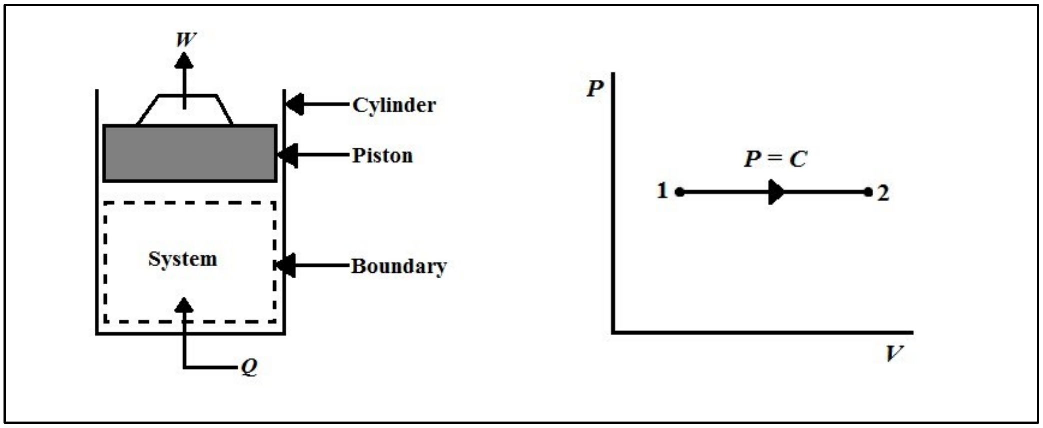
$\boldsymbol{P_2 = 5.367}$ **bar** **Ans.**

Applying the first law of thermodynamics for a constant volume process:

$Q = mC_v(T_2 - T_1) = 1 \times 0.718 \times (473 - 423)$

$\boldsymbol{Q = 35.9}$ **kJ** **Ans.**

2. Constant pressure (Isobaric) process: consider a cylinder with a piston carrying perfect gases as shown in figure below. When heat Q is supplied to the system, its temperature will rise and it will expand, forcing the piston to move upward. Thus a displacement work is done by the system against a constant force. The (P-V) diagram of the process is shown in figure below.



Work done by the system:

$$W = \int_1^2 PdV = P(V_2 - V_1) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.29)$$

Applying the first law of thermodynamics:

$Q - W = \Delta U$

$$Q = U_2 - U_1 + P(V_2 - V_1) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.30)$$

$$Q = (U_2 + PV_2) - (U_1 + PV_1) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.31)$$

Since $H = U + PV$, then:

$$Q = H_2 - H_1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.32)$$

$$Q = mC_p(T_2 - T_1) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.33)$$

For a unit mass, we get:

$$q = C_p(T_2 - T_1) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.34)$$

It can be seen that during an isobaric process, the heat transfer is equal to the change in enthalpy.

Example (5.4): When a stationary mass of gas was compressed without friction at constant pressure, its initial state of 0.4 m^3 and 0.105 MPa was found to change to a final state of 0.2 m^3 and 0.105 MPa. There was a transfer of 42.5 kJ of heat from the gas during the process. How much did the internal energy of the gas change?

Solution:

Since we have a constant pressure process, then work done by the gas is:

$W = P(V_2 - V_1) = 0.105 \times 10^3 \times (0.2 - 0.4) = -21 \text{ kJ}$

It can be seen that the compression work is negative, since it is subjected on the system, while expansion work is positive, since it is subjected by the system.

Applying the first law of thermodynamics:

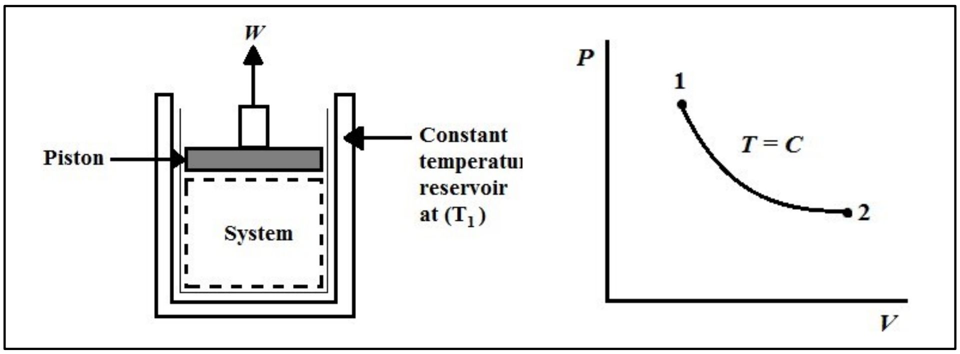
$Q - W = \Delta U$

$\Delta U = (-42.5) - (-21)$

$\boldsymbol{\Delta U = -21.5}$ **kJ** **Ans.**

3. Constant temperature (Isothermal) process: an isothermal process is shown in figure below. It consists of a constant temperature reservoir at temperature T_1 surrounding a piston cylinder arrangement. Assume that a perfect gas is at any instant, at the temperature of the system T_1, is contained inside the cylinder. At thermal equilibrium state, the temperature of the system and the surroundings is the same. Hence, there is no transfer of heat across the boundary.

