

CH-6

* Belts : الأحزمة

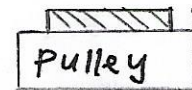
* Function الوظيفه

Belts are used to transmit power from one shaft to another by pulleys, which rotates at the same or different speeds.

* Types of belts : أنواع الأحزمة

I) Flat belt : الحزام المسطح

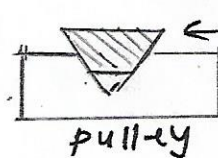
It is used to transmit power from one pulley to another, when the distance between them not more than (8) meters $\rightarrow$ fig (a)



$\leftarrow$ flat belt.

II) V-belt حزام V Fig (a)

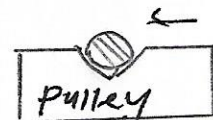
It is used to transmit power, when the distance between two pulleys is very small $\rightarrow$ fig (b)



$\leftarrow$ V-belt

III) Circular belt (rope) الحبال Fig (b)

It is used to transmit high power between two pulley, when the distance between them more than (8) meters $\rightarrow$ fig (b)



$\leftarrow$ rope

* Types of belt drives according fig (b)

I) Light drive : نظام خفيف

It is use to transmit small power, with speed about 10 m/s

II) Medium drive : نظام متوسط

It is used to transmit medium power, with speed over 10 m/s.

III) Heavy drive : نظام ثقيل

It is used to transmit large power, with speed about (22 m/s),

* Selection of a belt drive: شروط اختيار الاطراف

- 1) Speed of driver and driven shafts. السرعة
- 2) Power to be transmitted. القدرة
- 3) Positive drive requirement. متطلبات النقل
- 4) Space available. الفراغ المتاح
- 5) Speed reduction ratio. نسبة السرعة
- 6) Center distance between shafts. المسافة المركزية
- 7) Shaft layout. وضع الاعمدة
- 8) Service condition. شروط الصيانة

* The conditions of the transmitted power depend on the following: شروط نقل الطاقة

- 1- Velocity of belt.
- 2- tension in belt.
- 3- The arc of contact.
- 4- Condition of work of belt (like, T, etc.)

* materials of belts مواد الاطراف

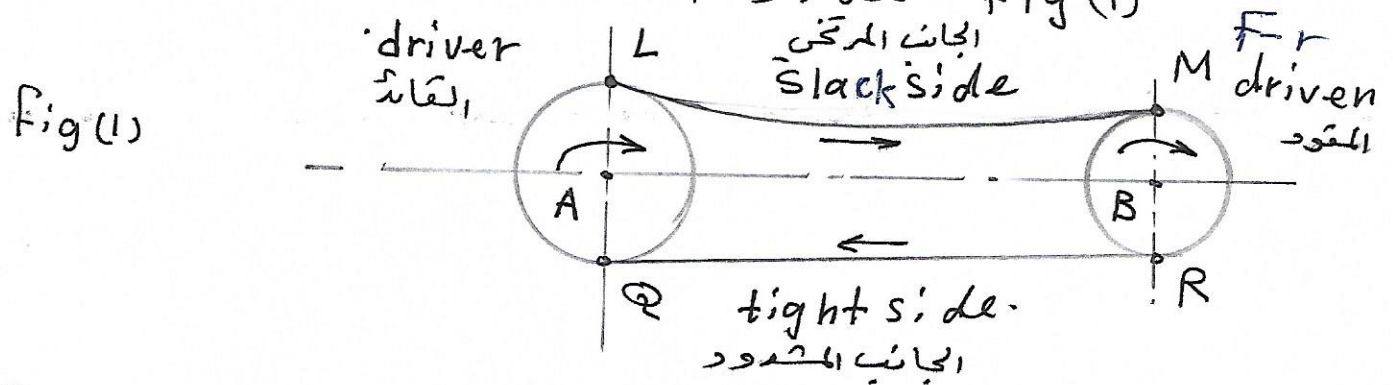
- 1) leather belts. من الجلد
2. Cotton or fabric belts. القطن
3. Rubber belts. المطاط
- 4- Balata belts. البصاني

* Types of belt drive according to the direction of rotation.

- I, open belt drive $\rightarrow$ in same direction
- II, Cross belt drive $\rightarrow$ in opposite direction.

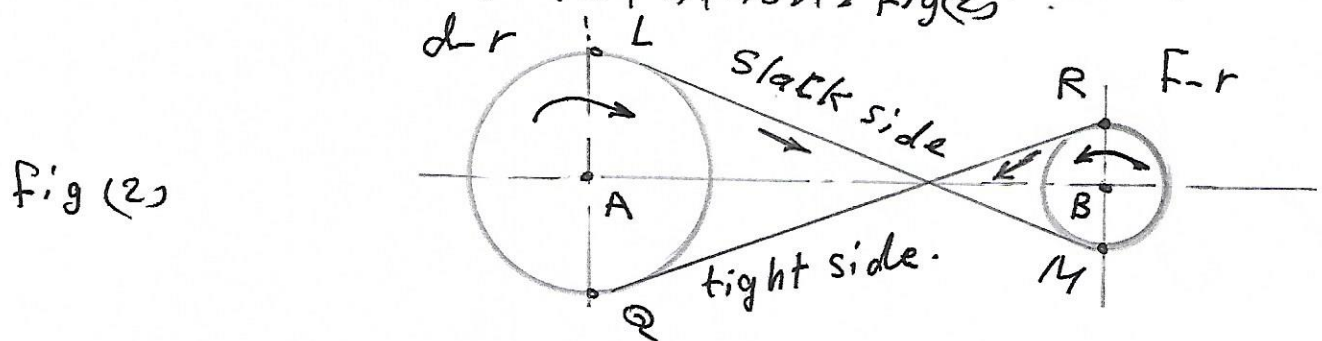
* Types of Flat belt drive:

