Medical Physics

The First Stage

Second Term –Fifth Lecture

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Thermal Expansion

Thermal Expansion of Solids and Liquids

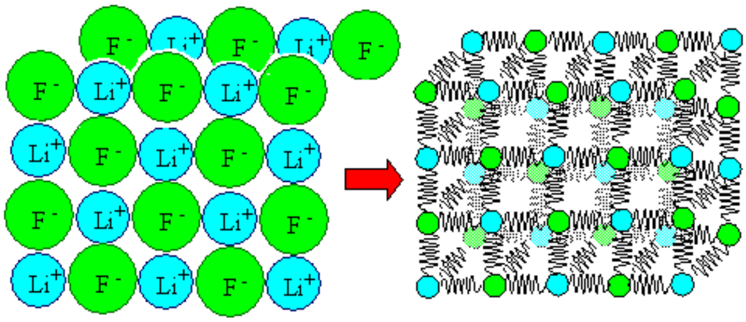
Our discussion of the liquid thermometer makes use of one of the best-known changes in a substance: as its temperature increases, its volume increases. This phenomenon, known as *thermal expansion*, plays an important role in numerous engineering applications. For example, thermal-expansion joints such as those shown in next figure must be included in buildings, concrete highways, railroad tracks, also have important role in brick walls, and bridges to compensate for dimensional changes that occur as the temperature changes.

Without these joints to separate sections of roadway on bridges, the surface would buckle due to thermal expansion on very hot days or crack due to contraction on very cold days.

The long, vertical joint is filled with a soft material that allows the wall to expand and contract as the temperature of the bricks changes.

Thermal expansion is a consequence of the change in the average separation between the atoms in an object.

To understand this concept, let's model the atoms as being connected by stiff springs as shown in adjacent figure.



At ordinary temperatures, the atoms in a solid oscillate about their equilibrium positions with an amplitude of approximately 10^{-11} m and a frequency of approximately 10^{13} Hz. The average spacing between the atoms is about 10^{-10} m. As the temperature of the solid increases, the atoms oscillate with greater amplitudes; as a result, the average separation between them increases. Consequently, the object expands.

<u>Thermal expansion for pharmaceutical industry and drugs forms</u>

Thermal expansion may also play a significant role in the computation of more fundamental properties. The pharmaceutical industry is increasingly looking toward computational methods to assist in drug development.

The aim is to shorten the time needed for establishing the physical properties of the crystalline forms that are relevant to designing the formulation and manufacturing process. The thermal expansion properties are very specific to the polymorph as well as the molecule, and the difference in the anisotropy can be significant.

The thermal expansion of the chosen solid form of the active pharmaceutical ingredient (API) can affect whether the formulated pills maintain their integrity over the wide range of temperatures involved in manufacture, storage, and transport throughout the world.

So, it is important to measure the thermal expansion of pharmaceutical crystals to assess what approximations in computational treatment of thermal expansion are appropriate for various applications.

The Unusual Behavior of Water

Liquids generally increase in volume with increasing temperature and have average coefficients of volume expansion about ten times greater than those of solids. Cold water is an exception to this rule as you can see from its density-versus-temperature curve shown in next figure.

As the temperature increases from 0ºC to 4ºC, water contracts and its density therefore increases. Above 4ºC, water expands with increasing temperature and so its density decreases. Therefore, the density of water reaches a maximum value of 1.000 g/cm^3 at 4ºC.

We can use this unusual thermal-expansion behavior of water to explain why a pond begins freezing at the surface rather than at the bottom. When the air temperature drops from, for example, 7ºC to 6ºC, the surface water also cools and consequently decreases in volume.

The surface water is denser than the water below it, which has not cooled and decreased in volume. As a result, the surface water sinks, and warmer water from below moves to the surface. When the air temperature is between 4ºC and 0ºC, however, the surface water expands as it cools, becoming less dense than the water below it. The mixing process stops, and eventually the surface water freezes. As the water freezes, the ice remains on the surface because ice is less dense than water.

Macroscopic Description of an Ideal Gas

The change in volume expansion equation $\Delta \boldsymbol{V} = \boldsymbol{\beta} \boldsymbol{V}_{\boldsymbol{i}} \Delta \boldsymbol{T}$ “were $\boldsymbol{\beta}$ is the average coefficient of volume expansion, $\boldsymbol{V}_{\boldsymbol{i}}$ is the initial volume” is based on the assumption that the material has an initial volume V_i before the temperature change occurs. Such is the case for solids and liquids because they have a fixed volume at a given temperature.

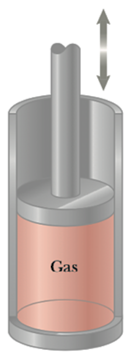
The case for gases is completely different. The interatomic forces within gases are very weak, and, in many cases, we can imagine these forces to be non-existent and still make very good approximations.

For a gas, it is useful to know how the quantities volume V, pressure P, and temperature T are related for a sample of gas of mass m. In general, the equation that interrelates these quantities, called the equation of state, is very complicated. If the gas is maintained at a very low pressure (or low density), however, the equation of state is quite simple and can be determined from experimental results. Such a low-density gas is commonly referred to as an ideal gas. We can use the ideal gas model to make predictions that are adequate to describe the behavior of real gases at low pressures.

It is convenient to express the amount of gas in a given volume in terms of the number of moles n. One mole of any substance is that amount of the substance that contains *Avogadro's number* $N_A = 6.022x10^{23}$ of constituent particles (atoms or molecules). The number of moles n of a substance is related to its mass m through the expression $n = \frac{m}{M}$ "where M is the molar mass of the substance."

