جامـــــعـة المـــــستـقـبـل
AL MUSTAQBAL UNIVERSITY

كلية العلوم

قسم الأمن السيبراني

Lecture: (7)

Function

Subject: Discrete Structures

First Stage: Semester II

Lecturer: BAQER KAREEM SALIM

Function:

In many instances we assign to each element of a set a particular element of a second set. For example, suppose that each student in a discrete mathematics class is assigned a letter grade from the set $\{A, B, C, D, F\}$. And suppose that the grades are A for Adams, C for Chou, B for Goodfriend, A for Rodriguez, and F for Stevens. This assignment is an example of a function. This assignment is an example of a function. The concept of a function is extremely important in mathematics and computer science.

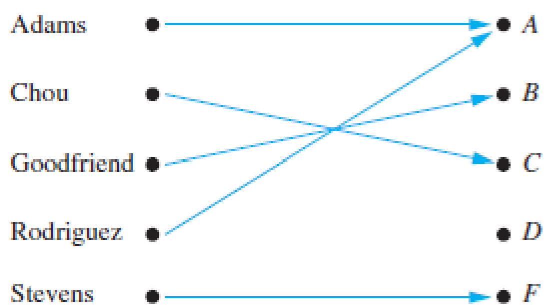


Fig. 1 Assignment of Grades in a Discrete Mathematics Class.

Function is a class of relation. it establishes the relationship between objects. For example, in computer system input is fed to the system in form of data or objects and the system generates the output that will be the function of input. So, function is the mapping or transformation of objects from one form to other.

Definition:

Let A and B be nonempty sets. A *function $F: A \to B$ is a rule which associates with **each** element of A a **unique** element in B.*

EXAMPLE 1

Let R be the relation with ordered pairs (Abdul, 22), (Brenda, 24), (Carla, 21), (Desire, 22), (Eddie, 24), and (Felicia, 22). Here each pair consists of a graduate student and this student's age. Specify a function determined by this relation.

Solution:

If *f* is a function specified by *R*, then

*f (*Abdul *)* = 22,

*f (*Brenda*)* = 24,

*f (*Carla*)* = 21,

*f (*Desire*)* = 22,

*f (*Eddie*)* = 24, and

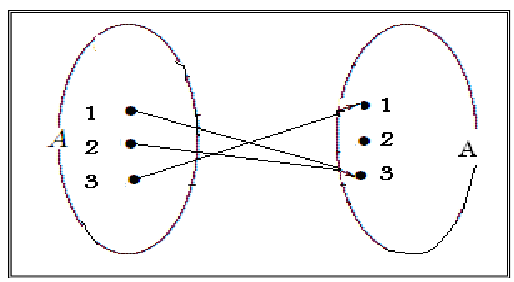
*f (*Felicia*)* = 22. (Here, *f (x)* is the age of *x*, where *x* is a student.)

EXAMPLE 2

Consider the function f (x) = x^3, i.e., f assigns to each real number its cube. Then the image of 2 is 8, and so we may write f (2) = 8.

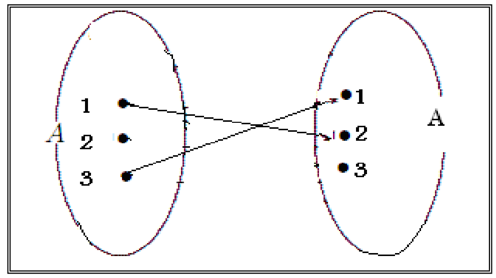
Example 3 :

consider the following relation on the set A={1,2,3} F = {(1,3),(2,3),(3,1)}

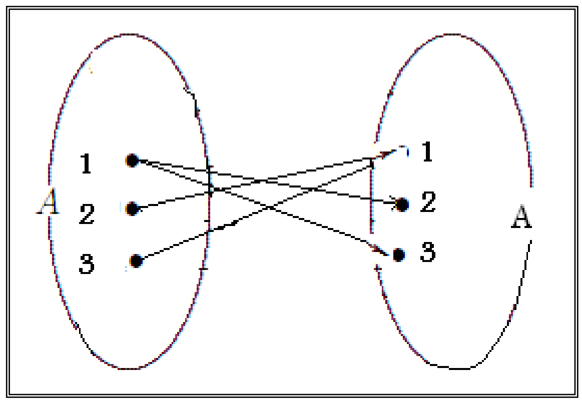


F is a function

G = {1,2},(3,1)}
G is not a function from A to A



H = {(1,3),(2,1),(1,2),(3,1)}
H is not a function.