I, open belt drive :
 It is used to transmit power between two parallel shafts rotating in the same direction. In this case because the tension of the driver, the belt have upper slack side and lower tension side, fig (1).



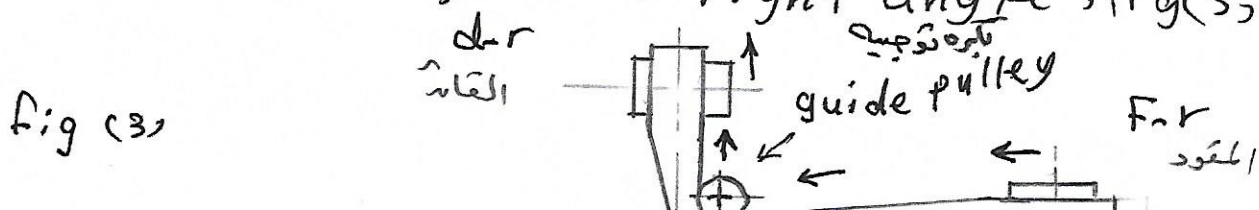
II, Crossed belt drive

It is used to transmit power between two parallel shafts rotating in opposite direction of rotation, figs.



III, Quarter turn belt drive

It is used to transmit power between two shafts arranged at right angle, fig (3).



IV - Belt drive with idler pulleys

This type with idler pulleys is used to transmit power between parallel shafts, when open drive cannot be used due to small angle of contact on the smaller pulley $\rightarrow$ Fig (4).
 يستخدم هذا النظام لنقل القدرة بين المحاور المتوازية عندما تكون زاوية التماس على البكرة الصغيرة قليلة جداً لا تكفي لاحتكاك انتقال النظام المفتوح.

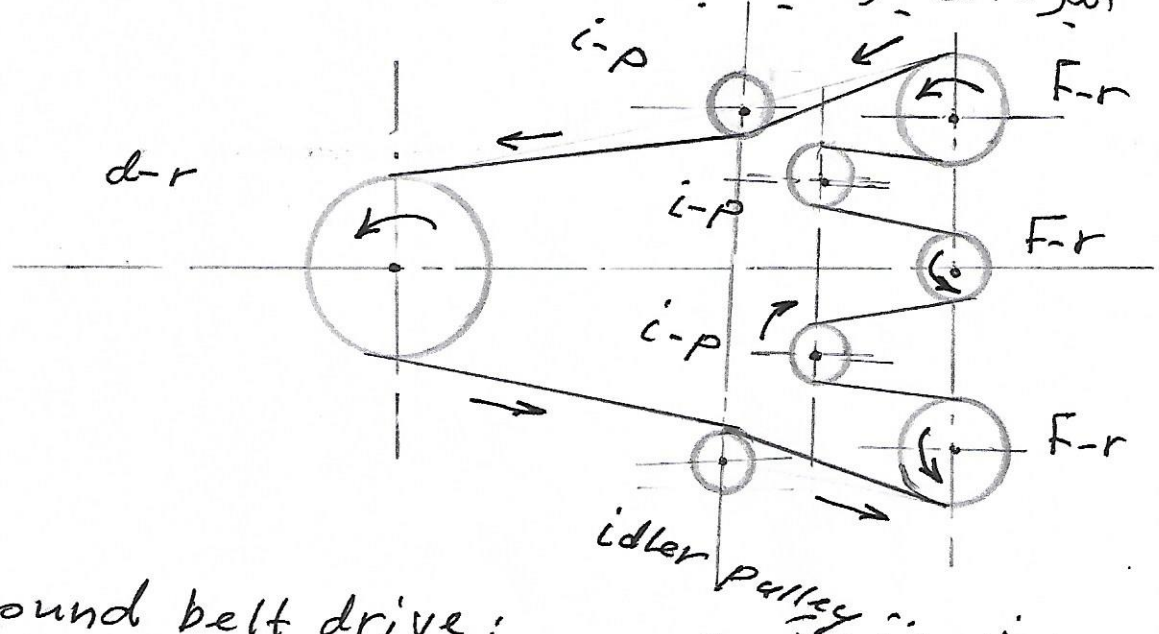


Fig (4) :

V - Compound belt drive :

It is used to transmit power from one shaft to another through number of pulleys, when the distance between them is so long. (figs).
 يستخدم لنقل القدرة لمسافات طويلة.