The molar mass of each chemical element is the atomic mass expressed in grams per mole.

Now suppose an ideal gas is confined to a cylindrical container whose volume can be varied by means of a movable piston as in adjacent figure. If we assume the cylinder does not leak, the mass (or the number of moles) of the gas remains constant. For such a system, experiments provide the following information:



- When the gas is kept at a constant temperature, its pressure is inversely proportional to the volume. (This behavior is described historically as Boyle's law.)
- When the pressure of the gas is kept constant, the volume is directly proportional to the temperature. (This behavior is described historically as Charles's law.)
- When the volume of the gas is kept constant, the pressure is directly proportional to the temperature. (This behavior is described historically as Gay– Lussac's law.)

These observations are summarized by the equation of state for an ideal gas:

$$PV = nRT$$

In this expression, also known as the ideal gas law, ***n*** is the number of moles of gas in the sample and ***R*** is a constant. Experiments on numerous gases show that as the pressure approaches zero, the quantity ***PV/nT*** approaches the same value ***R*** for all gases. For this reason, R is called the universal gas constant.

In SI units, in which pressure is expressed in pascals (1 Pa = 1 N/m^2) and volume in cubic meters, the product ***PV*** has units of newton. meters, or joules, and ***R*** has the value (R = 8.314 J/mol. K).

The ideal *gas law* states that if the volume and temperature of a fixed amount of gas do not change, the pressure also remains constant.

The ideal gas law is often expressed in terms of the total number of molecules ***N***. Because the number of moles ***n*** equals the ratio of the total number of molecules and *Avogadro's number* N_A, we can write:

$$PV = nRT = \frac{N}{N_A} RT = N k_B T$$

where $\boldsymbol{k_B}$ is **Boltzmann's constant**, which has the value:

$$k_B = \frac{R}{N_A} = 1.38 \times 10^{-23} J/K$$

It is common to call quantities such as ***P, V***, and ***T*** the thermodynamic variables of an ideal gas. If the equation of state is known, one of the variables can always be expressed as some function of the other two.

Quick Quiz:

(Q1) What is the power radiated per square centimeter from skin at a temperature of 306 K (33°C)?

Answer: W = $\sigma e T^4$ = (5.7 $\times 10^{-12}$) $(306)^4$ =0.05 W/cm^2

(Q2) What is the power radiated from a nude body 1.75m^2 (1.75$\times$ 10^4 cm^2) in area?

Answer: W = $\sigma e T^4$ = (0.05) (1.75$\times$ 10^4 cm^2) =875W.

(Q3) A common material for cushioning objects in packages is made by trapping bubbles of air between sheets of plastic. Is this material more effective at keeping the contents of the package from moving around inside the package on (a) a hot day, <u>(b) a cold day</u>, or (c) either hot or cold days?

(Q4) On a winter day, you turn on your furnace and the temperature of the air inside your home increases. Assume your home has the normal amount of leakage between inside air and outside air. Is the number of moles of air in your room at the higher temperature (a) larger than before, (b) smaller than before, or <u>(c) the same as before</u>?

(Q5) A spray can containing a propellant gas at twice atmospheric pressure (202 kPa) and having a volume of 125cm^3 is at 22°C. It is then tossed into an open fire. (Warning: Do not do this experiment; it is very dangerous.) When the temperature of the gas in the can reaches 195°C, what is the pressure inside the can? Assume any change in the volume of the can is negligible.

Answer: Conceptualize: Intuitively, you should expect that the pressure of the gas in the container increases because of the increasing temperature.

Categorize: We model the gas in the can as ideal and use the ideal gas law to calculate the new pressure.

Analyze Rearrange:

$$\frac{PV}{T} = nR \ldots\ldots\ldots..(eq.1)$$

No air escapes during the compression, so n, and there-

fore nR, remains constant. Hence, set the initial value of the left side of Equation (1) equal to the final value:

$$\frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f} \ldots\ldots\ldots..(eq.2)$$

Because the initial and final volumes of the gas are assumed to be equal, cancel the volumes:

$$P_f = \left(\frac{T_f}{T_i}\right)P_i = \left(\frac{468K}{295K}\right)(202KPa) = 320KPa$$

Finalize: The higher the temperature, the higher the pressure exerted by the trapped gas as expected. If the pressure increases sufficiently, the can may explode. Because of this possibility, you should never dispose of spray cans in a fire.

(Q6) Suppose we include a volume change due to thermal expansion of the steel can as the temperature increases. Does that alter our answer for the final pressure significantly?

Answer Because the thermal expansion coefficient of steel is very small, we do not expect much of an effect on our last answer.

You can try to solve the next questions.

(Q7) A piece of copper is dropped into a beaker of water. (a) If the water's temperature rises, what happens to the temperature of the copper? (b) Under what conditions are the water and copper in thermal equilibrium?

(Q8) Is it possible for two objects to be in thermal equilibrium if they are not in contact with each other? Explain.

(Q9) A nurse measures the temperature of a patient to be BIO 41.5°C. (a) What is this temperature on the Fahrenheit scale? (b) Do you think the patient is seriously ill? Explain.

(Q10) The temperature difference between the inside and the outside of a pharmacy on a cold winter day is 57.0°F. Express this difference on (a) the Celsius scale and (b) the Kelvin scale.

(Q11) A pair of eyeglass frames is made of epoxy plastic. At room temperature (20.0°C), the frames have circular lens holes 2.20 cm in radius. To what temperature must the frames be heated if lenses 2.21 cm in radius are to be inserted in them? The average coefficient of linear expansion for epoxy is 1.30×10^{-4} $(°C)^{-1}$.