Classification of functions:
(One-to-one ,onto and invertible functions) :

Some functions never assign the same value to two different domain elements. These functions are said to be one-to-one.

1) **One –to-one** :

a function $F:A\rightarrow B$ is said to be one-to-one if different elements in the domain (A) have distinct images.

Or If $F(a) = F(a')$ $\Rightarrow$ $a = a'$

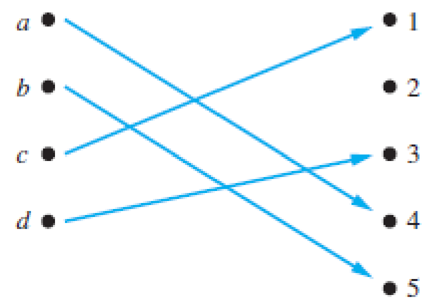


Fig 2: A One-to-One Function.

2) **Onto** :

F:A→B is said to be an onto function if each element of B is the image of some element of A.

$\forall\ b \in B \quad \exists \quad a \in A : F(a) = b$

EXAMPLE

Let *f* be the function from {*a*, *b*, *c*, *d*} to {1, 2, 3} defined by

$f(a) = 3$,

$f(b) = 2$,

$f(c) = 1$, and

$f(d) = 3$.

Is *f* an **onto** function?

Solution:

Because all three elements of the codomain are images of elements in the domain, we see that *f* is onto. This is illustrated in Figure 3.

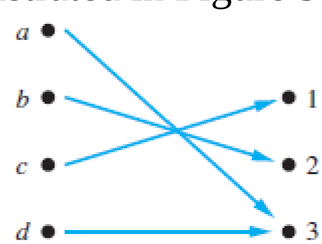


Fig. 3 An Onto Function

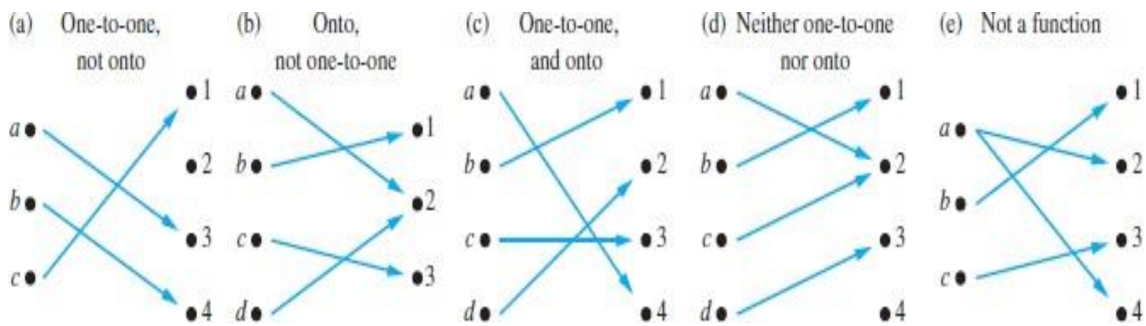


Fig 4 . Examples of Different Types of Correspondences.

3) **Invertible (One-to-one correspondence)**

F:A→ B is invertible if and only if F is **both** one-to-one and onto.