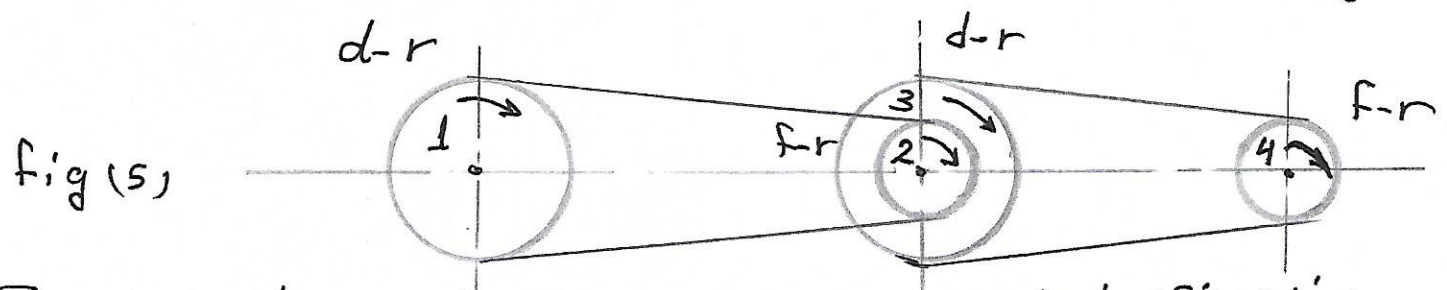


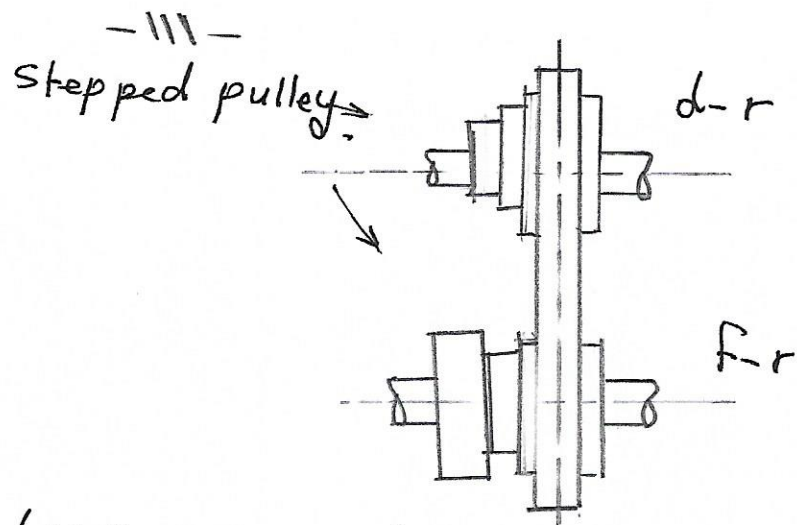
fig (5)

VI - Stepped (Cone) pulley drive.

It is used for changing the speed of follower when the speed of driver is constant.

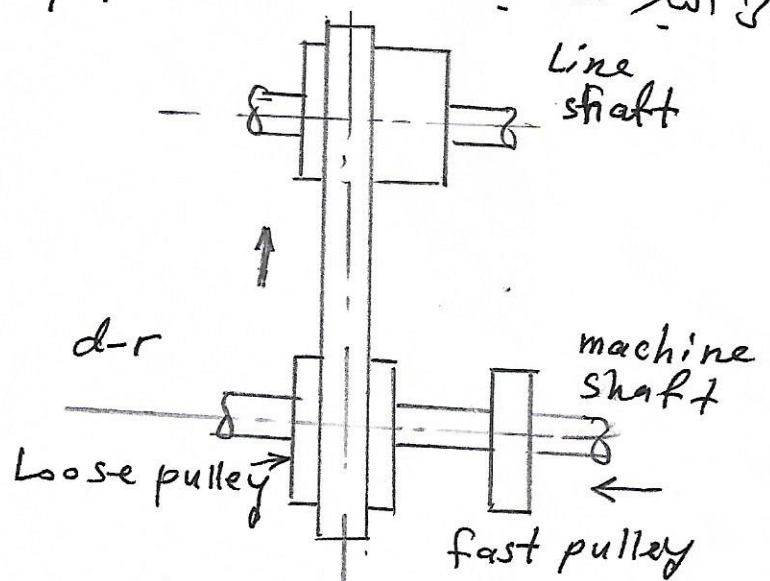
fig (6) . يستخدم لتغيير السرعة المتابعة عندما تكون سرعة الدافع ثابتة.

Fig (6)



VII, Fast and loose pulley drive.
 It is used to transmit power from driver to follower with speed or without speed by pushing the belt on the Loose pulley by mean - fig(7).
 نستخدم لنقل القدرة من الشايف الى المتوابع
 بسرعه او بدون سرعه عن طريق نقل الحزام
 الى الشايف او المتابع بواسطه البكره.

