

## 4. Perfect Gas

A gas is made of molecules that move around with random motion. In a perfect gas, the molecules may collide but they have no tendency to stick together or repel each other. In reality, there is a slight force of attraction between gas molecules but this is so small that gas laws formulated for an ideal gas work quite well for a real gas.

### Equation of State

Any equation that relates the pressure, temperature and volume of a substance is called an **equation of state**. Property relations that involve other properties of a substance at equilibrium states are also referred to as equations of state. There are several equations of state, some simple and others very complex. The simplest and best known equation of state for substances in the gas phase is the ideal gas equation of state. This equation predicts the ( $P$ - $V$ - $T$ ) behavior of a gas quite accurately within some properly selected region. The ideal gas equation of state is expressed as:

$$PV = mRT$$

or

$$Pv = RT \quad (\text{in terms of specific volume})$$

where  $R$  is the gas constant, which has a different value for each gas. Table 1 shows the values of the gas constant for different gases. The equation of state can also be expressed in terms of the number of moles instead of the mass as follows:

$$PV = NR_oT$$

where  $N$  is the number of moles, and  $R_o$  is the universal gas constant which has a constant value for all gases:

$$R_o = 8.314 \text{ kJ/kmol.K}$$

$$R_o = 1545.37 \text{ ft. lbf/kmol.R}$$

$$R = \frac{R_o}{M} \quad \text{and} \quad M = \frac{m}{N}$$

where  $M$  is the molar mass (also called molecular weight) of the gas.

For a constant mass, the properties of an ideal gas at two different states are related to each other by:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

**Note:** in these equations  $T$  is the absolute temperature (i.e. substituted in Kelvin or Rankine).

**Boyle's law** states that the pressure of a given mass of an ideal gas is inversely proportional to its volume at a constant temperature. It is expressed as:

$$PV = C \quad \text{or} \quad P_1V_1 = P_2V_2$$

**Charles's law** states that the volume of an ideal gas at constant pressure is directly proportional to the absolute temperature. It is expressed as:

$$\frac{V}{T} = C \quad \text{or} \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

**Gay-Lussac's law** states that, for a given mass and constant volume of an ideal gas, the pressure exerted on the sides of its container is directly proportional to its absolute temperature. It is expressed as:

$$\frac{P}{T} = C \quad \text{or} \quad \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

**Example (4.1):** An amount of gas has a pressure of 350 kPa, a volume of 0.03 m<sup>3</sup> and a temperature of 35°C. If  $R = 0.29$  kJ/kg.K, calculate the mass of the gas and the final temperature if the final pressure is 1.05 MPa and the volume remains constant.

Solution:

The absolute temperature:  $T_1 = 35 + 273 = 308$  K

Applying the equation of state for the initial conditions:

$$P_1V_1 = mRT_1$$

$$350 \times 0.03 = m \times 0.29 \times 308 \rightarrow m = \frac{350 \times 0.03}{0.29 \times 308}$$

$$m = 0.12 \text{ kg} \quad \text{Ans.}$$

Applying the equation of state between two conditions at constant volume:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{350}{308} = \frac{1.05 \times 10^3}{T_2} \rightarrow T_2 = \frac{1.05 \times 10^3 \times 308}{350}$$

$$T_2 = 924 \text{ K} \quad \text{Ans.}$$

**Example (4.2):** A tank has a volume of 0.5 m<sup>3</sup> and contains 10 kg of an ideal gas having a molecular weight of 24. The temperature is 25°C. What is the pressure of the gas?

Solution:

The absolute temperature:  $T = 25 + 273 = 298 \text{ K}$

$$R = \frac{R_o}{M} = \frac{8.314}{24} = 0.35 \text{ kJ/kg. K}$$

Applying the equation of state:

$$PV = mRT$$

$$P \times 0.5 = 10 \times 0.35 \times 298 \rightarrow P = \frac{10 \times 0.35 \times 298}{0.5}$$

$$\mathbf{P = 2086 \text{ kPa} \quad \quad \text{Ans.}}$$

## Exercises

**Problem (4.1):** Helium in a steel tank is at 250 kPa, 300 K with a volume of  $0.1 \text{ m}^3$ . It is used to fill a balloon. When the pressure drops to 150 kPa, the flow of helium stops by itself. If all the helium still is at 300 K, how big a balloon is produced? Take the gas constant of helium to be  $2.0769 \text{ kJ/kg.K}$ .

Ans. ( $0.06667 \text{ m}^3$ )

**Problem (4.2):** Air in an internal combustion engine has  $227^\circ\text{C}$ , 1000 kPa with a volume of  $0.1 \text{ m}^3$ . Now combustion heats it to 1500 K in a constant volume process. What is the mass of air and how high does the pressure become?

Ans. (0.697 kg, 3000 kPa)

**Problem (4.3):** Air in an automobile tire is initially at  $-10^\circ\text{C}$  and 190 kPa. After the automobile is driven awhile, the temperature gets up to  $10^\circ\text{C}$ . Find the new pressure. You must make one assumption on your own.

Ans. (204.4 kPa)

**Problem (4.4):** A rigid tank of  $1 \text{ m}^3$  contains nitrogen gas (molecular weight 28) at 600 kPa, 400 K. By mistake someone lets 0.5 kg flow out. If the final temperature is 375 K what is the final pressure?

Ans. (506.9 kPa)

**Problem (4.5):** A  $1 \text{ m}^3$  rigid tank contains propane (molecular weight 44) at 100 kPa, 300 K and connected by a valve to another tank of  $0.5 \text{ m}^3$  with propane at 250 kPa, 400 K. The valve is opened and the two tanks come to a uniform state at 325 K. What is the final pressure?

Ans. (139.9 kPa)

**Problem (4.6):**  $1 \text{ m}^3$  rigid tank with air at 1 MPa, 400 K is connected to an air line. The valve is opened and air flows into the tank until the pressure reaches 5 MPa, at which point the valve is closed and the temperature inside is 450 K.

- 1) What is the mass of air in the tank before and after the process?
- 2) The tank eventually cools to room temperature, 300 K. What is the pressure inside the tank then?

Ans. (8.711 kg, 38.715 kg, 3.33 MPa)