CH-7-

* CaMS : الحدبة عنصر دوار يعطي حركة ترددية او تذبذبية الى عنصر آخر يدعى بالتابع، ولهما خط تماس واحد ويكونان زوج عالي

Introduction:

A CaM is rotating Machine element, which gives reciprocating or oscillating motion to another element known as follower.

Can and follower have a line Contact and constitute a higher pair, الحدبة تدور بسرعة منتظمة بواسطة المحرك وحركة التابع تحدد لاحقاً حسب شكل الحدبة

The Cam rotate by engine with uniform Speed, but the motion of the follower is predetermined according to the shape of the Cam.

The Cams are widly used for operating the inlet and out let valves of internal Combustion engine, in automatic leath, textile machines, paper cutting machines..etc.

(محرك الاحتراق — مخرطة — ماكنة نسيج — ماكنة تقطيع الورق)

* Classification of followers : تصنيف التوابع

The followers may classified as duscussed below : ممكن ان تصنف حسب مايلي:

I) According to the surface in contact : حسب سطح التماس

1, knife edge follower : التابع ذات الحافة السكينية.

In which the follower has a knife edge in contact, and has a sliding motion in contact and low wear → as in fig (a).

2 - Roller follower : التابع ذات الرولة.

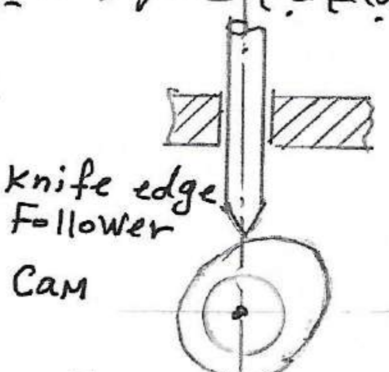
In which the follower has a roller end, and has a rolling motion in contact, which reduce the wear greatly. It is used morly in gas and oil engine, and air craft engine → fig (b).

3 - <u>Flat face or (mushroom) follower</u>: التابع المسطح النهاية

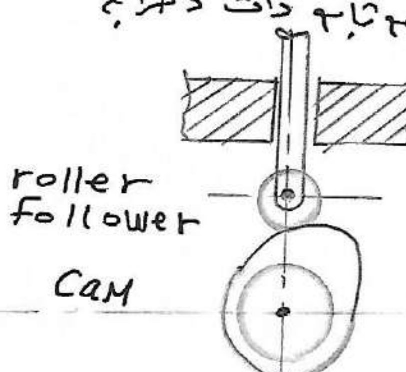
In which the follower has a flat face end in contact, it has a sliding motion in contact. as shown in fig(c), and reduce the wear morly, when the the follower has off-set axis with a Cam as shown in fig (d).

4) <u>Spherical faced follower</u>: التابع ذات النهاية الكروية

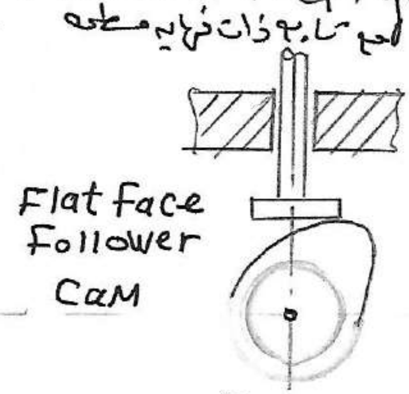
In which the follower has a spherical face end. it is used to minimise the stresses in the Contact surface as shown in fig(e)(f).



Fig(a)
Cam with knife edge Follower



Fig(b)
Cam with roller Follower



Fig(c)
Cam with flat face Follower

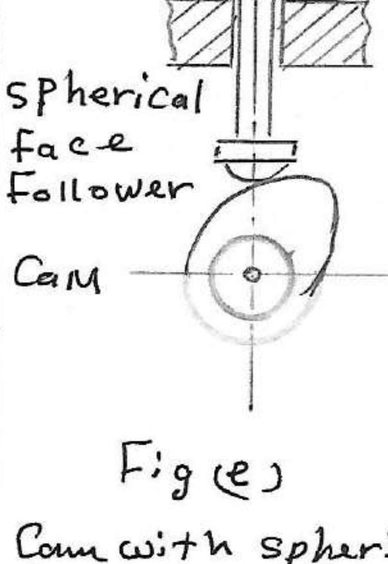
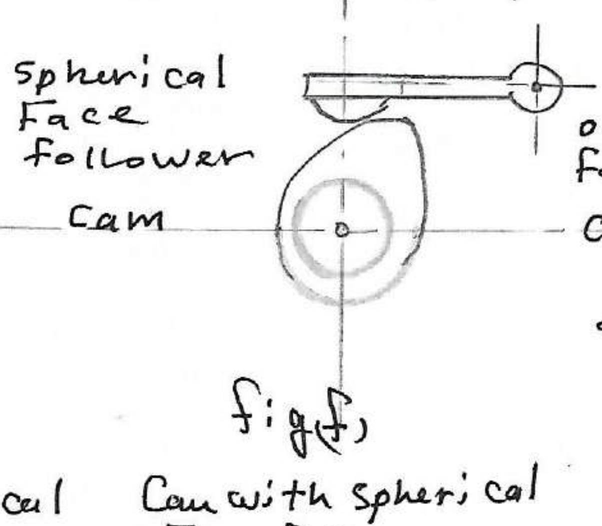
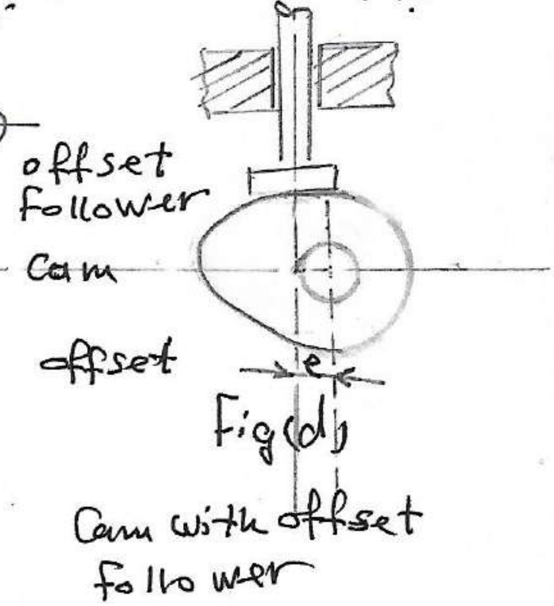


Fig (e)
Cam with spherical Face follower



fig(f)
Cam with spherical Face follower



Fig(d)
Cam with offset follower

التصنيف طبقاً لحركة التابع

II) According to the motion of the follower.

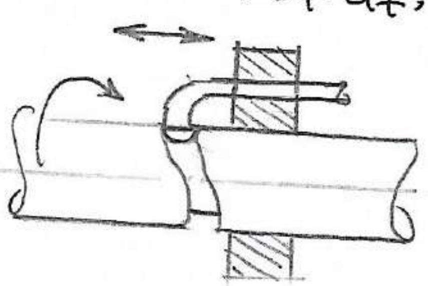
a) Reciprocating or translating follower: ذات الحركة الترددية او الانتقالية

In which the follower reciprocates in guides, as the Cam rotates uniformly, fig(g). التابع مجرى

فيها التابع يتردد في مجرى، بينما الحدبة تدور بسرعة منتظمة .

b) Oscillating or rotating follower: التابع ذات الحركة التذبذبية

In which the follower is oscillating and the Cam rotating uniformly, fig(h). فيها التابع يتذبذب والحدبة تدور بسرعة منتظمة



Fig(g)
Cylindrical cam with reciprocating follower

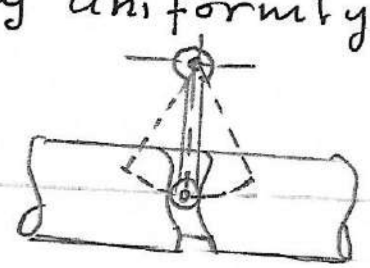


fig (h)
Cylidrical Cam with oscillating follower

طبقاً لمسار حركة التابع

III) According to the bath of motion of the follower:

a) Radial follower: تابع مركزي

In which the axis of the follower pass through the Center of the Cam. عندما يمر محور التابع خلال مركز الحدبة
as in fig a, b, c, d)

b) off-set follower; تابع غير مركزي

In which the axis of the follower pass out the Center of the Cam. عندما يمر محور التابع خارج مركز الحدبة
Fig (f).

* Classification of Cams: تصنيف الحدبات

1) Radial or disc Cam. الحدبة القطرية او القرصية

In which the follower reciprocates of oscille-tes ⊥ to the Cam axis. فيها التابع يتردد او يتذبذب عمودياً على محور الحدبة
Fig (a, -- f).

2) Cylindrical Cam : الحدبات الاسطوانية

In which the follower reciprocates or oscill-
-ates in direction parallel to the axis of
the Cam, fig (g, h) and the follower rides in a groove in the cylinder.

فيها التابع يتحرك ترددياً او تذبذبياً بموازاة محور الحدبة.
وفيها التابع يتحرك بمجرى في الاسطوانة.

* Terms of radial Cam : المصطلحات الفنية للحدبة القطرية

1- Base circle : الدائرة الاساسية

It is the Smallest Circle Camu be drawn to the Cam profile.

اصغر دائرة يمكن رسمها على سطح الحدبة

2- Trace point : النقطة المرجعية لمنحني الخطوة.

It is the reference point on the follower is used to generat the pitch curve.

نقطة على سطح التابع وتستخدم لتوليد منحني الخطوة.

3- pressure angle ; زاوية الضغط

It is the angle between the direction of the follower motion and the and a normal to the pich curve.

الزاوية المحصورة بين محور التابع والعمود على منحني الخطوة.

4- pitch point ; نقطة الخطوة.

