



$$\begin{aligned}
 u &= \sin x & du &= \cos x dx & dx &= \frac{du}{\cos x} \\
 &= x + 2 \int u \cos x \cdot \frac{du}{\cos x} \\
 &= x + 2 \int u du \\
 &= x + 2 \cdot \frac{u^2}{2} + c \\
 &= x + u^2 + c & & & &= x + \sin^2 x + c
 \end{aligned}$$



Evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}} dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin x)^{-\frac{1}{2}} \cos x dx$$

$$u = \sin x , \quad du = \cos x dx , \quad dx = \frac{du}{\cos x}$$



Evaluate $\int \frac{-e^{x^{-1}}}{x^2} dx$

$$= - \int x^{-2} e^{x^{-1}} dx$$

$$u = x^{-1}, \quad du = -1 \cdot x^{-2} dx$$

$$= - \int x^{-2} e^u \cdot \frac{du}{-x^2}$$

$$= \int e^u du = e^u + c = e^{x^{-1}} + C$$



Evaluate $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$

$$u = e^x - e^{-x}, \quad du = e^x - e^{-x} \cdot (-1)$$

$$du = e^x + e^{-x} dx, \quad dx = \frac{du}{e^x + e^{-x}}$$



$$\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx = \int \frac{e^x + e^{-x}}{u} \cdot \frac{du}{e^x + e^{-x}} \\ = \int \frac{du}{u} = \ln(u) + c = \ln(e^x - e^{-x})$$



Evaluate $\int e^x \sqrt{1 - e^{2x}} dx$

$$u = 1 - e^{2x}, \quad du = -2e^{2x} dx, \quad dx = \frac{du}{-2e^{2x}} \\ = \int e^x \sqrt{u} \cdot \frac{du}{-2e^{2x}} \\ = -\int u^{\frac{1}{2}} du = -\frac{2}{3} u^{\frac{3}{2}} + c = -\frac{2}{3} (1 - e^{2x})^{\frac{3}{2}} + c$$



Evaluate $\int \frac{(\ln x)^2}{x} dx$

$$u = \ln x, \quad du = \frac{1}{x} dx, \quad dx = x du \\ \int \frac{u^2}{x} \cdot x du = \int u^2 du \\ = \frac{u^3}{3} + C$$



Evaluate $\int_0^1 (1 + e^x)^2 e^x dx$

$$u = 1 + e^x , \quad du = e^x dx , \quad dx = \frac{du}{e^x}$$

$$\int_0^1 (1 + e^x)^2 e^x dx = \int_0^1 (u)^2 e^x \cdot \frac{du}{e^x}$$

$$\int_0^1 (u)^2 \cdot du = \left[\frac{u^3}{3} \right]_0^1 = \frac{1}{3} [u^3]_0^1$$

$$= \frac{1}{3} [1 + e^x]_0^1 = \frac{1}{3} [1 + e^1] - [1 + e^0]$$

= ...

Good Luck ..