Fig (7)



* Velocity ratio for open flat belt drive

Peripheral velocity of the belt on the driver and driven pulley be:

$$V_1 = \frac{\pi d_1 n_1}{60} \rightarrow \text{for } d-r$$

$$V_2 = \frac{\pi d_2 n_2}{60} \rightarrow \text{for } f-r$$

$$\text{But } V_1 = V_2 \Rightarrow \pi d_1 n_1 = \pi d_2 n_2$$

∴ Velocity ratio with no slip -

$$R_v = \frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{--- (1) or } = \frac{d_1 + t}{d_2 + t} \quad \text{with thickness of belt.}$$

* Velocity ratio for Compound belt drive

$$V_1 = \pi d_1 N_1, \quad V_2 = \pi d_2 N_2, \quad V_3 = \pi d_3 N_3, \quad V_4 = \pi d_4 N_4$$

$$\text{But } V_1 = V_2 = V_3 = V_4 \Rightarrow \frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

$$R_v = \frac{N_4}{N_1} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

* Slip of belt :

When the frictional grip between belt and pulley is insufficient, we get slip of belt, and this reduce the velocity ratio and power and induce heat between belt and pulley.

Let $S_1\%$, $S_2\%$ a slip between d-r and f-r
∴ Velocity of passing belt over d-r per second

$$V_1 = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \times \frac{S_1}{100} = \frac{\pi d_1 N_1}{60} \left(1 - \frac{S_1}{100}\right) \quad \text{--- (1)}$$

and for f-r

$$V_2 = \frac{\pi d_2 N_2}{60} = \frac{V_1}{2} - \frac{V_1}{2} \times \frac{S_2}{100} = \frac{V_1}{2} \left(1 - \frac{S_2}{100}\right) \quad \text{--- (2)}$$

By sub V_1 from (1) in (2) we get - Because $V_1 = V_2$

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60} \left(1 - \frac{S_1}{100}\right) \left(1 - \frac{S_2}{100}\right)$$

$$\therefore \frac{N_2}{N_1} = \frac{d_1}{d_2} \left(1 - \frac{S_2}{100} - \frac{S_1}{100} + \frac{S_1}{100} \times \frac{S_2}{100}\right) \rightarrow \text{very small}$$

By neglecting $\frac{S_1}{100} \times \frac{S_2}{100} \rightarrow \text{very small}$

$$\therefore \frac{N_2}{N_1} = \frac{d_1}{d_2} \left(1 - \frac{s_1}{100} - \frac{s_2}{100} \right) = \frac{d_1}{d_2} \left(1 - \frac{s_1 + s_2}{100} \right) \rightarrow S$$

But $S = s_1 + s_2 \rightarrow$ total slip.

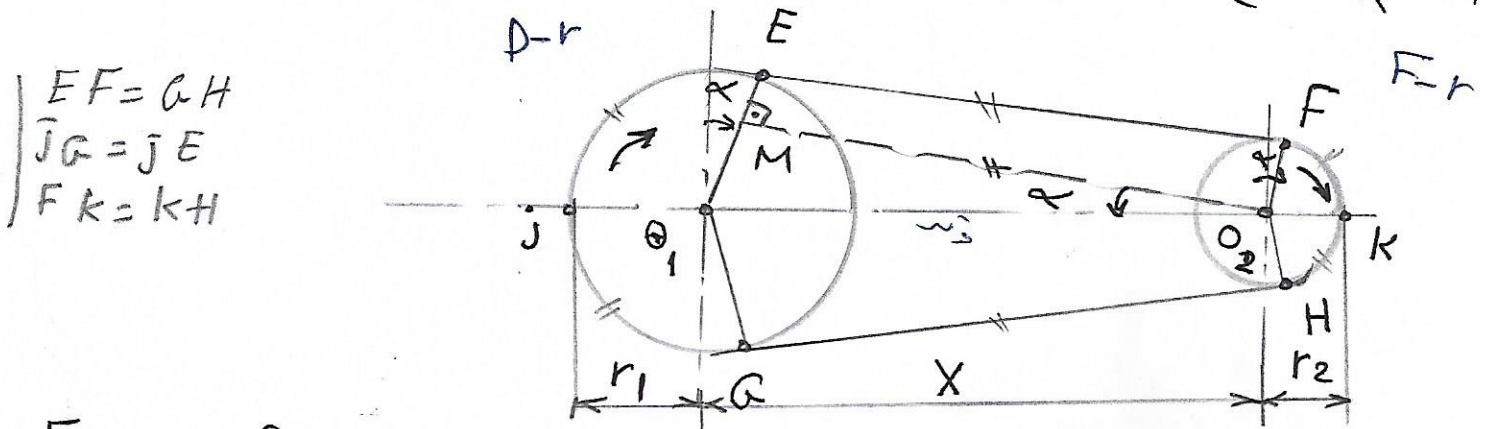
$$\therefore \boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2} \left(1 - \frac{S}{100} \right)} \quad \text{--- (3)}$$

by taking thickness of belt

$$\therefore \frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t} \left(1 - \frac{S}{100} \right)$$

* length of an open belt drive

حساب طول النظام المفتوح



From fig we know that the two pulleys rotate in same direction.

let $\begin{cases} r_1, r_2 & \text{radius of Large and small pulley} \\ X & \text{center distance} \\ L & \text{Length of belt} = ? \end{cases}$