A point on the pitch curve having max-m pressure angle

نقطة على منحني الخطوة ذات اكبر زاوية ضغط

5/ pitch circule : دائرة الخطوة

It is a circle drawn From the Center of the Cam through the pitch point.

دائرة ترسم من مركز الحدبة خلال نقطة الخطوة

6- pitch curve : منحني الخطوة

It is a curve generated by the trace point of the moving follower.

منحني يتولد بالنقطة الراسمة للتابع المتحرك

7- Prime circle

الدائرة الاولية

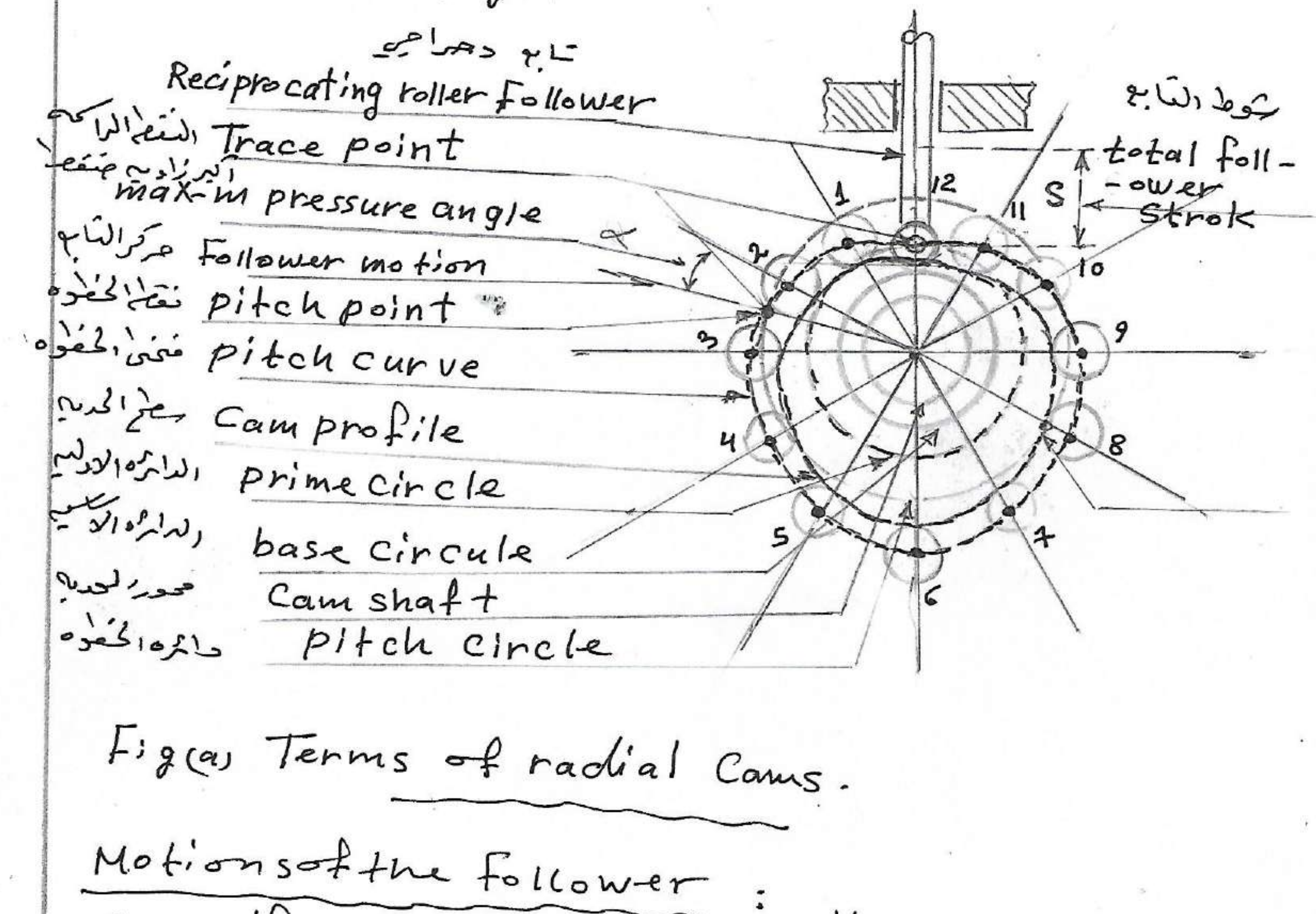
It is the smallest circle can be drawn from the centre of the Cam and be tangent to the pitch curve.

هو اصغر دائرة ترسم من مركز الكامة ومماس لمنحنى الخطوة.

8- lift or strok :

هي البعد الاعظم للتابع من ادنى الى اعلى موقع له.

It is the max-m travel of the follower from the lowest to the topmost position. as shown in fig(a).



Fig(a) Terms of radial Cams.

Motions of the follower :

I) Uniform velocity — الحركة بسرعة منتظمة

II) Simple harmonic motion. — الحركة التوافقية

III) Uniform acceleration and retardation. — الحركة بتعجيل او تباطؤ منتظم

IV) Cycloidal motion. — الحركة الدائرية

الإزاحة والسرعة والتعجيل عندما يتحرك التابع بسرعة منتظمة

I) Displacement, velocity and acceleration, when the follower moves with uniform velocity

For example, we take knife-edged follower.

1, uniform displacment (motion) إزاحة الحركة المنتظمة

it is a Simple equation of straight-line. or uniform motion.

$y = C\theta$ --- (1)

where: y - represent follower displacement in mm,

θ - Cam angle in radians, $\theta_{rad} = \theta_{deg} * \frac{\pi}{180}$,

C - Constant determined from boundary condition.

If we take:

S_o - out strok in (mm) - or total distance to rise,

θ_o - Cam angle in rad, when the Cam rotate to effect this rise.

$\therefore S_o = C\theta_o \Rightarrow C = \frac{S_o}{\theta_o}$ -- (2)

by sub-ing C from (2) in (1) → we get

$y = \frac{S_o}{\theta_o}\theta$ --- (3) → displacement eq-n.

when: $\theta = \theta_o$ → for out strok.

$\therefore \boxed{y_o = S_o}$ → displacement for outstrok

and when $\theta = \theta_R$

$\therefore \boxed{y_R = S_R}$ → displacement for returnstoke.

2) Velocity of uniform m-n. سرعة الحركة المنتظمة

may be obtain by finding the first derivative with respect to time → for (3)

$\therefore v = \dot{y} = \frac{d}{dt}\left(\frac{S_o}{\theta_o}\theta\right) = \frac{S_o}{\theta_o}\cdot\frac{d\theta}{dt} = \frac{S_o}{\theta_o}\omega$ -- (4)

تعجيل الحركة المنتظمة

3) Acceleration of uniform motion.

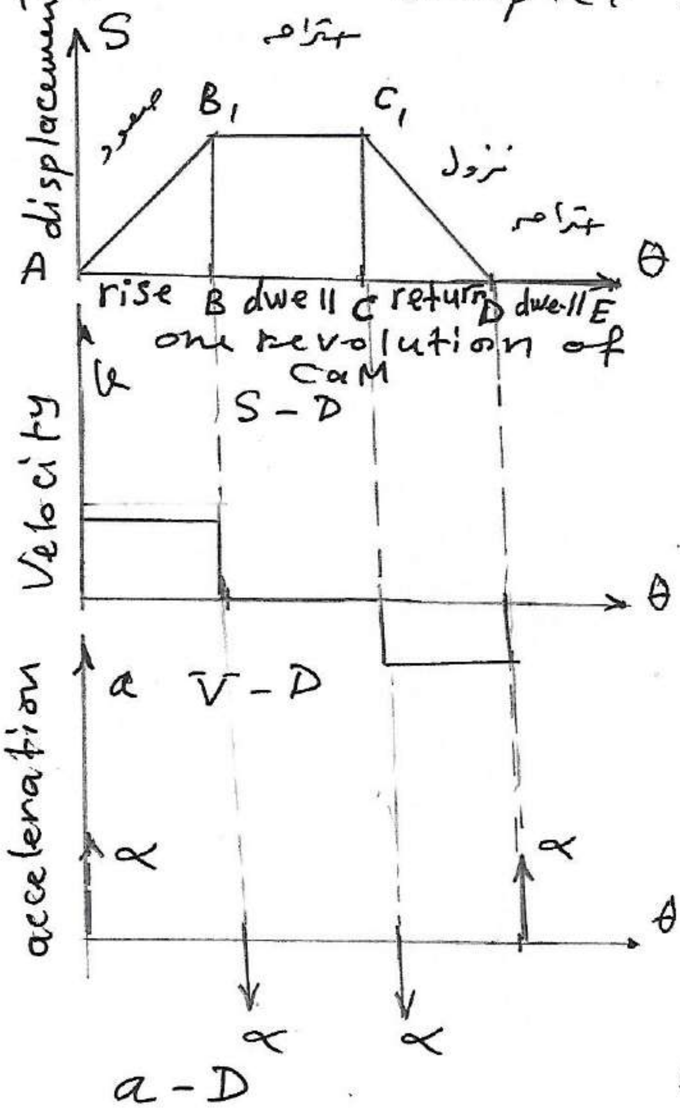
It is obtain by finding the second derivative of Displacement, or first derivative of Velocity.

