Al-Mustaqbal University

College of Sciences

Intelligent Medical System Department

جامـــــعة المـــســتـقبـل
AL MUSTAQBAL UNIVERSITY

كلية العلوم

قـســـــــــــم الانظمة الطبية الذكية

Lecture: (4)

Subject: Boolean Algebra
Level: First
Lecturer: Dr. Ahmad T. Jaiad

1. Boolean Algebra Definition

Boolean algebra is the mathematics of digital logic in which the values of the variables are the truth values **true** and **false**, usually denoted 1 and 0, respectively. Basic knowledge of Boolean algebra is indispensable to the study and analysis of logic circuits. **Variable, complement, and literal** are terms used in Boolean algebra. A variable is a symbol (usually an italic uppercase letter or word) used to represent an action, a condition, or data. Any single variable can have only a **1 or a 0** value. The complement is the inverse of a variable and is indicated by **a bar** over the variable (overbar).

- ❖ Connecting symbols used in Boolean algebra

Symbol	Name	Performance
=	Equal sign	This refers to the sign of **equality** as in mathematics.
.	Multiplication sign	It refers to the **AND** operation
+	Plus sign	This refers to the **OR** operation
‘ or ‾	Inversion sign	This operation performs a **complement** of the input given to the logic gate.

2. Boolean Operations and Expressions

A. Boolean Addition: is equivalent to the OR operation. In logic circuits, a sum term is produced by an OR operation with no AND operations involved. Some examples of sum terms are A + B, A + $\overline{B}$, A + B + $\overline{C}$, and A + B + C + $\overline{D}$.

EXAMPLE 1: Determine the values of A, B, C, and D that make the sum term $A + \overline{B} + C + \overline{D}$ equal to 0.

Solution: For the sum term to be 0, each of the literals in the term must be 0. Therefore, $A = \mathbf{0}$, $\overline{B} = 0$, $C=0$, *and* $\overline{D} = 0$ *so that* $B= 1$, *and* $D = \mathbf{1}$

$$A + \overline{B} + C + \overline{D} = 0 + \overline{1} + 0 + \overline{1} = 0 + 0 + 0 + 0 = 0$$

B. Boolean Multiplication: is equivalent to the AND operation. In logic circuits, a product term is produced by an AND operation with no OR operations involved. Some examples of product terms are AB, A$\overline{B}$, ABC, and A$\overline{B}$C$\overline{D}$.

EXAMPLE 2: Determine the values of A, B, C, and D that make the product term $A\overline{B}C\overline{D}$ equal to 1.

Solution: For the product term to be 1, each of the literals in the term must be 1. Therefore, $A = \mathbf{1}$, $\overline{B} = \mathbf{1}$, $C = \mathbf{1}$, and $\overline{D} = \mathbf{1}$ so that, $B = \mathbf{0}$, and $D = \mathbf{0}$.

$$A\overline{B}C\overline{D} = 1 . \overline{0} . 1 . \overline{0} = 1 . 1 . 1 . 1 = 1$$

3. Laws of Boolean Algebra

The laws of Boolean algebra are listed below:-

1. Distributive laws

According to this law, if we perform the OR operation of two or more variables and then perform the AND operation of the result with a single variable, then the result will be similar to performing the AND operation of that single variable with each two or more variable and then perform the OR operation of that product. This law explains the process of factoring. For three variables, the distributive law is written as:

$$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$$

$$A + (B \cdot C) = (A + B) \cdot (A + C)$$

Example

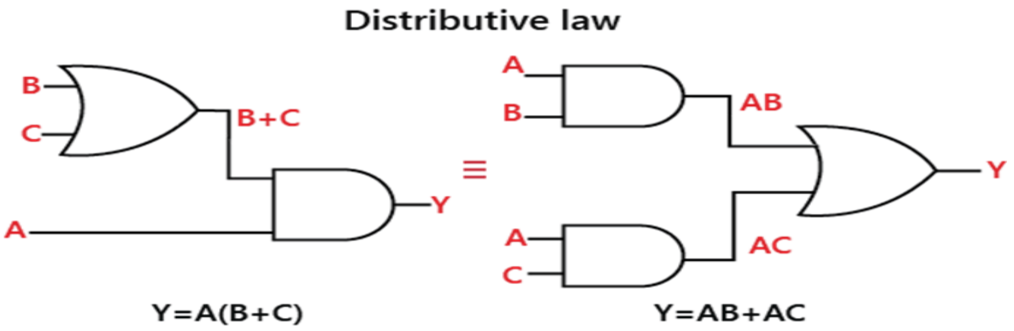


Fig.(1) Application of distributive law before and after .

2. Associative laws

The associative law of addition is written as follows for three variables:

$$A + (B + C) = (A + B) + C$$

Example

Figure (2) illustrates this law as applied to 2-input OR gates.