In order to calculate the length of belt, at first we draw line parallel to EF from O_2 .

where E, G, F, H , points at which the belt leave the large and small pulley

$\therefore O_2M \perp O_1E \rightarrow$ because the tangent $\perp$ radius so that the parallel line. and let $\alpha = \angle MO_2O_1$ in radians.

and triangle $\Delta O_1 M O_2$ a right triangle.
The length of belt be:

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2 (\text{Arc } JE + EF + \text{Arc } FK) \quad \text{--- (1)}$$

From $\Delta O_1 O_2 M$ right triangle.

$$\sin \alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - EM}{O_1 O_2} = \frac{r_1 - r_2}{x}$$

$$\text{But } \sin \alpha = \alpha = \frac{r_1 - r_2}{x}$$

and Arc $\widehat{JE} = r_1 \times (\frac{\pi}{2} + \alpha)$ - α - very small.

$$\text{Arc } \widehat{FK} = r_2 \times (\frac{\pi}{2} - \alpha)$$

$\widehat{JE} = r_1 \alpha$  (2)
* طول القوس = الزاوية * نصف القطر

$EF = M O_2$, and from $\Delta O_1 M O_2$ - مربع الدائرة = مجموع مربعي أضلعي

$$\therefore EF = M O_2 = \sqrt{(O_1 O_2)^2 - (O_1 M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$EF = x \sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

By expand this equation by binomial theorem $\text{بفتح المعادلة بالمتوالية الهندسية كفتح$

$$EF = x \left[1 - \frac{1}{2} \left(\frac{r_1 - r_2}{x}\right)^2 + \dots \right] = x - \frac{(r_1 - r_2)^2}{2x} \quad \text{--- (4)}$$

By sub (2) (3) (4) in (1) $\rightarrow$ we get.

$$L = 2 \left[r_1 \left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \left(\frac{\pi}{2} - \alpha\right) \right]$$

$$= 2 \left[r_1 \frac{\pi}{2} + r_1 \alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \frac{\pi}{2} - r_2 \alpha \right]$$

$$= 2 \left[\frac{\pi}{2} (r_1 + r_2) + \alpha (r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x} \right]$$

$$= \pi (r_1 + r_2) + 2\alpha (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

But $\frac{r_1 - r_2}{x} = \alpha \rightarrow$ from $\sin \alpha$

$$\therefore L = \pi (r_1 + r_2) + 2 \left(\frac{r_1 - r_2}{x}\right) \times (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$L = \pi(r_1 + r_2) + 2 \frac{(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$

$$L = \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$

* Length of a cross belt drive

In the cross belt drive, the two pulleys rotate in opposite direction.

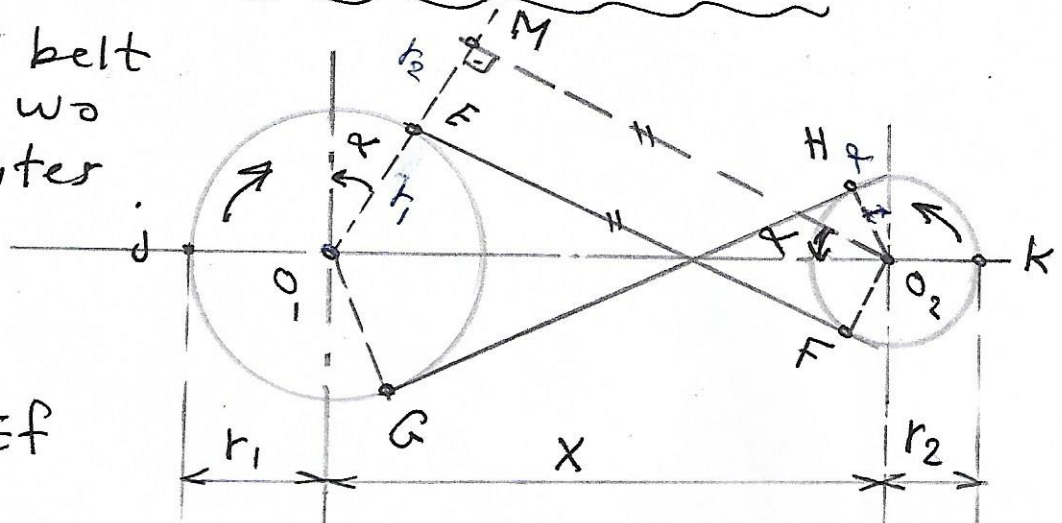


Fig (1)
draw $O_1M \parallel EF$
and $\perp O_1M$

Length of cross belt drive may be determine as follows:

$$L = \widehat{GJE} + EF + \widehat{FKH} + HG$$

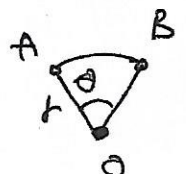
$$= 2(\widehat{JE} + EF + \widehat{FK}) \quad \text{--- (1)}$$

From triangle ΔO_1O_2M

$$\sin \alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + EM}{O_1O_2} = \frac{r_1 + r_2}{x}$$