F:A→ B is invertible if its inverse relation f^{-1} is a function F:B →A

$$F^{-1} : \{(b,a) \ \forall \ (a,b) \in F\}$$

EXAMPLE

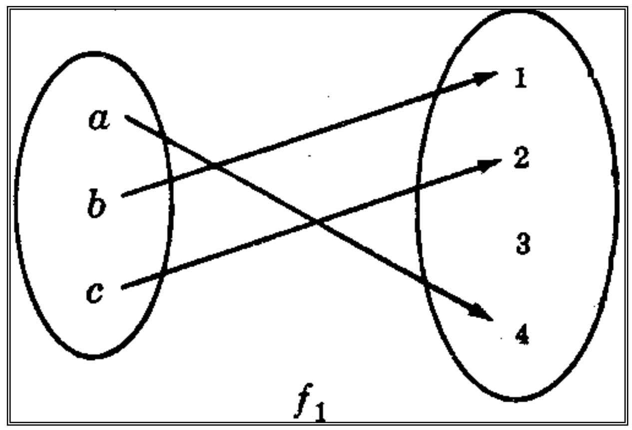
Let *f* be the function from {*a*, *b*, *c*, *d*} to {1, 2, 3, 4} with
f (a) = 4,
f (b) = 2,
f (c) = 1, and
f (d) = 3. Is *f* an **invertible**?

Solution:

The function *f* is one-to-one and onto.
It is one-to-one because no two values in the domain are assigned the same function value.
It is onto because all four elements of the codomain are images of elements in the domain. Hence, *f* is a invertible.

Figure 4 displays four functions where

the first is one-to-one but not onto,
the second is onto but not one-to-one, the
third is both one-to-one and onto, and the
fourth is neither one-to-one nor onto.
The fifth correspondence in Figure 4 is not a function, because it sends an element to two different elements.

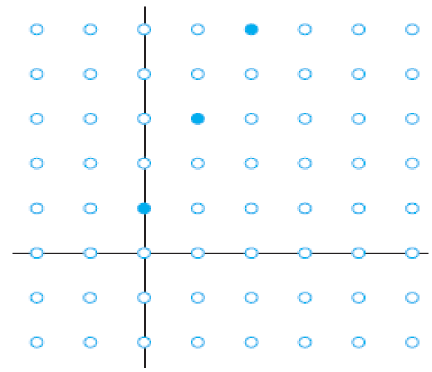


one to one but not onto
(3∈B but it is not the image under f1)

Graph of a function:
We can associate a set of pairs in $A \times B$ to each function from A to B. This set of pairs is called the **graph** of the function and is often displayed pictorially to aid in understanding the behavior of the function.

EXAMPLE
Display the graph of the function $f(n) = 2n + 1$ from the set of integers to the set of integers.

Solution:
The graph of f is the set of ordered pairs of the form $(n, 2n + 1)$, where n is an integer.

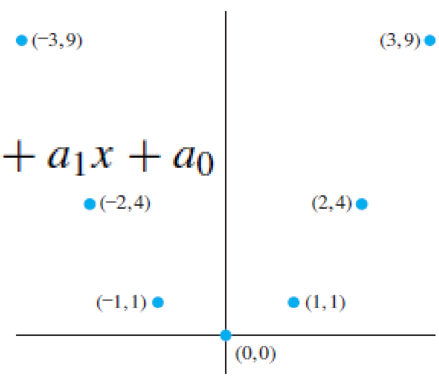


EXAMPLE

Display the graph of the function $f(x) = x^2$ from the set of integers to the set of integers.

Solution:

The graph of f is the set of ordered pairs of the form $(x, f(x)) = (x, x^2)$, where x is an integer.



By a *real polynomial function*, we mean a function f: $\mathbf{R} \rightarrow \mathbf{R}$ of the form

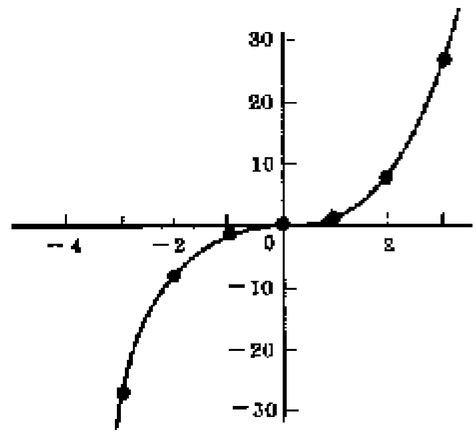
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where the a_i are real numbers. Since **R** is an infinite set, it would be impossible to plot each point of the graph. However, the graph of such a function can be approximated by first plotting some of its points and then drawing a smooth curve though these points. The table points are usually obtained from a table where various values are assigned to x and the corresponding value of f(x) computed.