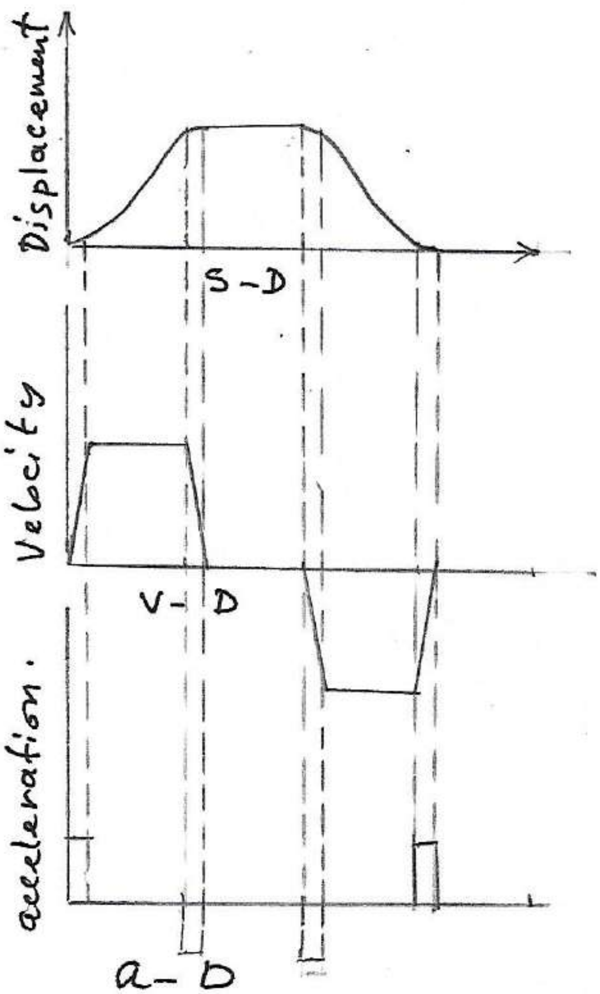
$$a = \ddot{y} = \frac{d}{dt}\left(\frac{S_o}{\theta_o}\theta\right)$$

or $$a = \ddot{y} = \dot{V} = \frac{d}{dt}\left(\frac{S_o}{\theta_o}\omega\right) = \frac{S_o}{\theta_o}\frac{d\omega}{dt} = 0 \quad -(5)$$

because of $\omega = const$ → angular Velocity of cam

The S-diagram, V-diagram and a-diagram are shown in fig (a), They are representing by (x, y) coordinate system, wher x-represent one complete revolution of cam, and y-represent complet strok of the follower





Fig(b): Modified, S-D, V-D, A-D

From fig(a) we see that the acceleration of out and return stroke is infinite, because the follower start from rest, and gain velocity with no time, and this is impracticable (غير عملي). there fore we must rounding the sharps corners of the displacement diagram → to a parabolic curves inorder to get low acceleration of the follower for given strokes, as shown in fig (b)

$a = \frac{dv}{dt}$, $dt = 0 \rightarrow a = \frac{dv}{0} = \infty$

مخططات الإزاحة والتعجيل عندما يتحرك التابع بتعجيل وتباطؤ منتظم

* II) Displacement, velocity and acceleration diagrams, when the follower moves with uniform acceleration and retardation.

* or with parabolic motion.

the equation of motion for parabolic consist of two half (rise and return)

1) equation of parabolic motion for first half of the motion (out strok)

$y = C\theta^2$ ——— ①

and this equation valid only for inflection point:

where $\begin{cases} y = \frac{S_o}{2} \\ \theta = \frac{\theta_o}{2} \end{cases}$ by sub - in ① → we get

$\frac{S_o}{2} = C\left(\frac{\theta_o}{2}\right)^2 \Rightarrow C = 2\frac{S_o}{\theta_o^2}$ by sub in ① →

or $y = 2 S_o\left(\frac{\theta}{\theta_o}\right)^2$ ——— ②

[illegible] equation for out strok

The velocity and acceleration obtain by finding the first and second derivatives for the equation of the displacement (2). as follow, by talcing.

$$\begin{cases} S_o = \text{constant} \\ \theta_o = \text{constant}. \\ \omega = \text{constant} \\ \theta - \text{changeable}. \end{cases}$$

2) Velocity :- السرعة

$$v = \dot{y} = \frac{d}{dt}\left[2S_o\left(\frac{\theta}{\theta_o}\right)^2\right] = 2\frac{S_o}{\theta_o^2}\frac{d\theta^2}{dt}$$

$$= 2\frac{S_o}{\theta_o^2}\cdot 2\theta \frac{d\theta}{dt} \rightarrow \omega = \frac{4S_o\omega\theta}{\theta_o^2}$$

$$\therefore \boxed{v = \frac{4S_o\omega\theta}{\theta_o^2}} \quad --- (3)$$

3 - acceleration

التعجيل

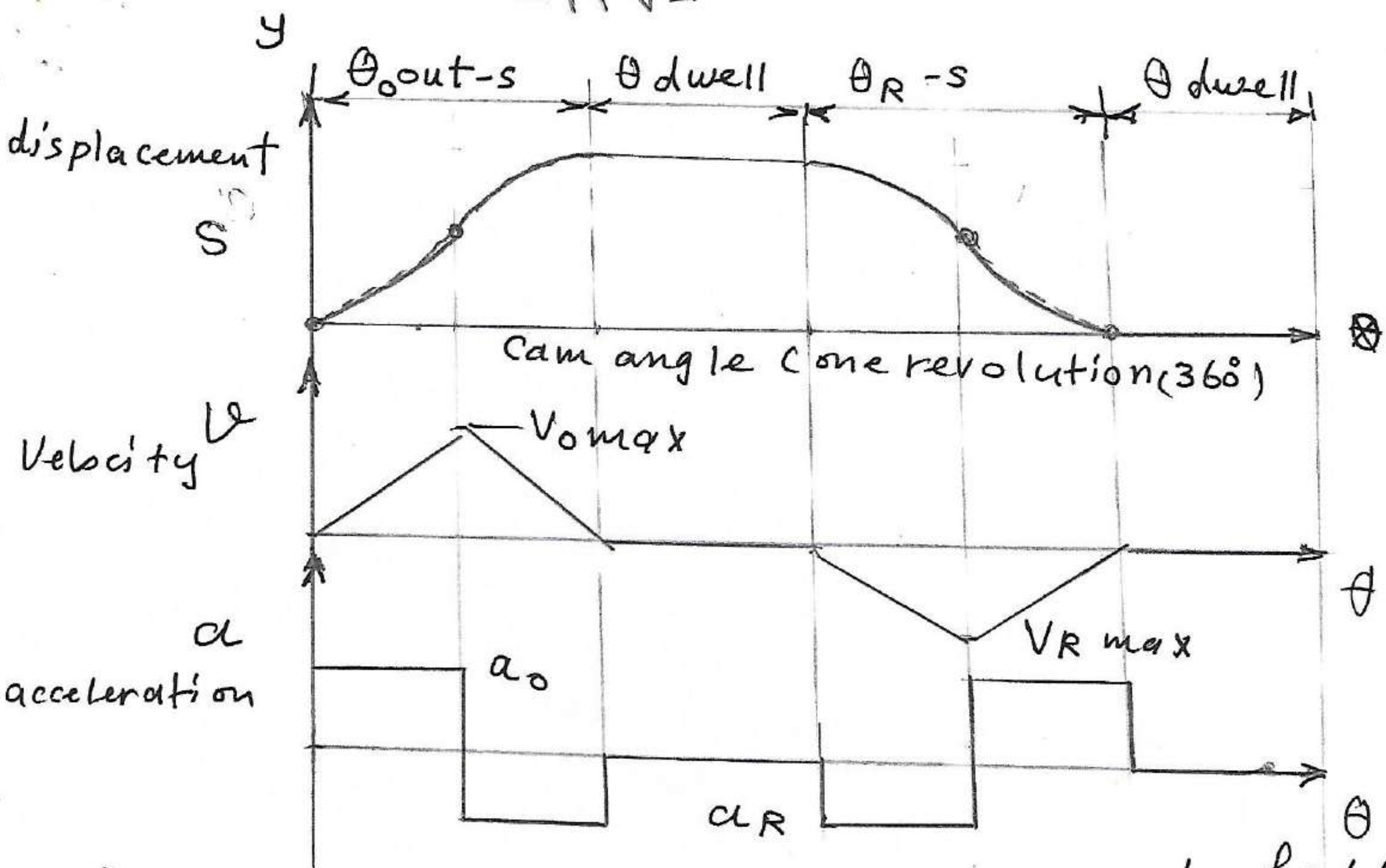
$$a = \ddot{y} = \dot{v} = \frac{d}{dt}\left(\frac{4S_o\omega\theta}{\theta_o^2}\right) = \frac{4S_o\omega}{\theta_o^2} \times \frac{d\theta}{dt} \rightarrow \omega$$

$$\therefore \boxed{a = \frac{4S_o\omega^2}{\theta_o^2}} \quad --- (4)$$

From eq-n (4) we see that a = constant and v_{max} occurs at the inflection point, where $\theta = \frac{\theta_o}{2}$ → for out strok.

$$\therefore v_{max_o} = \frac{4S_o\omega}{\theta_o^2}\left(\frac{\theta_o}{2}\right) = \frac{2S_o\omega}{\theta_o}$$

$$\therefore \boxed{v_{max_o} = \frac{2S_o\omega}{\theta_o}} \quad --- (5)$$



fig(a): S-D, V-D, a-D for parabolic motion

2- displacement equation for the second half motion (return strok).

$$y = C_1 + C_2\theta + C_3\theta^2 \quad \text{--- (6)}$$

For determining the const C_1, C_2, C_3 may be get by solving these equations:

assume that $\theta = \theta_R$, $y = S_R$ and by sub in (6) we get

$$\therefore S_R = C_1 + C_2\theta_R + C_3\theta_R^2 \quad \text{--- (7)}$$

when $\theta = \theta_R \longrightarrow v = 0$

$$\therefore v = 0 = \dot{y} = \frac{d}{dt}(C_1 + C_2\theta + C_3\theta^2) =$$

$$= 0 + C_2\frac{d\theta}{dt} + C_3\,2\theta\frac{d\theta}{dt}$$

$\therefore\ v = 0 = C_2 \omega + 2 C_3 \theta_R \omega$ ——— (8)

but V_{max} occurs when $\theta = \frac{\theta R}{2}$ at the inflection point.

or $\therefore\ v_{max} = \frac{2 S_R \omega}{\theta R} = C_2 \omega + 2 C_3 \omega \theta_R$ ---- (9)

by solving equation (7), (8), (9) → we get.