Since $\sin \alpha \approx \alpha$ very small

$$\therefore \sin \alpha = \alpha = \frac{r_1 + r_2}{x} \quad \text{--- (2)}$$



$$\widehat{AB} = r\theta =$$

$$\text{Arc } \widehat{JE} = r_1 \left(\frac{\pi}{2} + \alpha \right) \quad \text{--- (3)}$$

$$\text{Arc } \widehat{FK} = r_2 \left(\frac{\pi}{2} + \alpha \right) \quad \text{--- (4)}$$

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2}$$

مربع الدائرة = مجموع مربع الضلعين
في المثلث القائم الزاوية

$$= \sqrt{x^2 - (r_1 + r_2)^2}$$

$$\therefore EF = x \sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

تفتح هذه العلاقة بفتح المثلث

Expanding this equation by binomial theorem.

$$\therefore EF = x \left[1 - \frac{1}{2} \left(\frac{r_1 + r_2}{x}\right)^2 + \dots \right] = x - \frac{(r_1 + r_2)^2}{2x} \quad \text{--- (5)}$$

By substituting (2), (3), (4), (5) in (1) we get

$$L = 2 \left[r_1 \left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \left(\frac{\pi}{2} + \alpha\right) \right]$$

$$= 2 \left[r_1 * \frac{\pi}{2} + r_1 \alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \frac{\pi}{2} + r_2 \alpha \right]$$

$$= 2 \left[\frac{\pi}{2} (r_1 + r_2) + \alpha (r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x} \right]$$

$$= \pi (r_1 + r_2) + 2\alpha (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

But $\sin \alpha = \frac{r_1 + r_2}{x}$ because very small $\frac{r_1 + r_2}{x}$

$$\therefore L = \pi (r_1 + r_2) + 2 \frac{(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

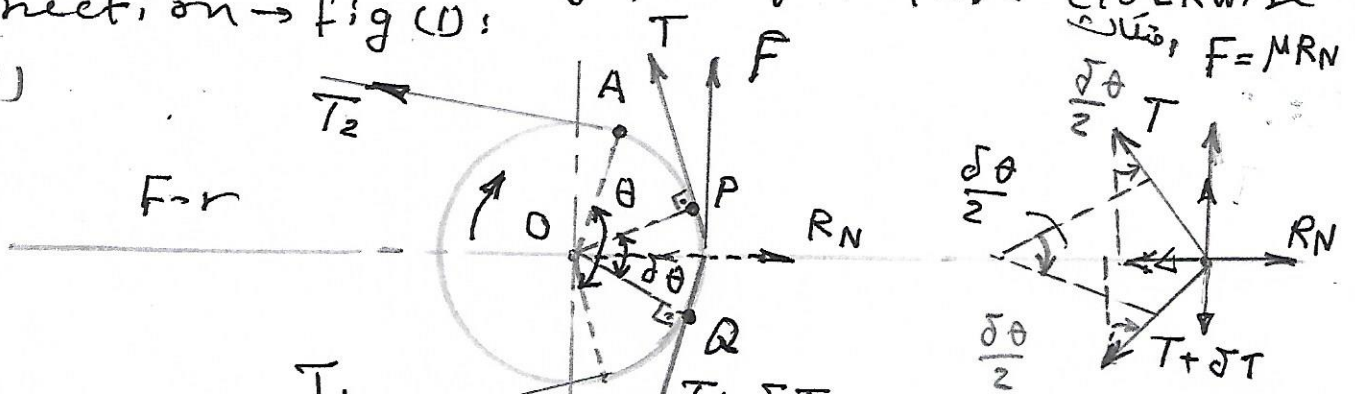
$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$L = \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}$$

* Ratio of driving tension for flat belt drive

assume the driving pulley rotate clockwise direction $\rightarrow$ fig (1):

Fig (1)



let $\left\{ \begin{array}{l} T_1 - \text{tension in tight side} \\ T_2 - \text{tension in slack side} \\ \theta - \text{Angle of contact in radians} \end{array} \right.$

If we consider a small arc PQ is in equilibrium under the following forces

$\left\{ \begin{array}{l} T \text{ at } P \\ T + \delta T \text{ at } Q \\ R_N - \text{normal reaction} \\ F = \mu \times R_N - \text{frictional force} \end{array} \right.$ اذا افترضنا الجزء PQ في حالة توازن تحت تأثير هذه القوى

→ and resolving these forces horizontally and take

$$\sum F_H = 0$$

$$\therefore R_N = (T + \delta T) \sin \frac{\delta \theta}{2} + T \sin \frac{\delta \theta}{2} \quad \dots (1)$$

Since $\sin \frac{\delta \theta}{2} = \frac{\delta \theta}{2} \rightarrow \text{very small}$

$$\therefore R_N = (T + \delta T) \frac{\delta \theta}{2} + T \times \frac{\delta \theta}{2} = \frac{T \cdot \delta \theta}{2} + \frac{\delta \theta \cdot \delta T}{2} + \frac{T \cdot \delta \theta}{2}$$