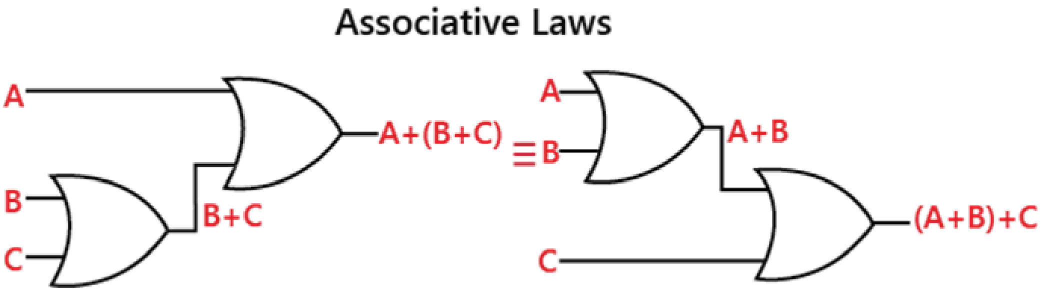


Fig.(2) Application of associative law of addition.

The associative law of multiplication is written as follows for three variables:

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

Example

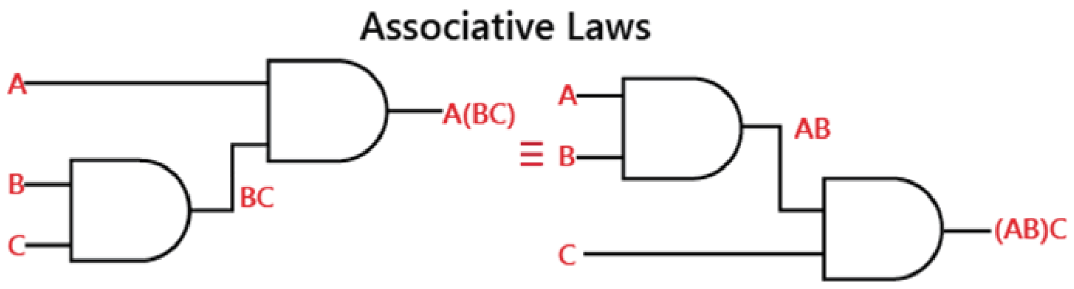


Fig.(3) Application of associative law of multiplication.

This law states that it makes no difference in what order the variables are grouped when ANDing more than two variables.

3. Commutative laws قوانين التباديل

The commutative law of addition for two variables is written as

$$A + B = B + A$$

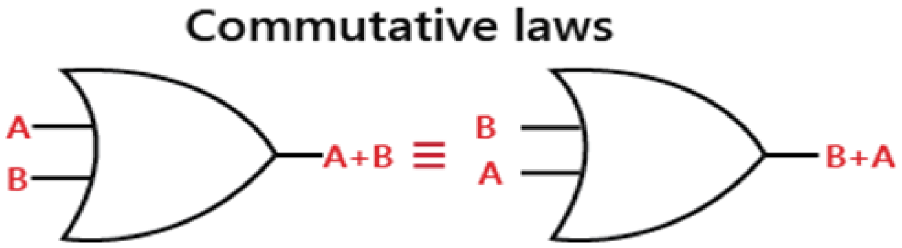


Fig.(4) Application of commutative law of addition.

The commutative law of multiplication for two variables is

$$A \cdot B = B \cdot A$$

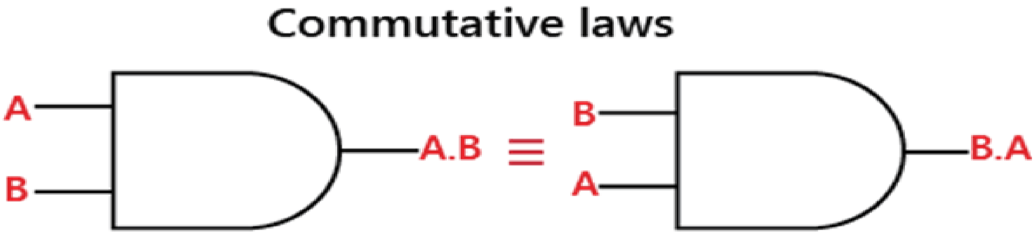


Fig.(5) Application of commutative law of multiplication.