$C_1 = S_R$ → By sub C_2, C_3 in (7) → C_1

$C_2 = \frac{4 S R}{\theta R}$ → By sub C_3 in (8) → C_2

$C_3 = -2 \frac{S_R}{\theta_R^2}$ → By sub C_2 in (8) and (9) → C_3

By sub these Constants in eq-n (6) and obtain the displacement eq-n.

1, $y = S_R \left[1 - 2 \left(1 - \frac{\theta}{\theta_R}\right)^2 \right]$ --- (10)

2) $v = \dot{y} = \frac{d}{dt} \left[S_R \left(1 - 2\left(1 - \frac{\theta}{\theta_R}\right)^2\right) \right]$

$\therefore\ v = \frac{d}{dt} \left[S_R - 2 S_R \left(1 - \frac{\theta}{\theta_R}\right)^2 \right]$

$= 0 - 2 S_R \cdot 2 \left(1 - \frac{\theta}{\theta_R}\right) * \left(0 - \frac{1}{\theta_R} \cdot \frac{d\theta}{dt}\right)$ ($\frac{d\theta}{dt} = \omega$)

$= 0 - 4 S_R \left(1 - \frac{\theta}{\theta_R}\right) \cdot - \frac{1}{\theta_R} \omega$

$$V = + \frac{4 S_R \omega}{\theta_R} \left(1 - \frac{\theta}{\theta_R}\right) \text{ ——— (11)}$$

at the inflection point.

$v \to v_{max}$, $\theta \to \frac{\theta R}{2}$

$$\therefore\ v_{max} = \frac{4 S_R \omega}{\theta_R} \left(1 - \frac{\theta_R}{2 \theta_R}\right) = \frac{2 S_R \omega}{\theta_R}$$ at the inflection point.

3) the acceleration be.

$$a = \ddot{y} = \frac{d}{dt}\left(\frac{4SRW}{\theta_R}\left(1-\frac{\theta}{\theta_R}\right)\right)$$

where $\omega = const$, θ – changeable.

$$\therefore \dot{a} = \frac{d}{dt}\left(\frac{4S_R W}{\theta R} - \frac{4S_R W \cdot \theta}{\theta_R^2}\right)$$

$$= 0 - \frac{4S_R W}{\theta_R^2}\frac{d\theta}{dt} = \frac{4S_R W^2}{\theta_R^2}$$

($\frac{d\theta}{dt} = \omega$)

$$\therefore \boxed{a = \frac{4S_R W^2}{\theta_R^2}} \quad \text{——— (١٢)}$$

الطريقة البيانية لبناء مخطط الازاحة للحركة القطع الناقص

* Graphical Method for drawing displacement, velocity and acceleration, when the follower move with uniform acceleration and retardation → (Parabolic-motion), as follow:

1- Divide the angular displacement for out strok (θ_o) in to any equal number (say 8).
قسم الإزاحة الزاوية للصعود الى عدد متساوي وليكن (٨)

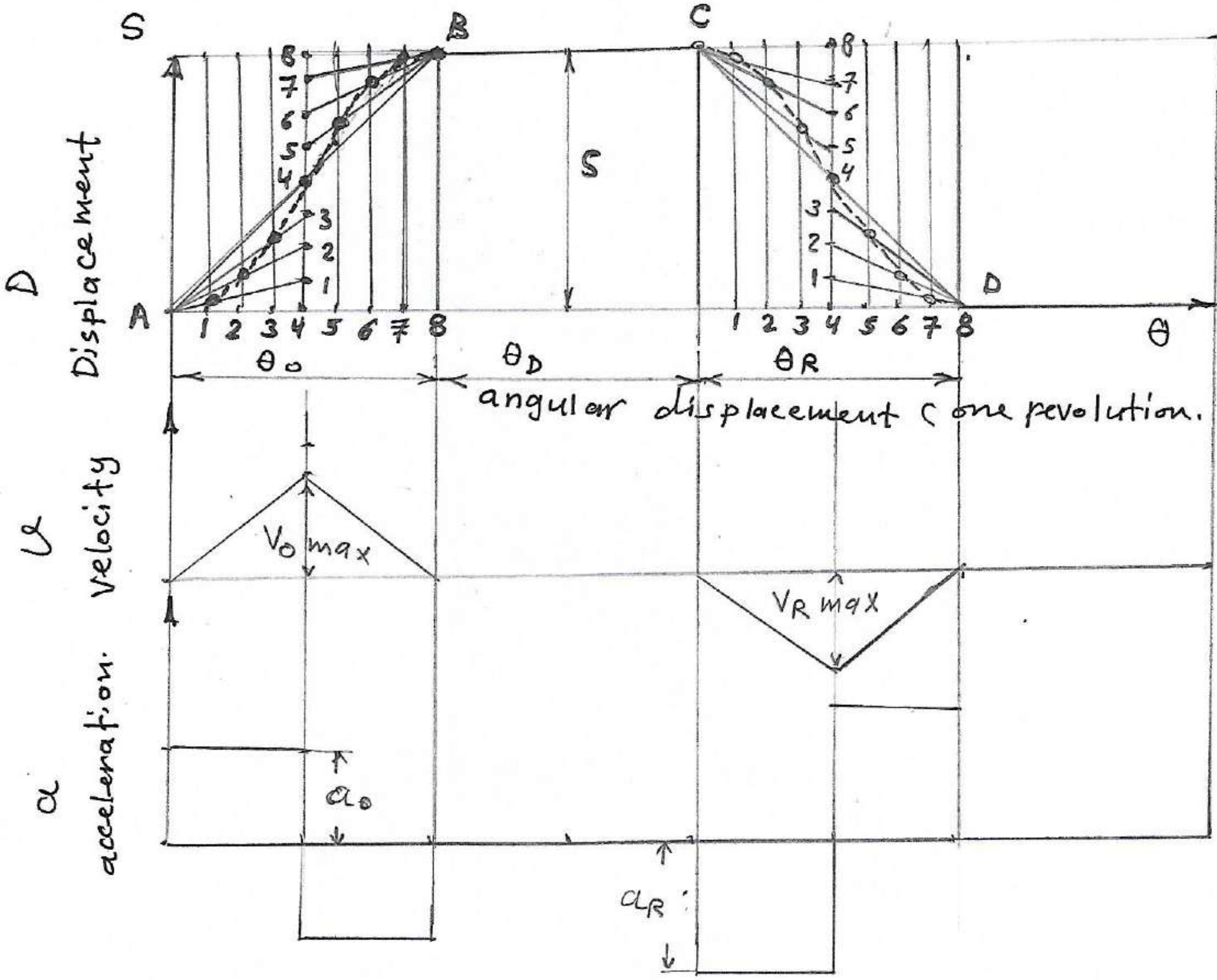
2- Divide the stroke (S_o) into the same equal number of parts (8).

3- Join point A with 1, 2, 3, 4 (B, C, D, E), and join point J with 5, 6, 7, 8 (F, G, H, J), then these point to obtain parabolic curve for out strok.

4- By similar way draw parabolic curve for return strok (θ_R, S_R).

5) Calculete the v_{max}, a_{max} for out strok, and v_{min}, a_{min} for return stroke, as follow; fig(٩)

* Graphical Method for drawing parabolic curve



Fig(a) : Graphical Method fordrawing, D, V, a diagram as Follow :

1- Divide θ_o into equal number, like 1, 2, 3, 4, 5, 6, 7, 8
2- Divide S_o into Same equal number , 1, 2, 3, 4, 5, 6, 7, 8
3- Join A with 1, 2, 3, 4
4- Join B with 5, 6, 7, 8
5- Find A Cross point
6- draw parabolic Curve through cross point For out strok.
7- by Same way for SR - strole.
8- Find $V_o max$, $V_R max$.

III) Displacement, velocity and acceleration diagrams, when the follower moves with simple harmonic motion.

1) Simple harmonic motion represent with following equation for displacement.

$$y = \frac{S_o}{2}\left(1 - \cos\frac{\pi\theta}{\theta_o}\right) \quad \text{---} (1) \rightarrow \text{for out stroke}$$

2) The velocity represent with a first derivative of displacement.

$$\therefore v = \dot{y} = \frac{d}{dt}\left(\frac{S_o}{2}\left(1 - \cos\frac{\pi\theta}{\theta_o}\right)\right)$$

$$= \frac{d}{dt}\left(\frac{S_o}{2} - \frac{S_o}{2}\cos\frac{\pi\theta}{\theta_o}\right) = 0 - \frac{S_o}{2}\left(-\frac{\pi}{\theta_o}\sin\frac{\pi\theta}{\theta_o}\right)\frac{d\theta}{dt} \quad \left(\frac{d\theta}{dt} \rightarrow \omega\right)$$

منحنى جيبي

$$v = +\frac{\pi S_o \omega}{2\theta_o}\sin\frac{\pi\theta}{\theta_o} \quad \text{---} (2) \rightarrow \text{Sine Curve}$$

3) acceleration represent second derivative of displacement of first der.ve of velocity.

$$\therefore a = \ddot{y} = \dot{v} = \frac{d}{dt}\left(\frac{\pi S_o \omega}{2\theta_o}\sin\frac{\pi\theta}{\theta_o}\right) =$$