By neglecting $\frac{\delta T \cdot \delta \theta}{2} \rightarrow \text{very small}$

$$\therefore R_N = T \cdot \delta \theta \quad \dots (2)$$

By resolving force vertically → we have

$$\therefore F = \mu \cdot R_N = (T + \delta T) \cos \frac{\delta \theta}{2} - T \cos \frac{\delta \theta}{2} \quad \dots (3)$$

But $\cos \frac{\delta \theta}{2} \approx 1 \rightarrow \text{very small}$

$$\therefore \mu R_N = T + \delta T - T = \delta T$$

$$\text{and } R_N = \frac{\delta T}{\mu} \quad \dots (4)$$

By equating R_N from (2) and (4) → we get

$$T \cdot \delta \theta = \frac{\delta T}{\mu} \rightarrow \frac{\delta T}{T} = \mu \cdot \delta \theta$$

∴ By integrating both sides and limit $T_2, T_1 \rightarrow 0 \rightarrow \theta$

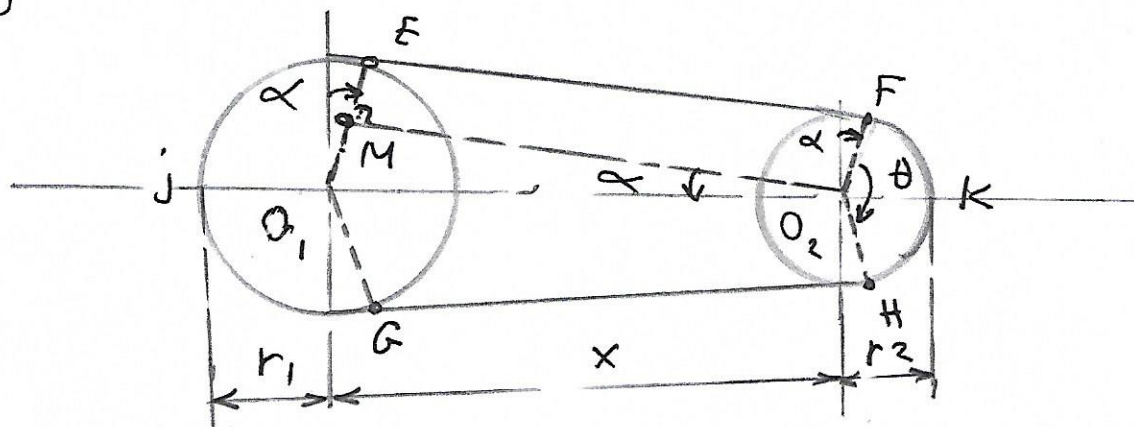
$$\therefore \int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^\theta \delta \theta \rightarrow \log_e \left[\frac{T_1}{T_2} \right] = \mu \theta \rightarrow \frac{T_1}{T_2} = e^{\mu \theta} \quad \dots (5)$$

$$\therefore 2.3 \log \left(\frac{T_1}{T_2} \right) = \mu \theta \rightarrow \text{by corresponding}$$

* Determining the angle of contact - driving pulley

- 1) angle of contact for open belt drive
fig (1)

fig (1)

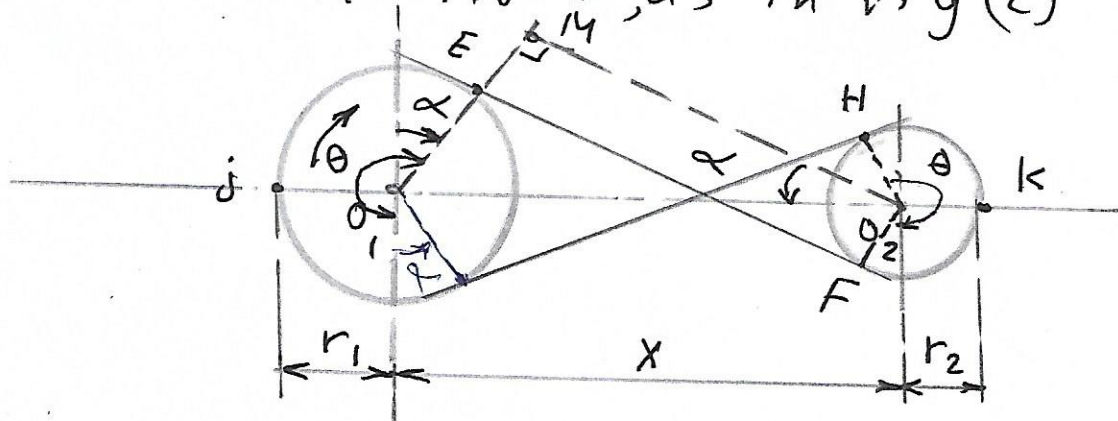


$$\sin \alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - ME}{O_1 O_2} = \frac{r_1 - r_2}{x} \quad \text{rad} \quad = (180 + 2\alpha) \frac{\pi}{180}$$

$\therefore$ angle of contact θ on small pulley

$$\theta = (180 - 2\alpha) \frac{\pi}{180} \text{ rad.} \quad \text{--- (1)}$$

- 2) For cross belt drive, as in fig (2)



$$\sin \alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + ME}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