Example: let f: R→R and $\mathbf{f(x)= x^3}$, find f(x)

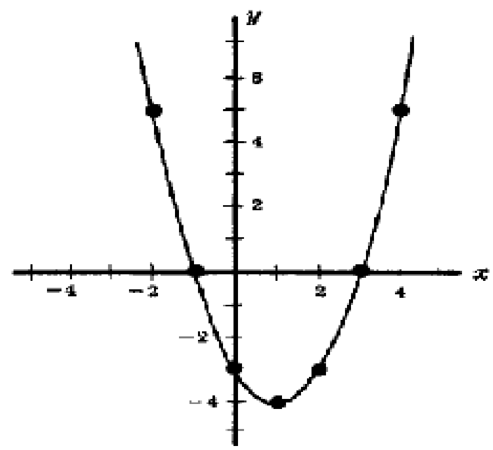
f(3) = $3^3 = 27$
f(-2) = $(-2)^3 = -8$

x	$f(x)$
−3	−27
−2	8
1	−1
0	0
1	1
2	8
3	27



Example : let f: R→R and $\boldsymbol{f(x) = x^2 - 2x - 3}$, , find f(x)

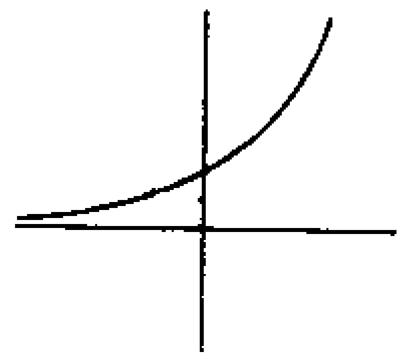
x	$f(x)$
−2	5
−1	0
0	−3
1	−4
2	−3
3	0
4	5



Geometrical Characterization of One-to-One and Onto Functions

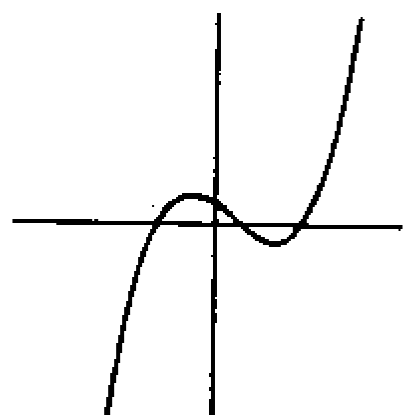
For the functions of the form f : R → R. the graphs of such functions may be plotted in the Cartesian plane and functions may be identified with their graphs, so the concepts of being one-to-one and onto have some geometrical meaning :

1- f :R → R is said to be <u>one-to-one</u> if there are no 2 distinct pairs (a1,b) and (a2,b) in the graph one-to-one or if each horizontal line intersects the graph of f in at most one point.



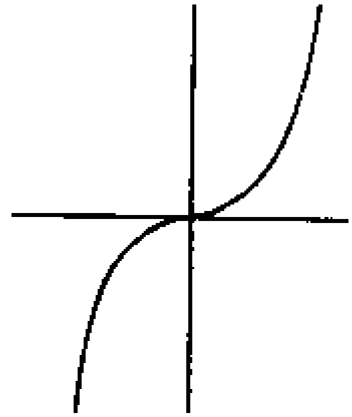
$f_2(x) = 2^x$

2- f :R → R is an onto function if each horizontal line intersects the graph of f at one or more points (at least once)

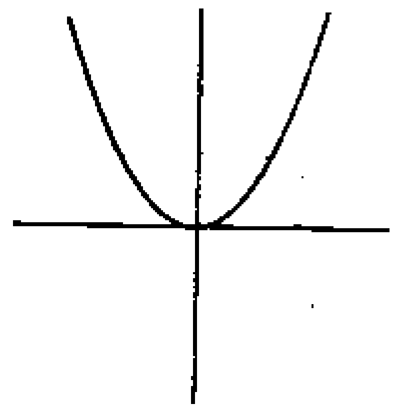


$f_3(x) = x^3 - 2x^2 - 5x + 6$

3- if f is both one-to-one and onto, i.e. invertible, then each horizontal line will intersect the graph of f **at exactly one point**.



$f_4(x) = x^3$



$f_1(x) = x^2$

f(x)NOT (ONE-TO-ONE) & NOT (ONTO)