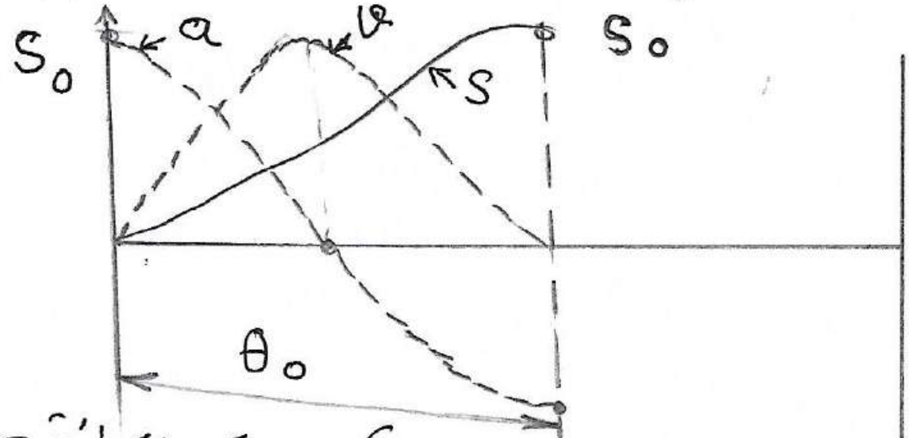
$$= \frac{\pi S_o \omega}{2\theta_o} * \frac{\pi}{\theta_o}\cos\frac{\pi\theta}{\theta_o}\frac{d\theta}{dt} \quad \left(\frac{d\theta}{dt} \rightarrow \omega\right)$$

منحنى جيب تمام

$$a = \frac{\pi^2 S_o \omega^2}{2\theta_o^2}\cos\frac{\pi\theta}{\theta_o} \quad \text{---} (3) \rightarrow \text{Cosine Curve}$$

* for return strok by same way,

representation of simple harmonic motion for out stroke show in fig(a).

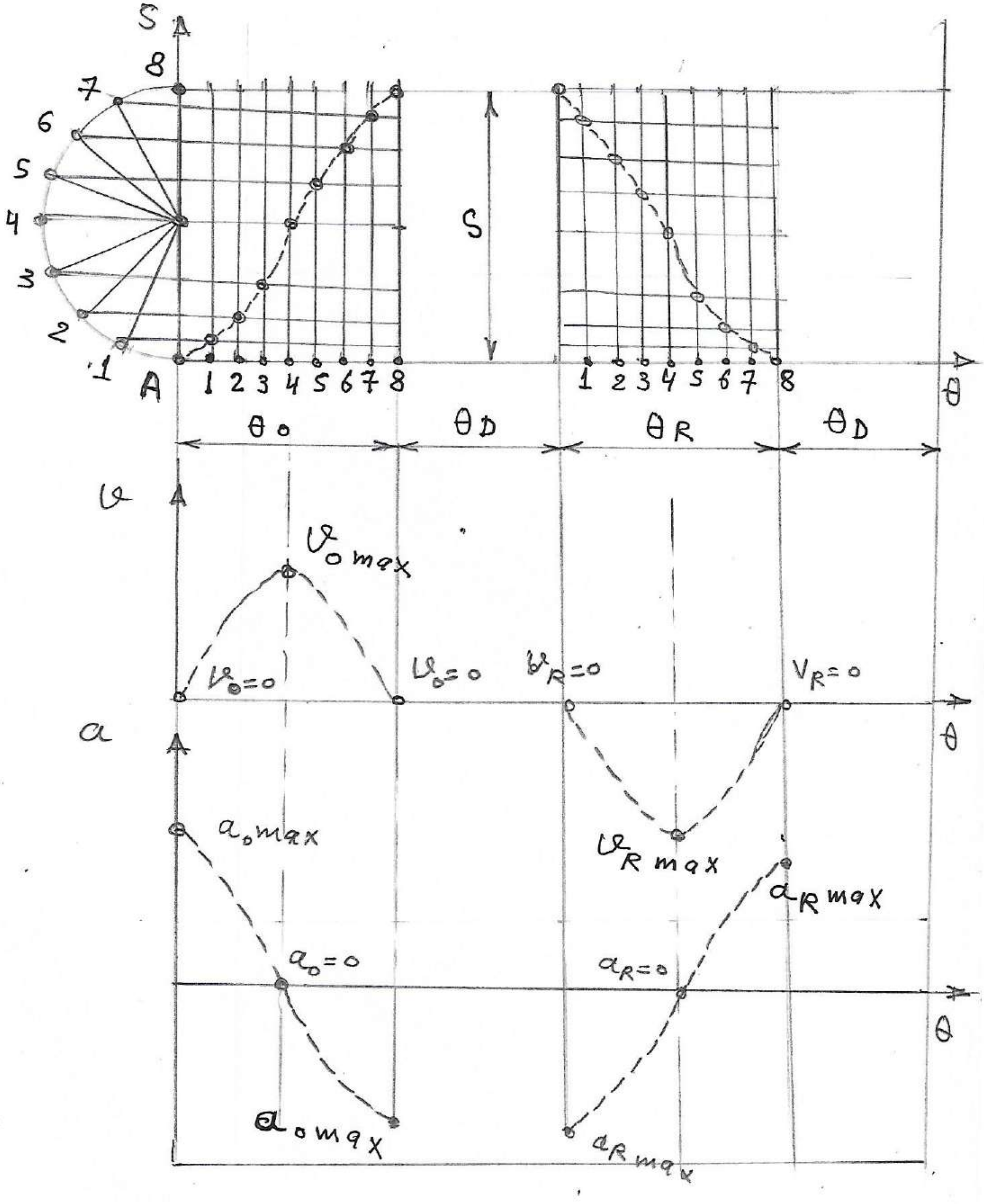


الطريقة البيانية لرسم مخطط الازاحة والسرعة والتعجيل للحركة التوافقية البسيطة .

* Graphical Method for drawing the displacement, velocity and acceleration when the follower move with simple harmonic motion: as follow

1- draw semi circle with diameter (S_0). ١- نرسم دائرة قطرها S_0
2- Divide the semi circle into any equal parts, for example (8 parts). ٢- تقسم الدائرة الى عدد متساوي
3- Divide the angular displacements of the cam into the same number of equal parts. ٣- تقسم θ_0 الى نفس العدد
4, Displacement diagram is obtained by the projected points, as bellow. ٤- نرسم منحنى الازاحة من نقاط التقاطع بالتقاطع
5) velocity diagram consist of sine curve, at begining and end → $V_0=0$, at midpoint → V_o max
6) acceleration diagram consist of Cosine curve. at beining and end → $a = a_{max}$, and at midpoint → $a_0 = 0$ as shown in fig(b),

Fig (b), Graphical Method.



* Examples

* Construction of cam profile for a radial cams

بناء مخطط الكمة للكمات القطرية

Construction of cam profile for different types of followers, with different types of motions, as discussed by the following examples.

بناء مخطط الكمة للانواع المختلفة من التوابع وللانواع المختلفة من الحركات سوف نتناولها بالامثلة التالية

design = بناء ⇒ Draw – ارسم

Example (1)

A cam is to be designed for a knife edge follower moves with uniform velocity for both strokes, and with the following data:

1- outstroke during 60° of cam rotation;
2- Dwell for next 30° of cam rotation;
3- Return stroke during next 60° of C.R;
4- Dwell for the remaining 210° of C.R;
5- Strok of the follower 40 mm; and
6- minimum radius of the cam is 50 mm.

Draw the profile of the cam when,
1) The axis of the follower passes through the axis of the cam shaft; and
2) ~~The~~ axis of the follower is offset by 20 mm of the axis of the cam shaft.
3) determine the max-M velocity and acceleration, when the cam rotate with 100 r.p.m and draw S-D, V-D, a-D diagram for (1) complete revolution.
4- Speed of cam 200 r.p.m.

Solution:

I) Draw displacement diagram as follow:
1- draw horizontal line AX = 360° and mark AS = 60°, ST = 30°, TP = 60°, PX = 210°,

2 - draw vertical Line $Ay = S = 40$ mm, and complete the rectangle.

3 - divide the angular displacement for out and return strok into any equal number (say 6). and draw vertical line through each number, with suitable scale.

4) since the follower moves with uniform velocity, then the displacement for out and return stroke ane straight Lines (AG, HP), and the S-D be complet, as shown bellow.

5/* take Scale → $2^\circ = 1$ mm, fig (a)