$\therefore$ angle of contact for small pulley

$$\theta = (180 + 2\alpha) \frac{\pi}{180} \text{ rad.} \quad \text{--- (2)}$$

* Centrifugal tension:

when the belt run on pulley with speed in which induce centrifugal force, which added to the tension, and name, as Centrifugal tension as in fig (1)

Length of Fig (1)

$$\rightarrow \text{Arc } PQ = r \cdot d\theta$$

and mass of $PQ = m \cdot r \cdot d\theta$ — (1)
By sub (1) in $F_c \Rightarrow$ we get

Centrifugal force on PQ

$$F_c = m r \omega^2 = m r d\theta \times \frac{v^2}{r} = m \cdot d\theta \cdot v^2$$

* By resolving centrifugal tension horizontally and equating with a centrifugal force we get

$$T_c \sin\left(\frac{d\theta}{2}\right) + T_c \cdot \sin\left(\frac{d\theta}{2}\right) = F_c = m d\theta \cdot v^2$$

since $\sin \frac{d\theta}{2} \approx \frac{d\theta}{2} \rightarrow$ very small

$$\therefore 2 T_c \left(\frac{d\theta}{2}\right) = m \cdot d\theta \cdot v^2$$

$$\therefore \boxed{T_c = m \cdot v^2} \text{ — — — (1)}$$

* Maximum tension in belt:

may be calculated from max-stress as follow

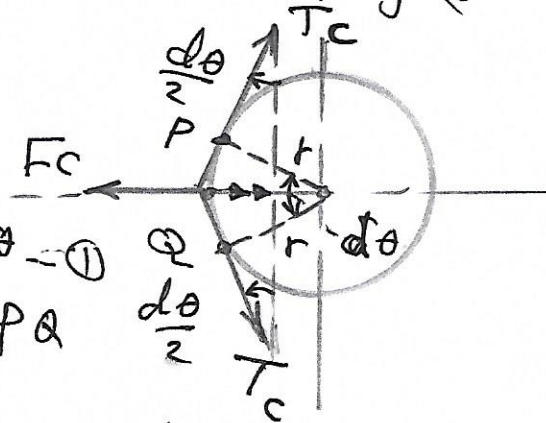
$$\sigma = \frac{T}{A} = \frac{T}{b \times t}$$

where $\left\{ \begin{array}{l} T - \text{tension} \\ b - \text{width} \\ t - \text{thickness} \end{array} \right.$

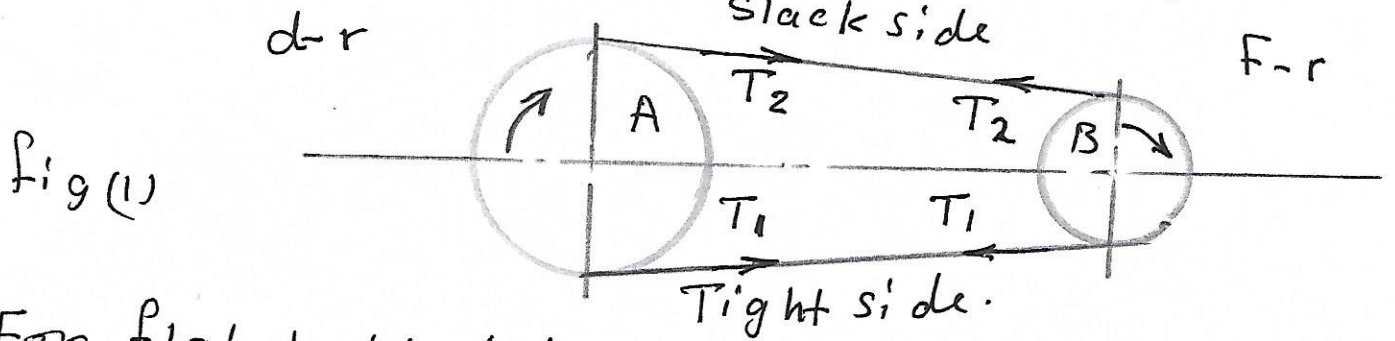
$$\therefore \boxed{T = \sigma \times b \times t} \text{ — — — (1)}$$

when we have centrifugal force:

$$\therefore \boxed{T = T_1 + F_c} \text{ — — — (2)}$$



* power transmitted by belts القوة المنقولة بالاعترقة



For flat belt drive

- T_1, T_2 - tension for tight and slack side.
- r_1, r_2 - radius of d-r and F-r pulley
- V - velocity of belt in m/s

driving force $\rightarrow F_d = T_1 - T_2$

the work done ^{in second} $W = (T_1 - T_2) \times \delta \rightarrow N \cdot m/s$ العمل في هذه الزمن

The power transmitted $P = \frac{(T_1 - T_2) \times \delta}{t} \rightarrow v$

Torque

$$P = (T_1 - T_2) \cdot V \rightarrow W \rightarrow 1 N \cdot m/s$$

$$T = (T_1 - T_2) \times r_1 \rightarrow \text{for d-r}$$

$$T = (T_1 - T_2) \times r_2 \rightarrow \text{for F-r}$$

القوة
العزم