If the piston now moves slightly downward, expansion of the gas takes place increasing its volume by dV and consequently the pressure and temperature of the system drop by an amount of dP and dT respectively. Therefore, heat will flow from the surroundings till the system reaches the original temperature T_1. The isothermal process will be possible only when the process is quasi-static. The (*P*-*V*) diagram of the isothermal expansion process is shown in figure below.



Applying the first law of thermodynamics:

$$Q - W = \Delta U = mC_v(T_2 - T_1)$$

Since ($T_1 = T_2$) for an isothermal process, then:

$$Q = W = \int_1^2 PdV \qquad \dots\dots\dots\dots\dots\dots\dots (5.35)$$

For an isothermal process, from Boyles's law, we have:

$$PV = C \rightarrow P = \frac{C}{V} \qquad \dots\dots\dots\dots\dots\dots\dots (5.36)$$

Substituting equation (5.33) in (5.32), we get:

$$= C\int_1^2 \frac{dV}{V} = C|\ln V|_1^2 = C\ln\left(\frac{V_2}{V_1}\right) \qquad \dots\dots\dots\dots\dots\dots\dots (5.37)$$

Since $P_1V_1 = P_2V_2 = mRT = C$, then:

$$Q = W = P_1V_1\ln\left(\frac{V_2}{V_1}\right) \qquad \dots\dots\dots\dots\dots\dots\dots (5.38)$$

or

$$Q = W = mRT\ln\left(\frac{V_2}{V_1}\right) \qquad \dots\dots\dots\dots\dots\dots\dots (5.39)$$

Example (5.5): Air enters a compressor at 10^5 Pa and 25°C having a volume of 1.8 m^3/kg, is compressed to 5×10^5 Pa isothermally. Determine:

1) Work done.
2) Change in internal energy.
3) Heat transferred.

Solution:

1) The work done by an isothermal process is:

$$w = P_1v_1\ln\left(\frac{V_2}{V_1}\right)$$

Since for an isothermal process $\frac{V_2}{V_1} = \frac{P_1}{P_2}$, then:

$$w = P_1v_1\ln\left(\frac{P_1}{P_2}\right) = 10^5 \times 10^{-3} \times 1.8 \times \ln\left(\frac{10^5}{5 \times 10^5}\right)$$

$\boldsymbol{w = -289.7}$ **kJ/kg** **Ans.**

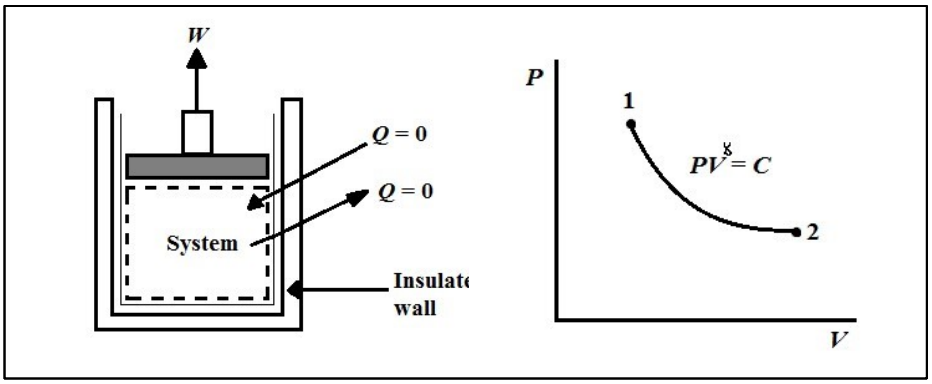
2) The change in internal energy for an isothermal process is:

$\Delta \boldsymbol{u} = \mathbf{0}$ **Ans.**

3) For an isothermal process:

$\boldsymbol{q} = \boldsymbol{w} = -\mathbf{289.7}$ **kJ/kg** **Ans.**

4. Adiabatic process: an adiabatic process is one in which the system undergoes no heat transfer with the surroundings, but the boundary of the system moves giving displacement work. The arrangement for the adiabatic process is shown in figure below. It consists of a piston cylinder arrangement where the cylinder is insulated from all sides to prevent heat transfer. Since $dQ = 0$, therefore dW is only due to dU. The (P-V) diagram for an adiabatic process is shown in figure below.



Applying the first law of thermodynamics:

$\delta Q - \delta W = dU$

For an adiabatic process $\delta Q = 0$

Also $\delta W = PdV$ and $dU = mC_v dT$

So

$0 - PdV = mC_v dT$

$PdV + mC_v dT = 0 \qquad \dots\dots\dots\dots\dots\dots\dots (5.40)$

From the equation of state: $PV = mRT$

Differentiating both sides, we get:

$PdV + VdP = mRdT$

$$dT = \frac{PdV + VdP}{mR} \qquad \dots\dots\dots\dots\dots\dots\dots (5.41)$$

Substituting (5.41) in (5.40), we get:

$$PdV + \frac{mC_v(PdV + VdP)}{mR} = 0 \qquad \dots\dots\dots\dots\dots\dots\dots (5.42)$$