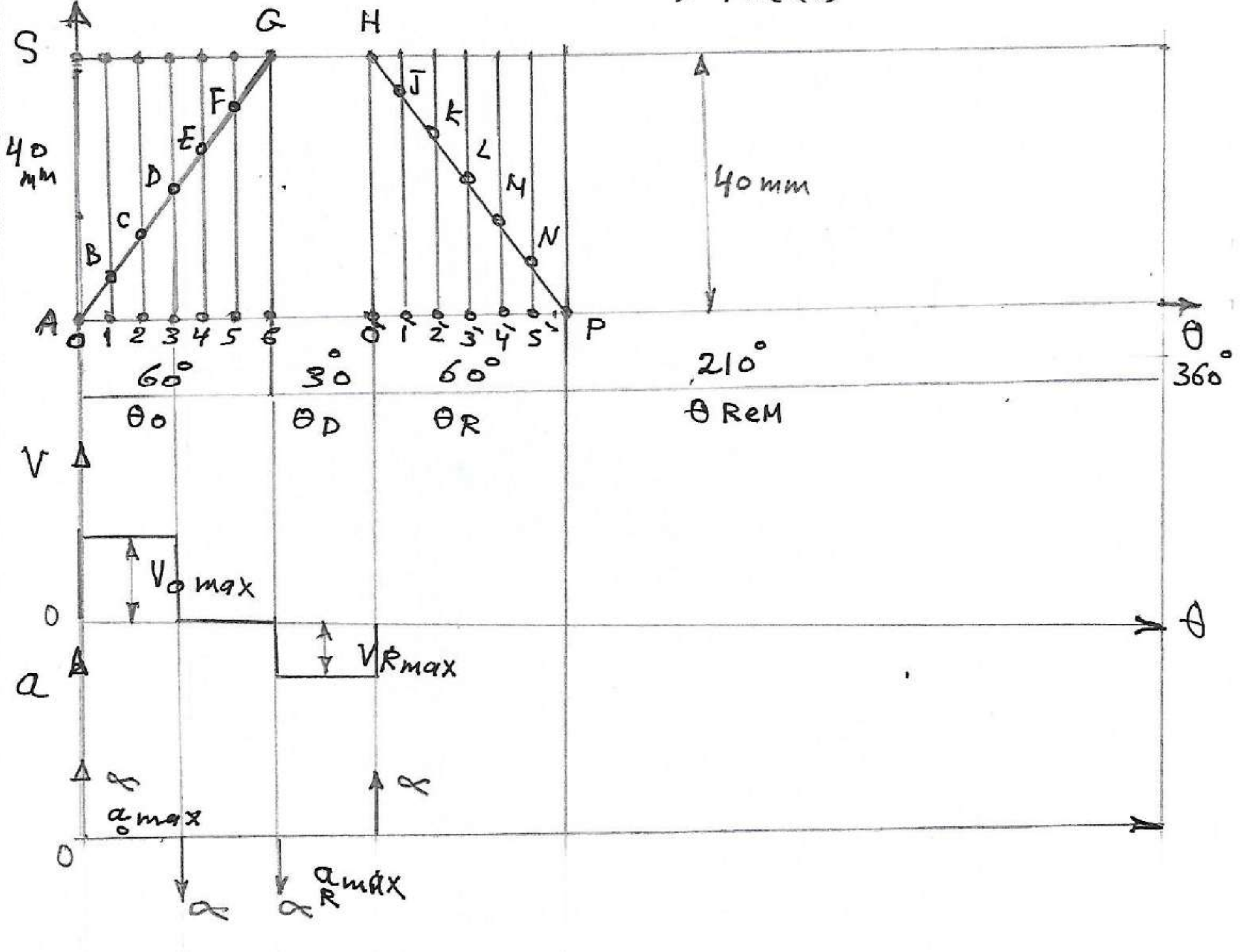
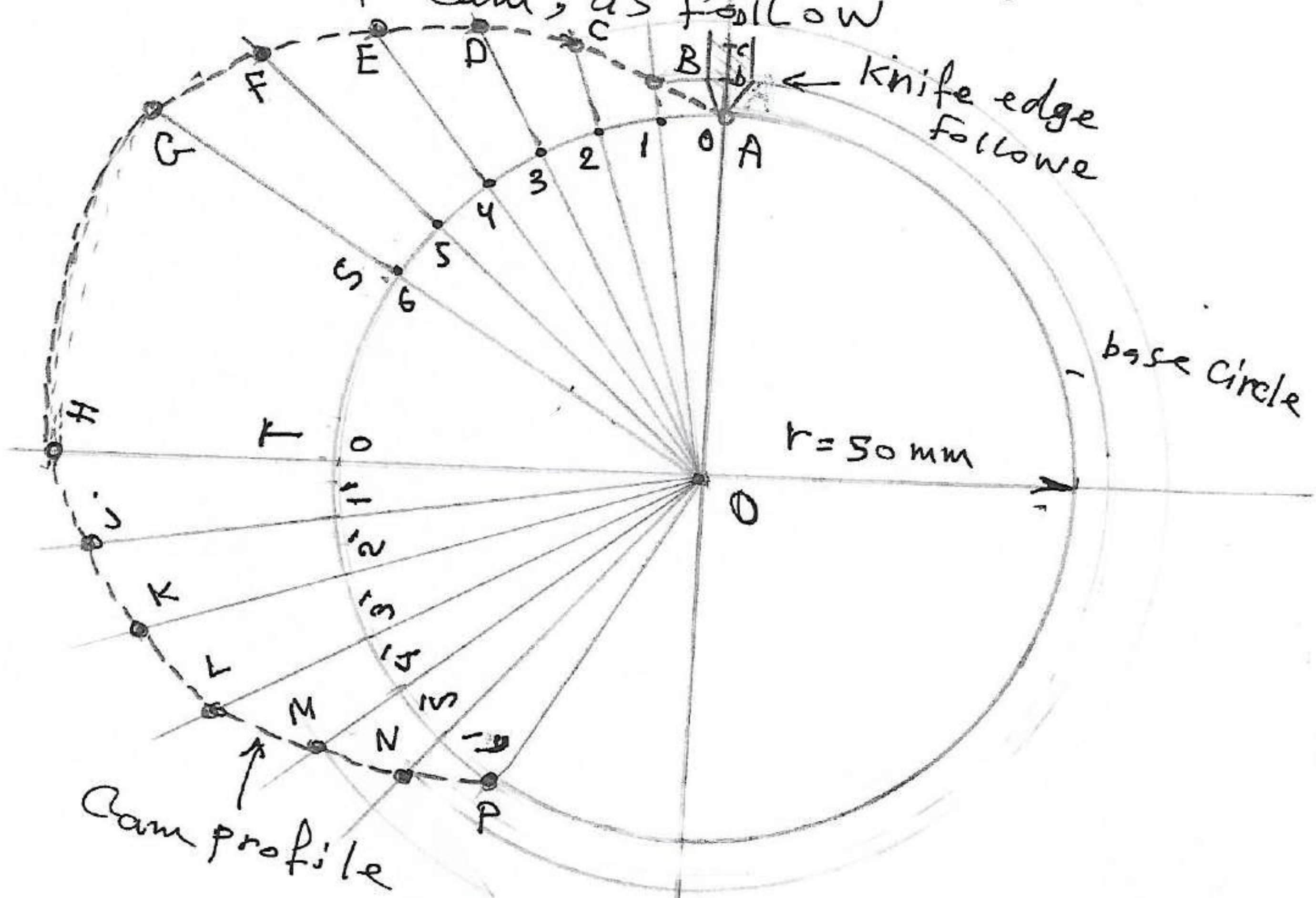


Fig (a) show: S-D, V-D, a-D.

II, Draw Cam profile, when the axi's of the follower pass through the center of cam, as follow

1) draw base circule with (r = 50 mm) and center o.
2) Mark point A, and from oA mark angle AOS = 60° for out stroke, ang SOT = 30° for dwell, ang TOP = 60° for return stroke.
3) divide the angle of out and return stroke into the same equal number of parts, as in the displacement diagram.
4) Join the point 1, 2, 3, - and 1̀, 2̀, 3̀, - with the center of cam o.
5) Added the lengths 1B, 2C, --6G to o1, o2, -oG on the cam for out stroke, and by same way added oH, --6P for return stroke.
6) Join the point A, B, C, E, --N, P to get the profile of cam, as follow



III) Draw Cam profile, when the axis of the foll-ower is offset by 20 mm from the axis of the cam shaft, as follow

1- draw base circle with r = 50 mm.

2, draw the axis of the follower with ofset 20 mm.

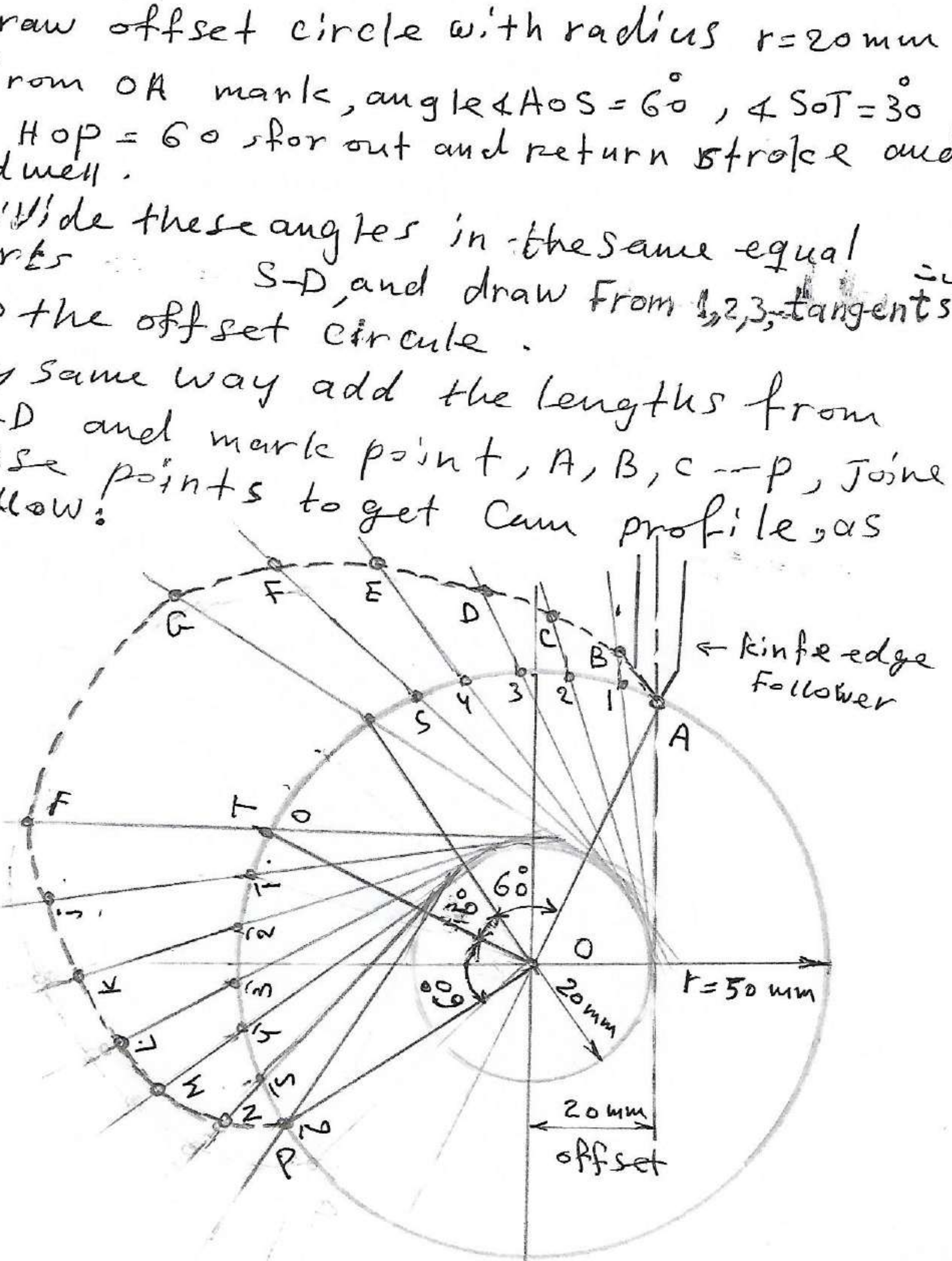
3, draw offset circle with radius r = 20 mm

4, from OA mark, angle ∢AOS = 60°, ∢SOT = 30°, ∢HOP = 60, for out and return stroke and dwell.

