. It's often much easier to solve the integral that results from this formula than it is to directly evaluate the original single function. Integration by parts gives us an efficient way to integrate functions that would otherwise be very difficult to integrate, and it's particularly helpful in tricky cases such as inverse trigonometric functions and logarithmic functions.*

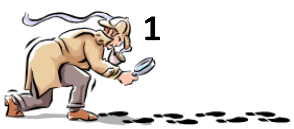
The some simple steps for how to integrate by parts:

1- Choose u and dv to separate the integrand into a product of functions.
2- Differentiate u to find du, and integrate dv to find v

3- $plug\ u\ ,v\ ,du\ ,and\ \ dv\ into\ the\ integration\ by\ parts\ formula$

$$\int u\,dv = uv - \int v\,du$$

4- Solve, and simplify where needed.



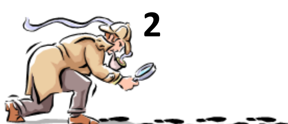
1

Evaluate $I = \int xe^{x}dx$

Choose $u = x$ **and** $dv = e^{x}\,dx$**, then** $du = dx$ **and** $v = e^{x}$**.**

$$\int xe^{x}\,dx = xe^{x} - \int e^{x}\,dx$$

$$= xe^{x} - e^{x} + C$$



2

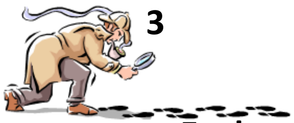
Evaluate $I = \int \ln(x)\,dx$

Solution :-

$u = \ln(x) \qquad dv = dx$

$du = \dfrac{dx}{x} \qquad v = x$

$$\Rightarrow I = x\ln(x) - \int x\frac{1}{x}dx = x\ln(x) - \int dx = x\ln(x) - x + c$$



3

Evaluate $I = \int \tan^{-1}(x)dx$

Solution :-

$$u = \tan^{-1}(x) \qquad dv = dx$$

$$du = \frac{dx}{1+x^2} \qquad v = x$$

$$\Rightarrow \quad I = x\tan^{-1}(x) - \int \frac{xdx}{1+x^2} = x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2) + c$$

Evaluate $\int x \ \sin x \ dx$

$$u = x \qquad dv = \sin(x)\, dx$$

$$du = 1\, dx \qquad v = -\cos x$$

$$\int x \ \sin x \ dx = -x\cos(x) - \int -\cos(x)dx$$

$$= -\cos(x) + \sin(x) + c$$

Evaluate $\int x^2 \sin x \; dx$

$$let \quad u = x^2 \quad and \qquad dv = \sin(x)\, dx$$

$$du = 2xdx \quad , \qquad v = -\cos(x)$$

$$\int x^2 \sin(x) dx = x^2 \, (-\cos(x)) - \int (-\cos(x)) \, 2x \, dx$$

$$= -x^2 \cos(x) + 2 \int x \cos(x) dx$$

Sometimes, you'll have to integrate by parts more than once in the same problem. In this case, we'll need to use it again to evaluate $\int x \cos(x) dx$

Let u = x and **dv = cos (x) dx**

du = 1 dx **v= sin (x)**

$$\int x \cos(x) dx = x \sin(x) - \int \sin(x) dx$$
$$= x \sin(x) - (-\cos(x))$$
$$= x \sin(x) + \cos(x) + c$$

We can use substitution to plug this value into our original integral formula to finish the problem.

$$\int x^2 \sin(x) dx = x^2 \cos(x) + 2\, x \sin(x) + 2 \cos(x) + c$$