Al-Mustaqbal University

Department of Medical Instrumentation Techniques Engineering

Mathematics I

LECTURE 4

Integration of Exponential and Logarithmic Functions

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Evaluate $\int \frac{sex\,3x\;\;tan\,3x}{16+\,sec^2}$

u = sec 3x du = 3sec3x tan3x

$$= \int \frac{sex\;3x\;tan\;3x}{16+\;u^2}\;.\;\frac{du}{3sex3x\;tan3x}$$

$$= \frac{1}{3}\int \frac{1}{4^2\;+u^2}\;du \;=\; \frac{1}{3}\;.\;\frac{1}{a}\;\tan^{-1}\left(\frac{u}{a}\right)+c$$

$$= \frac{1}{12}\;\tan^{-1}\left(\frac{sec\;3x}{4}\right)+c$$

Evaluate $\int (sinx + cosx)^2$

$$\int (sin\;x + cox\;x)^2 = \int \left(sin^2x + 2sinxcos\;x + cos^2x\right)dx$$

$$= \int \left(sin^2x + cox^2x\right) + 2\;sin\;x\;cosx\;)dx$$

$$= \int (1 + 2sinx\;cosx\;)dx$$

$$= \int 1 + 2\;\int sinx\;cos\;x\;dx$$

$$= x + 2\;\int sin\;x\;cosx\;dx$$

$$u = sinx \qquad du = cos\, xdx \qquad dx = \frac{du}{cox}$$

$$= \quad x + 2\int u \quad cosx \quad .\frac{du}{cosx}$$

$$= x + 2\int u \; du$$

$$= x + 2\,.\frac{u^2}{2} + c$$

$$= x + u^2 + c \qquad = x \; + sin^2 x + c$$

Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}} dx$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin x)^{\frac{-1}{2}} \cos x \; dx$$

$$u = sin\, x \quad , \quad du = cosx\, dx \quad , \quad dx = \frac{du}{cos\, x}$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (u)^{\frac{-1}{2}} \cos x \, . \, \frac{du}{\cos x} = \left[\frac{(u)^{\frac{1}{2}}}{\frac{1}{2}}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \left[2\sqrt{\sin x}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 2\sqrt{\sin\frac{\pi}{2}} - 2\sqrt{\sin\frac{\pi}{4}} = 2\sqrt{1} - 2\sqrt{\frac{1}{2}}$$

$$= 2 - \sqrt{2}\cdot\sqrt{2}\cdot\frac{1}{\sqrt{2}} = 2 - \sqrt{2}$$

Evaluate $I = \int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} dx$

Solution :- **Let** $u = \sin^{-1}(x) \Rightarrow du = \frac{1}{\sqrt{1-x^2}} dx \Rightarrow dx = \sqrt{1-x^2}\, du$

$$I = \int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} dx = \int \frac{u}{\sqrt{1-x^2}} \sqrt{1-x^2}\, du \Rightarrow I \int u du = \frac{u^2}{2} + c = \frac{[\sin^{-1}(x)]^2}{2} + c$$