5, divide these angles in the same equal parts S-D, and draw From 1,2,3, tangents

6, to the offset circule.

7, by same way add the lengths from S-D and mark point, A, B, C -- P, Joine these points to get Cam profile, as bellow:



→ max-m velocity for outstroke, and return stroke

$$V_{o\,max} = \frac{S_o}{\theta_o}\,\omega = \frac{0,04}{60 \times \frac{\pi}{180}} \times \frac{2\pi n}{60} = 1,2\ m/s$$

$$V_{R\,max} = \frac{S_R}{\theta_R}\,\omega = \frac{0,04}{60 \times \frac{\pi}{180}} \times \frac{2\pi n}{60} = 1,2\ m/s.$$

→ max-m acceleration for out and return stroke.

$$a_{o\,max} = \frac{S_o}{\theta_o}\,\frac{d\omega}{dt} = \infty$$

$$a_{R\,max} = \infty \quad , \text{beause } \omega = \text{const}$$

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* Example (2) :

A Cam is to be designed for a knife edge follower with the following data:

1- Cam lift = 40 mm during 90° of cam rotat-ation with simple harmonic motion.

2- Dwell to the next 30°,

3, During the next 60° of Cam rotation, the foll-ower returns to its original position with simple harmonic motion.

4- Dwell during the remaining 180°.

Draw the profile of the Cam when:

a) the line of stroke of the follower passes through the Center of the Cam shaft, and

b) the line of stroke is offset 20 mm from the axis of the Cam shaft.

The radius of the base circle of the Cam is 30 mm, Determine the max-m velocity and acceleration of the follower during its out and return stroke, if the Cam rotes at 240 r.p.m.

* Solution :

Solution :

I, Draw displacement diagram with steps as discussed bellow:

1, Draw horizontal line $AX = 360°$, with scale $2°$ = 1 mm, and mark $AS = 90°$ for outstroke, $SR = 30°$ for dwell, $RP = 60°$, and $PX = 180°$ dwell.

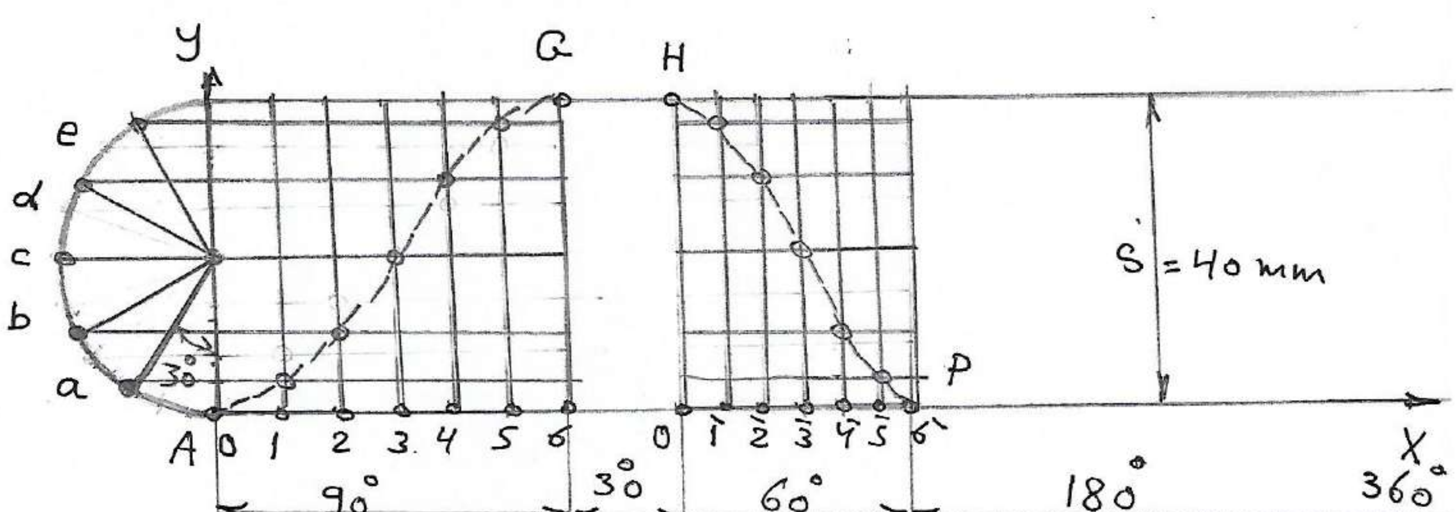
2) Draw verticle line AY = 40 mm stroke, and Complete the rectangle.

3) Divide the angular displacement and stroke to same equal number of parts (say = 6).

4) For Simple harmonic motion draw semi circle with diameter(S), and devide into same equal number.

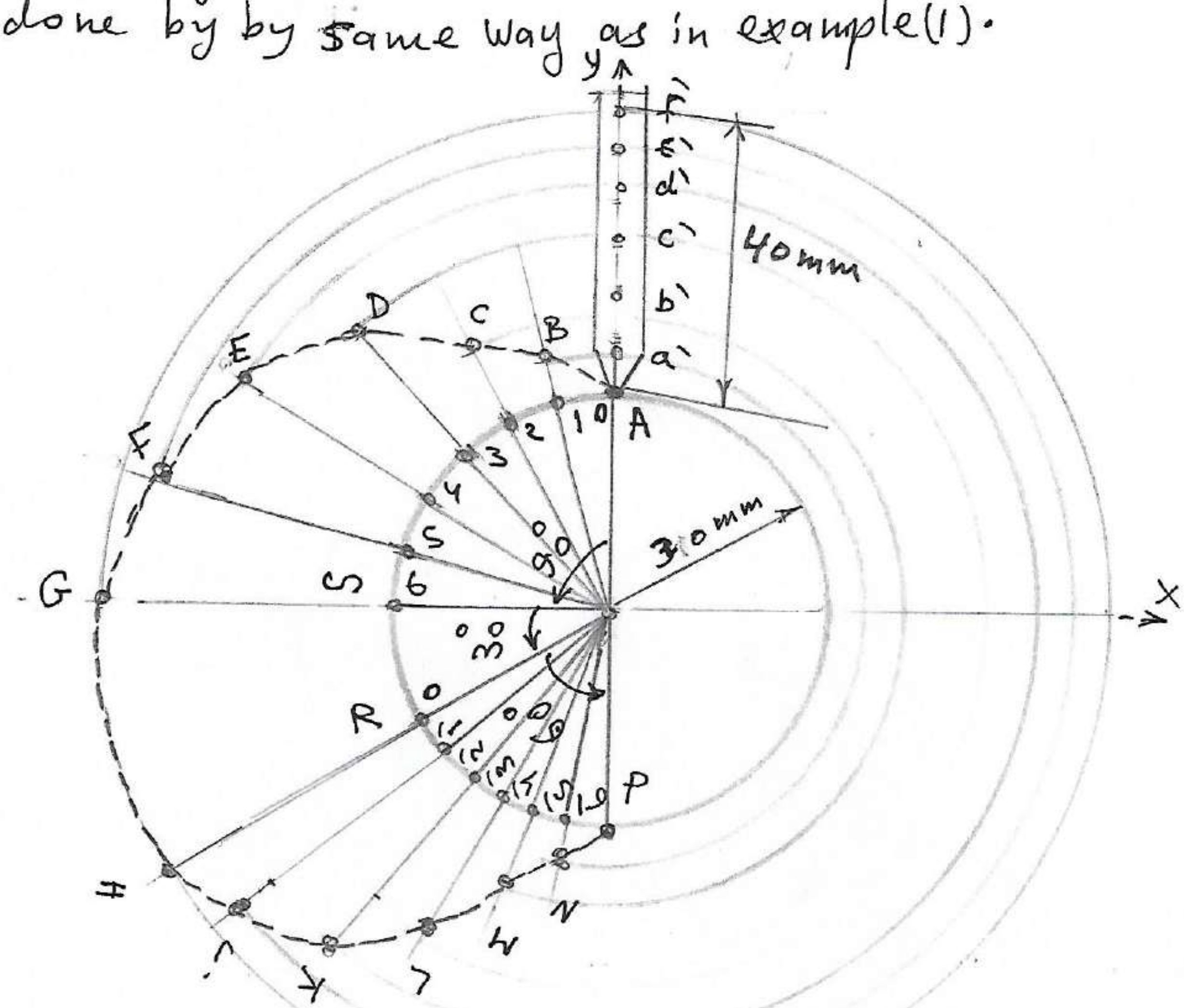
5) Locate the intersecting point (A, B, -- G) for outstrok from the vertical lines (1, 2, 3, -6) and horizontal lines a, b, c — e), by same way for return stroke.