4. Inverse laws

$$A + A' = 1$$

$$A \cdot A' = 0$$

5. Zero and one laws

$$A + 1 = 1$$

$$A \cdot 0 = 0$$

6. Identity laws

$$A + 0 = A$$

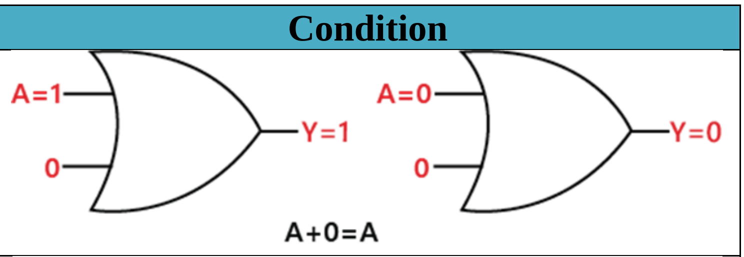
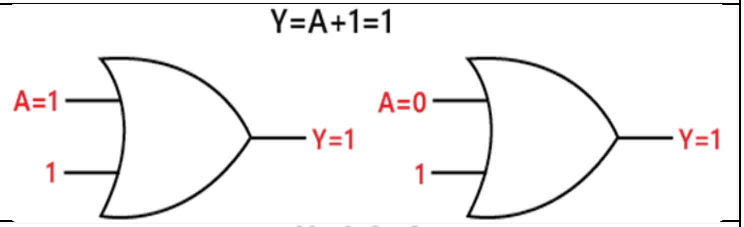
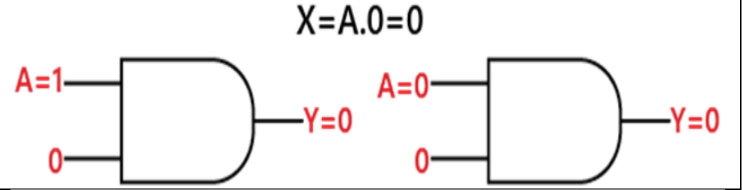
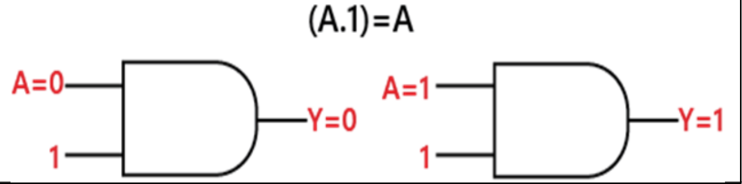
$$A \cdot 1 = A$$

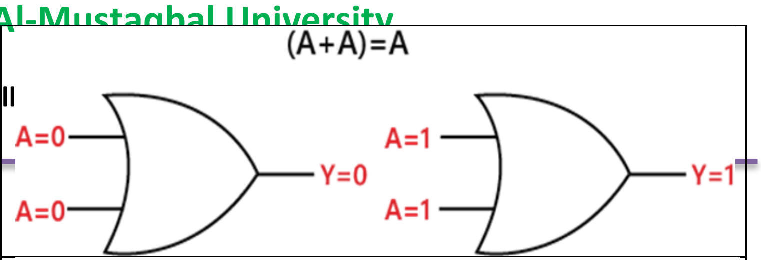
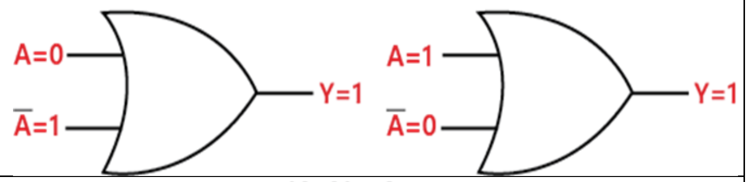
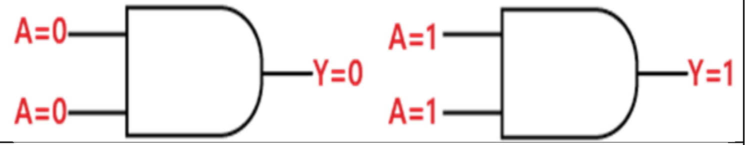
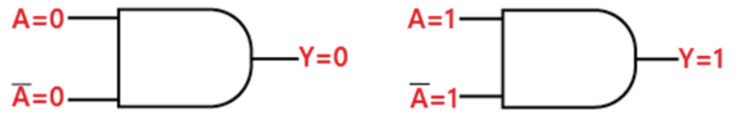
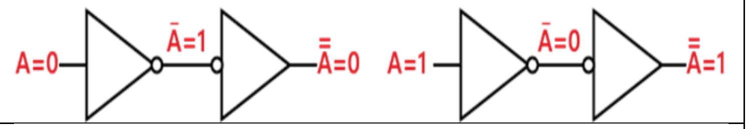
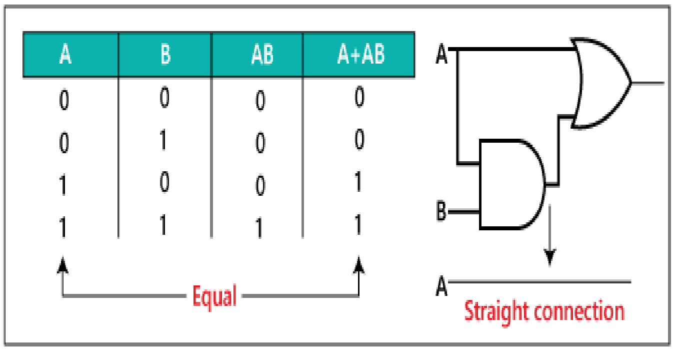
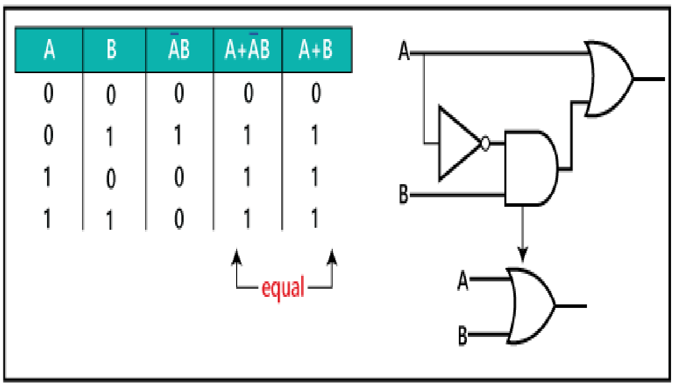
❖ Other relations for simplification

