جامعة المستقبل
AL MUSTAQBAL UNIVERSITY

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Lecture: (9)

Degree

Subject: Discrete Structures

First Stage: Semester II

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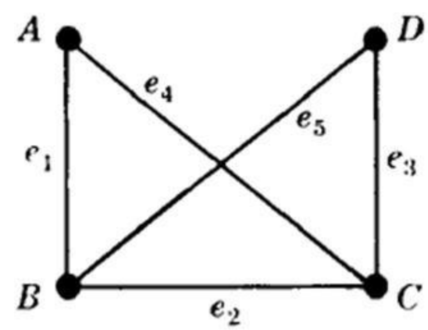
Degree

The degree of a vertex v [deg(v)], is equal to the number of edges which are incident on v. since each edge is counted twice in counting the degrees of the vertices of a graph.

Theorem: The sum of the degrees of the vertices of a graph is equal to twice the number of edges. Let $G = (V, E)$ be an undirected graph with m edges. Then

$$2m = \sum_{v \in V} deg(v).$$

For example, in the figure (5) we have



deg(A) = 2,
deg(B) = 3,
deg(C) = 3,
deg(D) = 2

The sum of the degrees = twice the number of edges = 2×5=10

EXAMPLE 1: How many edges are there in a graph with 10 vertices each of degree six?
Solution: Because the sum of the degrees of the vertices is 6

× 10 = 60, it follows that $2m = 60$
where m is the number of edges. Therefore, $m = 30$.

A vertex is said to be **even** or **odd** according as its degree is an even or odd number. Thus A and D are even vertices whereas B and C are odd vertices.
This theorem also holds for multigraphs where a loop is counted twice towards the degree of its endpoint. For example, in Fig (6) we have deg (D) = 4 since the edge e6 is counted twice; hence D is an even vertex.
A vertex of degree zero is called an isolated vertex.

Connectivity:
Many problems can be modeled with paths formed by traveling along the edges of graphs. For instance, the problem of determining whether a message can be sent between two computers using intermediate links can be studied with a graph model. Problems of efficiently planning routes for mail delivery, garbage pickup, diagnostics in computer networks, and so on can be solved using models that involve paths in graphs.

a walk is a sequence of edges that begins at a vertex of a graph and travels from vertex to vertex along edges of the graph. As the path travels along its edges, it visits the vertices along this walk, that is, the endpoints of these edges.
A **walk** in a multigraph G consists of an alternating sequence of vertices and edges of the form:

v0, e1, v1, e2, v2,……., en-1,vn-1,en,vn

where each edge e_i contains the vertices v_{i-1} and v_i (which appear on the sides of ei in the sequence).

Length of walk : is the number n of edges. When there is no ambiguity, we denote a path by its sequence of vertices

(v0, v1, . . . , vn).

Closed walk: the walk is said to be closed if v0 = vn .
Otherwise, we say that the walk is from v0 to vn.

Trail: is a walk in which all edges are distinct.
Path: is a walk in which all vertices are distinct.
Cycle: is a closed walk such that all vertices are distinct except v1 = vn, A cycle of length k is called a k-cycle.

EXAMPLE 1

In the simple graph shown in Figure 8:

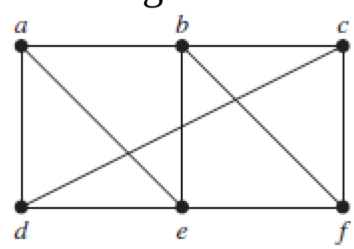


Figure 8

a, *d*, *c*, *f* , *e* is a path of length 4, because {*a*, *d*}, {*d*, *c*}, {*c*, *f* }, and {*f*, *e*} are all edges. However,

d, *e*, *c*, *a* is not a path, because {*e*, *c*} is not an edge. Note that

b, *c*, *f* , *e*, *b* is a circuit of length 4 because {*b*, *c*}, {*c*, *f* }, {*f*, *e*}, and {*e*, *b*} are edges, and this path begins and ends at *b*.

The walk *a*, *b*, *e*, *d*, *a*, *b*, which is of length 5, is not path because it contains the edge {*a*, *b*} twice.

Example: Consider the graph in figure (9), then

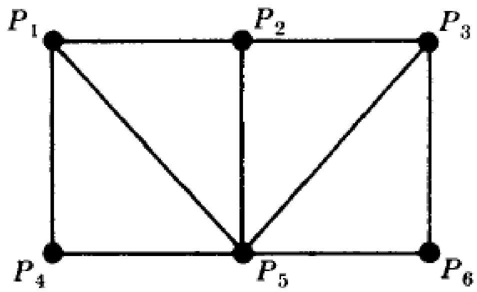


Figure (9)

The sequence: (P4, P1, P2, P5, P1, P2, P3, P6) is a walk from P4 to P6. It is not a trail since the edge {P1,P2} is used twice.

The sequence: (P4, P1, P5, P3, P2, P6) Is not a walk since there is no edge {P2, P6}.

The sequence: (P4, P1, P5, P2, P3, P5, P6) is a trail since no edge is used twice; but it is not a path since the vertex P5 is used twice.
The sequence: (P4, P1, P5, P3, P6) Is a path from P4 to P6.

The shortest path from P4 to P6 is (P4, P5, P6) which has length = 2 (2 edges only)
The distance between vertices u & v d(u,v) is the length of the shortest path d(P4,P6) = 2.