$$\eta_{Carnot} = 1 - \frac{Q_{rej}}{Q_{add}} \quad \dots\dots\dots\dots\dots\dots\dots (7.17)$$

As the heat addition takes place at high temperature, while heat rejection takes place at low temperature, so writing these heat interactions as Q_{High}, Q_{Low} we get:

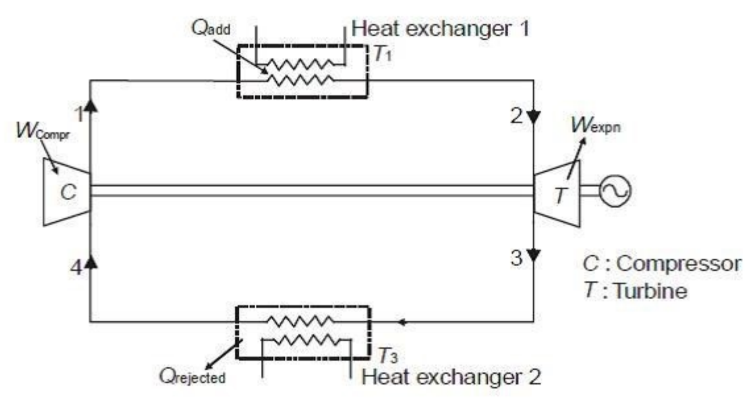
$$\eta_{Carnot} = 1 - \frac{Q_{Low}}{Q_{High}} \quad \dots\dots\dots\dots\dots\dots\dots (7.18)$$

The piston-cylinder arrangement shown and discussed for realizing Carnot cycle is not practically feasible as:

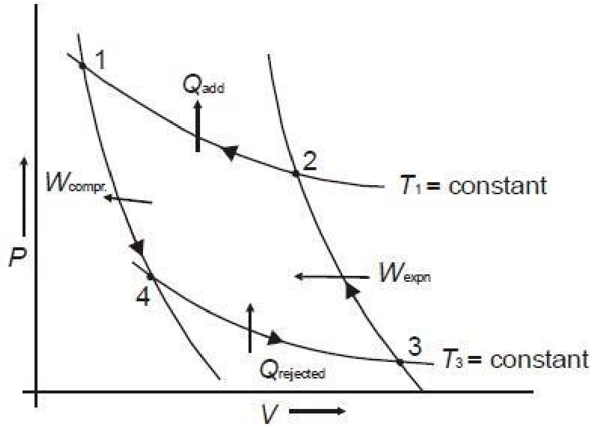
1. Frequent change of cylinder head i.e. of insulating type and diathermic type for adiabatic and isothermal processes is very difficult.
2. Isothermal heat addition and isothermal heat rejection are practically very difficult to be realized.
3. Reversible adiabatic expansion and compression are not possible.
4. Even if near reversible isothermal heat addition and rejection is to be achieved then time duration for heat interaction should be very large i.e. infinitesimal heat interaction occurring at dead slow speed. Near reversible adiabatic processes can be achieved by making them to occur fast. In a piston-cylinder reciprocating engine arrangement such speed fluctuation in a single cycle is not possible.

Carnot heat engine arrangement is also shown with turbine, compressor and heat exchangers for adiabatic and isothermal processes. Fluid is compressed in compressor adiabatically, heated in heat exchanger at temperature T_1, expanded in turbine adiabatically, cooled in heat exchanger at temperature T_3 and sent to compressor for compression. Here also following practical difficulties are confronted:

1. Reversible isothermal heat addition and rejection are not possible.
2. Reversible adiabatic expansion and compression are not possible.



Carnot cycle can also operate reversibly as all processes constituting it are of reversible type. Reversed Carnot cycle is shown below.



Heat engine cycle in reversed form as shown above is used as an ideal cycle for refrigeration and called "Carnot refrigeration cycle".

Thermodynamic Temperature Scale

The zeroth law of thermodynamics provides a basis for temperature measurement, but that a temperature scale must be defined in terms of a particular thermometer substance and device. A temperature scale that is independent of any particular substance, which might be called an **absolute temperature scale**, would be most desirable. In the preceding paragraph we noted that the efficiency of a Carnot cycle is independent of the working substance and depends only on the reservoir temperatures. This fact provides the basis for such an absolute temperature scale called the **thermodynamic scale**. Since the efficiency of a Carnot cycle is a function only of the temperature, it follows that:

$$\eta_{thermal} = 1 - \frac{Q_L}{Q_H} = 1 - f(T_H, T_L) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.19)$$

There are many functional relations that could be chosen to satisfy the relation given in equation (7.19). For simplicity, the thermodynamic scale is defined as:

$$\frac{Q_H}{Q_L} = \frac{T_H}{T_L} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.20)$$

Substituting this definition into equation (7.19), results in the following relation between the thermal efficiency of a Carnot cycle and the absolute temperatures of the two reservoirs:

$$\eta_{thermal} = 1 - \frac{T_L}{T_H} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.21)$$

Example (7.1): A heat engine produces net power output of 30 MW and the rate of waste heat rejected to a nearby river is 50 MW. Determine the heat transferred to the heat engine from the furnace and its thermal efficiency.