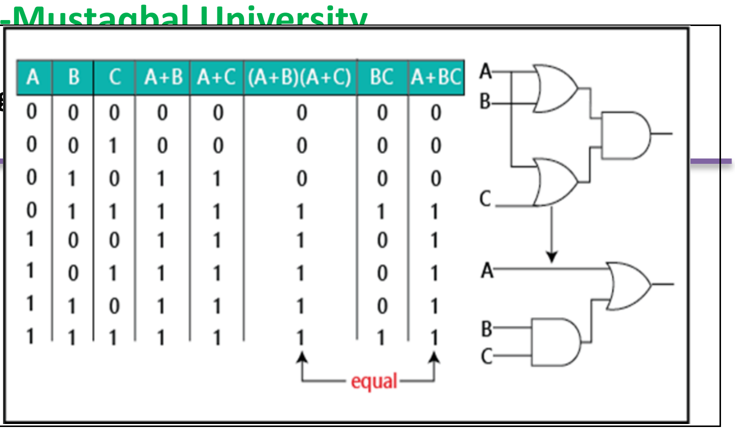
$$A + (A \cdot B) = A$$
$$A \cdot (A + B) = A$$
$$A + A = A$$
$$A . A = A$$
$$A'' = A$$
$$A + A' \cdot B = A + B$$
$$A \cdot (A' + B) = A \cdot B$$
$$A \cdot B + A \cdot B' = A$$
$$(A + B) \cdot (A + B') = A$$
$$A \cdot B + A' \cdot C + B \cdot C = A \cdot B + A' \cdot C$$
$$(A + B) \cdot (A' + C) \cdot (B + C) = (A + B) \cdot (A' + C)$$

4. Rules of Boolean algebra

There are the following rules of Boolean algebra, which are mostly used in manipulating and simplifying Boolean expressions. These rules plays an important role in simplifying Boolean expressions.

No	Rule	Condition
1.	A+0=A	
2.	A+1=1	
3.	A.0=0	
4.	A.1=A	

<table>
<tr><td><b>5.</b></td><td><b>A+A=A</b></td><td>(A+A)=A
 </td></tr>
<tr><td><b>6.</b></td><td><b>A+A'=1</b></td><td>(A+A′)=1
 </td></tr>
<tr><td><b>7.</b></td><td><b>A.A=A</b></td><td>(A.A)=A
 </td></tr>
<tr><td><b>8.</b></td><td><b>A.A'=0</b></td><td>(A.A′)=0
 </td></tr>
<tr><td><b>9.</b></td><td><b>A = (A')'</b></td><td>A=(A′)′
 </td></tr>
<tr><td><b>10.</b></td><td><b>A+AB=A</b></td><td><table><tr><th>A</th><th>B</th><th>AB</th><th>A+AB</th></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> </td></tr>
<tr><td><b>11.</b></td><td><b>A+A'B=A+B</b></td><td><table><tr><th>A</th><th>B</th><th>$\bar{A}$B</th><th>A+$\bar{A}$B</th><th>A+B</th></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td><td>1</td><td>1</td></tr></table> </td></tr>
</table>

12.	(A+B)(A+C)=A+BC	

A	B	C	A+B	A+C	(A+B)(A+C)	BC	A+BC
0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	1	1	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1