Evaluate $I = \int \frac{e^x}{1+e^{2x}}\,dx \;\Rightarrow\; \int \frac{e^x}{1+(e^x)^2}\,dx$

Solution :- **Let** $u = e^x \;\Rightarrow\; du = e^x dx \;\Rightarrow\; dx = \frac{du}{e^x}$

$$I = \int \frac{e^x}{1+(e^x)^2}\,dx = \int \frac{e^x}{1+(u)^2}\,\frac{du}{e^x} \Rightarrow I = \int \frac{du}{1+u^2} = \tan^{-1}(u) + c = \tan^{-1}(e^x) + c$$

Evaluate $I = \int \frac{[\ln(x)]^2}{x}\,dx$

Solution :- **Let** $u = \ln(x) \;\Rightarrow\; du = \frac{1}{x}\,dx \;\Rightarrow\; dx = x\,du$

$$I = \int \frac{[u]^2}{x}\,x\,dx = \int u^2\,du \Rightarrow I = \frac{u^3}{3} = \frac{[\ln(x)]^3}{3} + c$$

Evaluate $\int \sqrt[3]{27e^{9x} + e^{12x}}\;\mathbf{dx}$

$$\int (\,27e^{9x} + e^{12x})^{\frac{1}{3}}\mathbf{dx} = \int (\,27e^{9x} + e^{3x+9x})^{\frac{1}{3}}\mathbf{dx}$$

$$= \int (\,27e^{9x} + e^{3x}\,e^{9x})^{\frac{1}{3}}\;\mathbf{dx}$$

$$= \int ((e^{9x})\,(27 + e^{3x}))^{\frac{1}{3}}\ dx$$

$$= \int (e^{9x})^{\frac{1}{3}}\,(\ 27 + e^{3x}\ )^{\frac{1}{3}}\ \ dx$$

$$= \int e^{3x}\,(\ 27 + e^{3x}\ )^{\frac{1}{3}}\,dx$$

$$u = \ 27 + e^{3x} \qquad , \ \ du = 3e^{3x}dx \qquad , \quad dx = \frac{du}{3e^{3x}}$$

$$\int e^{3x}\,(\ 27 + e^{3x}\ )^{\frac{1}{3}}\,dx \quad = e^{3x}\ \ u^{\frac{1}{3}}\ \ .\frac{du}{3e^{3x}}$$

$$= \frac{1}{3}\ \int\ u^{\frac{1}{3}}\ du$$

$$= \ \frac{1}{3}\ \frac{u^{\frac{4}{3}}}{4/3} \quad + \ \ C$$

$$= \ \frac{1}{3}\,.\frac{3}{4} \quad 27 + e^{3x}\ )^{\frac{4}{3}}$$

Evaluate $\int_0^{\sqrt{3}} x\, e^{(\frac{x^2}{3})}$

$$u = \frac{x^2}{3} \quad , \quad \frac{du}{dx} = \frac{1}{3} \,.\, 2x$$

$$du = \frac{2}{3}\, x\, dx \quad , \quad dx = \frac{3}{2}\, x\, du$$

$$\int_0^{\sqrt{3}} x\, e^u \,.\, \frac{3}{2}\, x\, du = \frac{3}{2} \int_0^{\sqrt{3}} e^u\, du$$

$$= \frac{3}{2}\, [e^u]_0^{\sqrt{3}} = \frac{3}{2} \left[e^{\frac{x^2}{3}}\right]_0^{\sqrt{3}}$$

$$= \frac{3}{2} \left[e^{\frac{(\sqrt{3})^2}{3}} - e^{\frac{0^2}{3}}\right] = \frac{3}{2}\, \left[e^1 - e^0\right]$$

$$= \frac{3}{2}\, [e - 1] = \approx\ 2.577 \ldots$$