Solution:

The river is the cold sink of the heat engine, while the furnace is the heat source.

$$W_{net} = Q_H - Q_L \rightarrow Q_H = W_{net} + Q_L = 30 + 50$$

$\boldsymbol{Q_H = 80}$ **kW** **Ans.**

The thermal efficiency of the heat engine is:

$$\eta_{th} = \frac{W_{net}}{Q_H} = \frac{30}{80}$$

$\boldsymbol{\eta_{th} = 37.5\%}$ **Ans.**

Example (7.2): Food freezer in a refrigerator is maintained at 4°C by removing its heat at a rate of 360 kJ/min. If the required power input to the refrigerator is 2 kW. Determine the coefficient of performance and the rate of heat rejection to the room containing the refrigerator.

Solution:

The room represents the high-temperature reservoir, while the freezer represents the low temperature reservoir to the refrigerator. The coefficient of performance of the refrigerator is:

$$(C.O.P.)_{ref.} = \frac{\text{Desired effect}}{\text{Net work}} = \frac{Q_L}{W_{net}} = \frac{360}{60 \times 2}$$

$\boldsymbol{(C.O.P.)_{ref.} = 3}$ **Ans.**

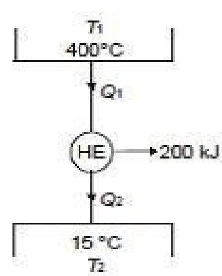
The rate of heat rejection to the room:

$$Q_H = Q_L + W_{net} = \frac{360}{60} + 2$$

$\boldsymbol{Q_H = 8}$ **kW** **Ans.**

Example (7.3): Determine the heat to be supplied to a Carnot engine operating between 400°C and 15°C and producing 200 kJ of work.

Solution:



$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \rightarrow \frac{Q_1}{Q_2} = \frac{673}{288} \quad \dots\dots\dots\dots\dots\dots (1)$$

$$W = Q_1 - Q_2 \rightarrow 200 = Q_1 - Q_2 \quad \dots\dots\dots\dots\dots\dots (2)$$

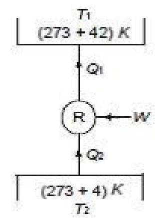
Solving (1) and (2) we get:

$Q_1 = 349.6 \text{ kJ}, Q_2 = 149.6 \text{ kJ}$

$\textbf{Heat to be supplied} = \mathbf{349.6\ kJ}$ **Ans.**

Example (7.4): A refrigerator operates on reversed Carnot cycle. Determine the power required to drive the refrigerator between the temperatures of 42°C and 4°C, if heat at the rate of 2 kW is extracted from the low temperature region.

Solution:



$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \rightarrow \frac{Q_1}{2} = \frac{315}{277} \rightarrow Q_1 = 2.274 \text{ kW}$$

$W = Q_1 - Q_2 = 2.274 - 2$

$\boldsymbol{W} = \mathbf{0.274\ kW}$ **Ans.**

Exercises

Problem (7.1): A household refrigerator with a C.O.P. of 1.2 removes heat from the refrigerated space at a rate of 60 kJ/min. Determine the electric power consumed by the refrigerator and the rate of heat transferred to the kitchen air.

Ans. (0.83 kW, 1.83 kW)

Problem (7.2): An air conditioner removes heat steadily from a house at a rate of 750 kJ/min, while drawing electric power at a rate of 6 kW. Determine the C.O.P. of this air conditioner and the rate of heat transfer to the outside air.

Ans. (2.08, 18.5 kW)

Problem (7.3): A Carnot heat engine operates between a source at 1300 K and a sink at 300 K. If the heat engine is supplied with heat at a rate of 800 kJ/min, determine the thermal efficiency and the power output of this heat engine.

Ans. (77%, 10.23 kW)

Problem (7.4): A Carnot heat engine receives 650 kJ of heat form a source of unknown temperature and rejects 250 kJ of it to a sink at 24°C. Determine the temperature of the source and the thermal efficiency of the heat engine.

Ans. (499.2°C, 61.5%)

Problem (7.5): A Carnot refrigerator operates in a room in which the temperature is 22°C and consumes 2 kW of power when operating. If the food compartment of the refrigerator is to be maintained at 3°C, determine the rate of heat removal from the food compartment.

Ans. (29 kW)

Problem (7.6): A refrigerator is to remove heat from the cooled space at a rate of 300 kJ/min to maintain its temperature at –8°C. If the air surrounding the refrigerator is at 25°C, determine the minimum power input required for this refrigerator.

Ans. (0.623 kW)

Problem (7.7): An air conditioning system operating on the reversed Carnot cycle is required to transfer heat from a house at a rate of 750 kJ/min to maintain its temperature at 24°C. If the outdoor air temperature is 35°C, determine the power required to operate this air conditioning system.

Ans. (0.46 kW)

Problem (7.8): A Carnot refrigerator operates in a room in which the temperature is 25°C. The refrigerator consumes 500 kW of power when operating and has a C.O.P. of 4.5.

Determine the rate of heat removal for the refrigerated space and the temperature of the refrigerated space.

Ans. (2250 kW, –29.2°C)

Problem (7.9): In a winter season when the outside temperature is –1°C, the inside of a house is to be maintained at 25°C. Estimate the minimum power required to run the heat pump of maintaining the temperature. Assume heating load as 125 MJ/h.

Ans. (3.02 kW)

Problem (7.10): A reversible heat engine operates between two reservoirs at 827°C and 27°C. The engine drives a Carnot refrigerator maintaining –13°C and rejecting heat to a reservoir at 27°C. Heat input to the engine is 2000 kJ and the net work available is 300 kJ. How much heat is transferred to the refrigerant and total heat rejected to the reservoir at 27°C?

Ans. (7504.58 kJ, 9204.68 kJ)