Multiplying both sides by R, we get:

$$RPdV + C_v(PdV + VdP) = 0 \qquad \dots\dots\dots\dots\dots\dots\dots (5.43)$$

Since $R = C_p - C_v$, then:

$$(C_p - C_v)PdV + C_v(PdV + VdP) = 0 \qquad \dots\dots\dots\dots\dots\dots\dots (5.44)$$

$$C_pPdV - C_vPdV + C_vPdV + C_vVdP = 0$$

$$C_p\,PdV + C_v\,VdP = 0 \qquad \dots\dots\dots\dots\dots\dots\dots (5.45)$$

Dividing by C_vPV, we get:

$$\left(\frac{C_p}{C_v}\right).\frac{dV}{V} + \frac{dP}{P} = 0 \qquad \dots\dots\dots\dots\dots\dots\dots (5.46)$$

Since $\gamma = \frac{C_p}{C_v}$, then:

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0 \qquad \dots\dots\dots\dots\dots\dots\dots (5.47)$$

Integrating, we get:

$$\ln P + \gamma \ln V = C \qquad \dots\dots\dots\dots\dots\dots\dots (5.48)$$

$$\ln(PV^\gamma) = C$$

$$PV^\gamma = C \qquad \dots\dots\dots\dots\dots\dots\dots (5.49)$$

For a unit mass:

$$Pv^\gamma = C \qquad \dots\dots\dots\dots\dots\dots\dots (5.50)$$

Now

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma \qquad \dots\dots\dots\dots\dots\dots\dots (5.51)$$

For a perfect gas:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} \cdot \frac{V_2}{V_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \cdot \frac{V_2}{V_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

So

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad \dots\dots\dots\dots\dots\dots\dots (5.52)$$

Also

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \dots\dots\dots\dots\dots\dots\dots (5.53)$$

The work done is derived as follows:

$$W = \int_1^2 P dV$$

Since $PV^{\gamma} = C \rightarrow P = \frac{C}{V^{\gamma}}$, then:

$$W = C\int_1^2 \frac{dV}{V^{\gamma}} = C\int_1^2 V^{-\gamma}\, dV$$

$$= C\left|\frac{V^{-\gamma+1}}{-\gamma+1}\right|_1^2 = \left[\frac{C}{1-\gamma}\right]\left[V_2^{1-\gamma} - V_1^{1-\gamma}\right]$$

Since $C = P_1V_1^{\gamma} = P_2V_2^{\gamma}$, then:

$$W = \frac{P_2V_2 - P_1V_1}{1-\gamma} = \frac{P_1V_1 - P_2V_2}{\gamma-1} \qquad \dots\dots\dots\dots\dots\dots\dots (5.54)$$

Since $PV = mRT$, then:

$$W = \frac{mR(T_1 - T_2)}{\gamma-1} \qquad \dots\dots\dots\dots\dots\dots\dots (5.55)$$

For a unit mass:

$$w = \frac{P_1v_1 - P_2v_2}{\gamma-1} \qquad \dots\dots\dots\dots\dots\dots\dots (5.56)$$

or

$$w = \frac{R(T_1 - T_2)}{\gamma-1} \qquad \dots\dots\dots\dots\dots\dots\dots (5.57)$$

Example (5.6): Air at 1.02 bar and 22°C, initially occupying a cylinder volume of 0.015 m^3, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate:

1) The final temperature.
2) The final volume.
3) The work done.
4) The heat transferred to or from the cylinder walls.

Solution:

The absolute temperature is: $T_1 = 22 + 273 = 295$ K

To find the mass of the air:

$$P_1V_1 = mRT_1 \rightarrow m = \frac{P_1V_1}{RT_1} = \frac{1.02 \times 10^5 \times 10^{-3} \times 0.015}{0.287 \times 295} = 0.018 \text{ kg}$$

1) The final temperature can be calculated as:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow T_2 = 295 \times \left(\frac{6.8}{1.02}\right)^{\frac{1.4-1}{1.4}}$$

$\boldsymbol{T_2 = 507.25}$ **K** **Ans.**

2) The final volume can be calculated as:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \rightarrow V_2 = V_1 \times \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} \rightarrow V_2 = 0.015 \times \left(\frac{1.02}{6.8}\right)^{\frac{1}{1.4}}$$

$\boldsymbol{V_2 = 0.00387}$ **m^3** **Ans.**

3) The work done is:

$$W = \frac{mR(T_1 - T_2)}{\gamma - 1} = \frac{0.018 \times 0.287 \times (295 - 507.25)}{1.4 - 1}$$

$\boldsymbol{W = -2.741}$ **kJ** **Ans.**

4) The heat transferred for an adiabatic process:

$\boldsymbol{Q = 0}$ **Ans.**

5. Polytropic process: During actual expansion and compression processes of gases, pressure and volume are often related by $PV^n = C$, where *n* and C are constants. A process of this kind is called a polytropic process. The (*P-V*) diagram for such a process is shown below. As mentioned, the general equation for polytropic processes is expressed as:

$PV^n = C$ ……………………(5.58)