6) Join the intersecting points A, B, C -- with smooth curve to get the required (S — D)

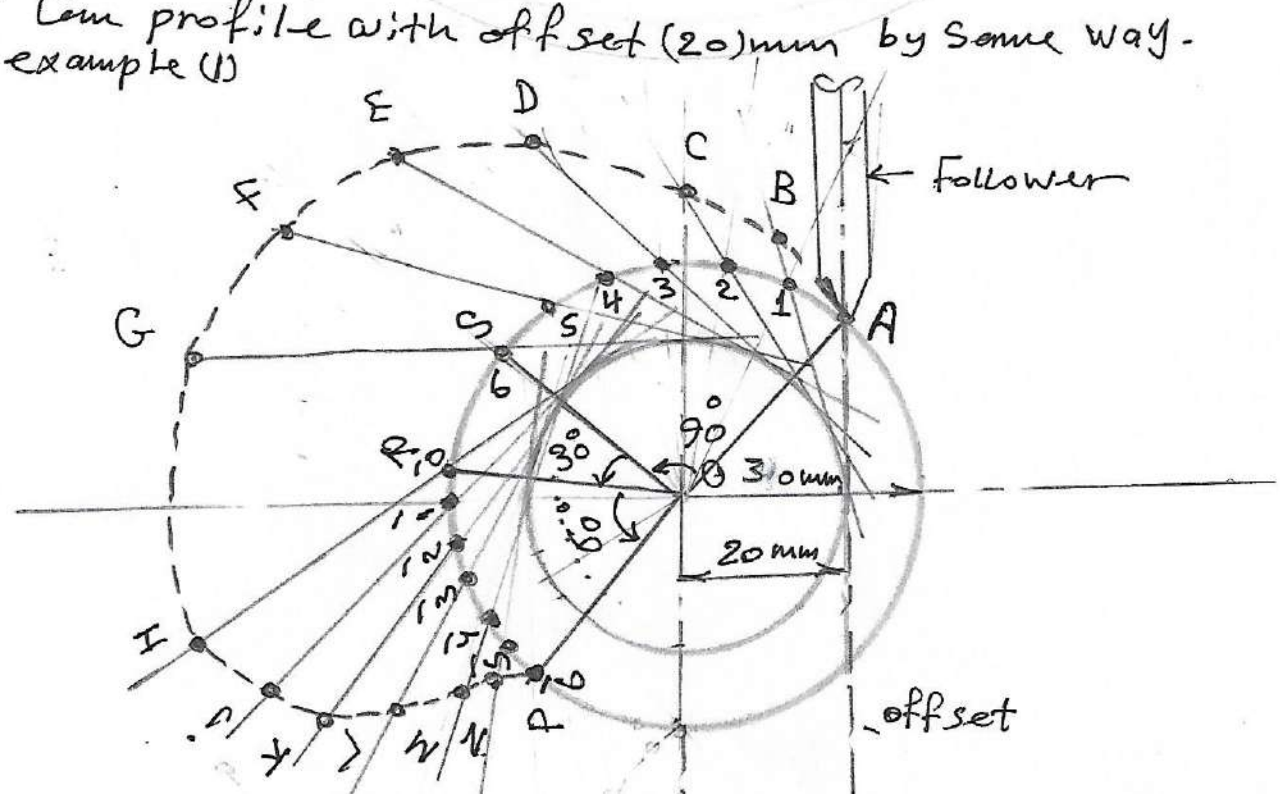


Fig(a) displacement diagram.

II, Draw pofile of cam, when the axis of follower passes through the center of the cam. This is done by by same way as in example(1).



III, Draw cam profile with offset (20)mm by same way. as in example (1)



IV, max-m velocity during out and return stroke

$$v_{o\,max} = \frac{\pi \omega S}{2\theta_o} \quad , \quad v_{R\,max} = \frac{\pi \omega S}{2\theta_R}$$

$$\theta_o = 90^\circ * \frac{\pi}{180} = 1,571 \text{ rad.}$$

$$\theta_R = 60 * \frac{\pi}{180} = 1.047 \text{ rad}$$

$$\omega = \frac{2\pi n}{60} = \frac{2 * 3,14 * 240}{60} = 25,14 \text{ rad/s}$$

$$\therefore v_{o\,max} = \frac{3,14 * 25,14 * 0,04}{2 * 1,571} = 1 \text{ m/s} \quad \text{, and}$$

$$v_{R\,max} = \frac{3,14 * 25,14 * 0,04}{2 * 1,047} = 1,51 \text{ m/s}$$

V, max-m acceleration for out and return stroke.

$$a_{o\,max} = \frac{\pi^2 \omega^2 S}{2\theta_o^2} \quad , \quad a_{R\,max} = \frac{\pi^2 \omega^2 S}{2\theta_R^2}$$

$$\therefore a_{o\,max} = \frac{(3,14)^2 * (25,14)^2 * 0,04}{2 * (1,571)^2} = 50,6 \text{ m/s}^2$$

$$a_{R\,max} = \frac{(3,14)^2 * (25,14)^2 * 0,04}{2 * (1,047)^2} = 113,8 \text{ m/s}^2$$

* Example (3)

A Cam, with a minimum radius of (25 mm), rotating clockwise at uniform speed, is to be designed to give a roller follower, at the end of a valve rod,

, motion described below:

1- To raise the valve through (50 mm) during (120°) rotation of the Cam.

2- To keep the valve fully raised through next (30°).

3- To lower the valve during next (60°), and

4) To keep the valve closed during rest of revolution (150°).

The diameter of the roller (20mm) and the diameter of the Cam shaft is (25 mm).

Draw the profile of the Cam when:

a) the line of stroke of the valve rod passes through the axis of the Cam shaft; and

b) the line of the stroke is offset (15 mm) from the axis of the Cam shaft.

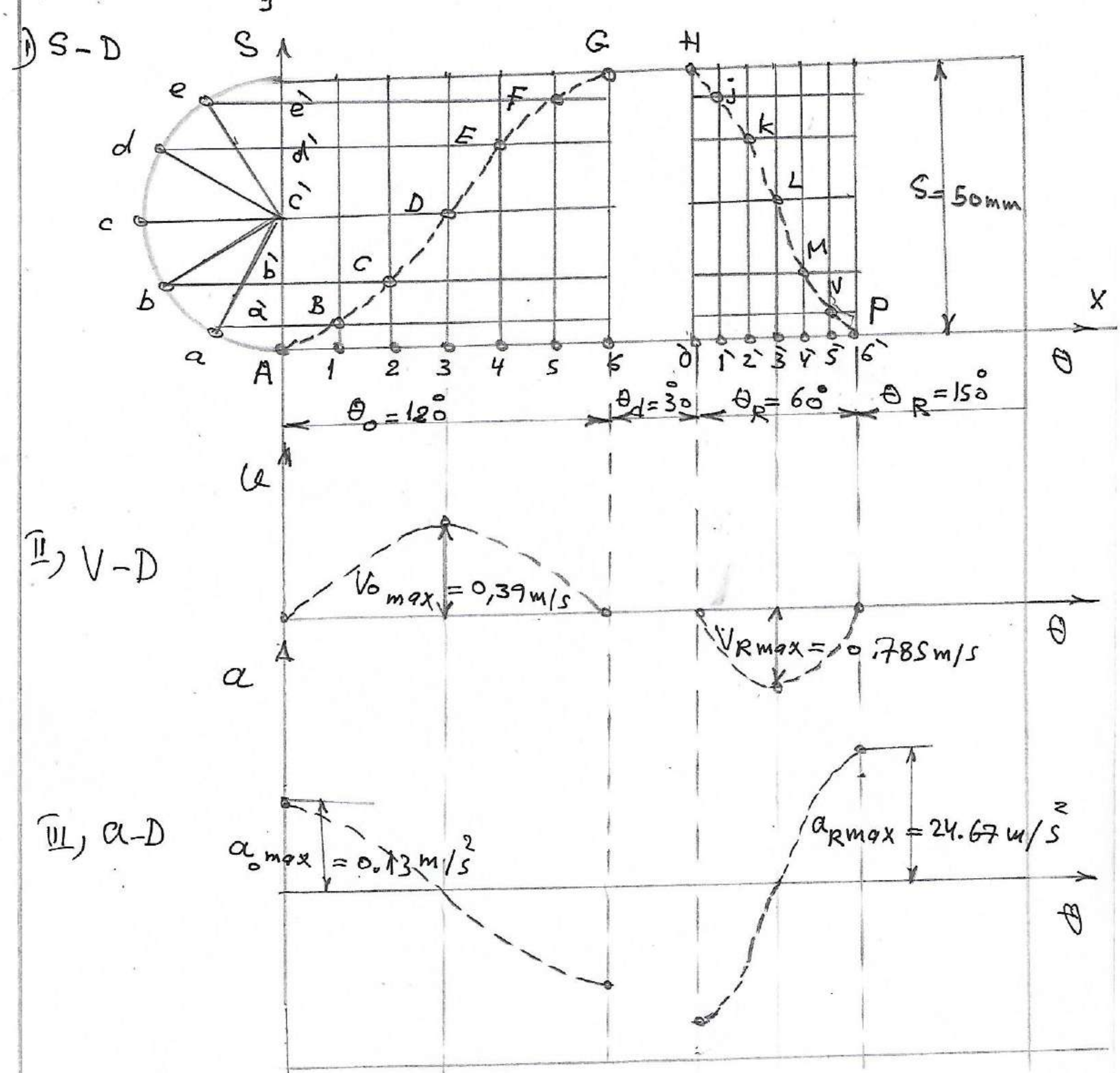
The displacement of the valve, while being raised and lowering, is to take place with simple harmonic motion. Determine the max-m acceleration of the valve, when the Cam shaft rotates at (100 r.p.m). Draw the displacement, the velocity, and the acceleration diagrams for one Complete revolution of the Cam.

* Solution:

I) Draw displacement diagram by same way, as in example (2) for simple harmonic motion, as shown.

∴ S-D for Complete revolution, velocity diagram V-D, and acceleration diagram. a-D -

Take Scale: 3° = 1mm



II) Change all angles into radians as follow

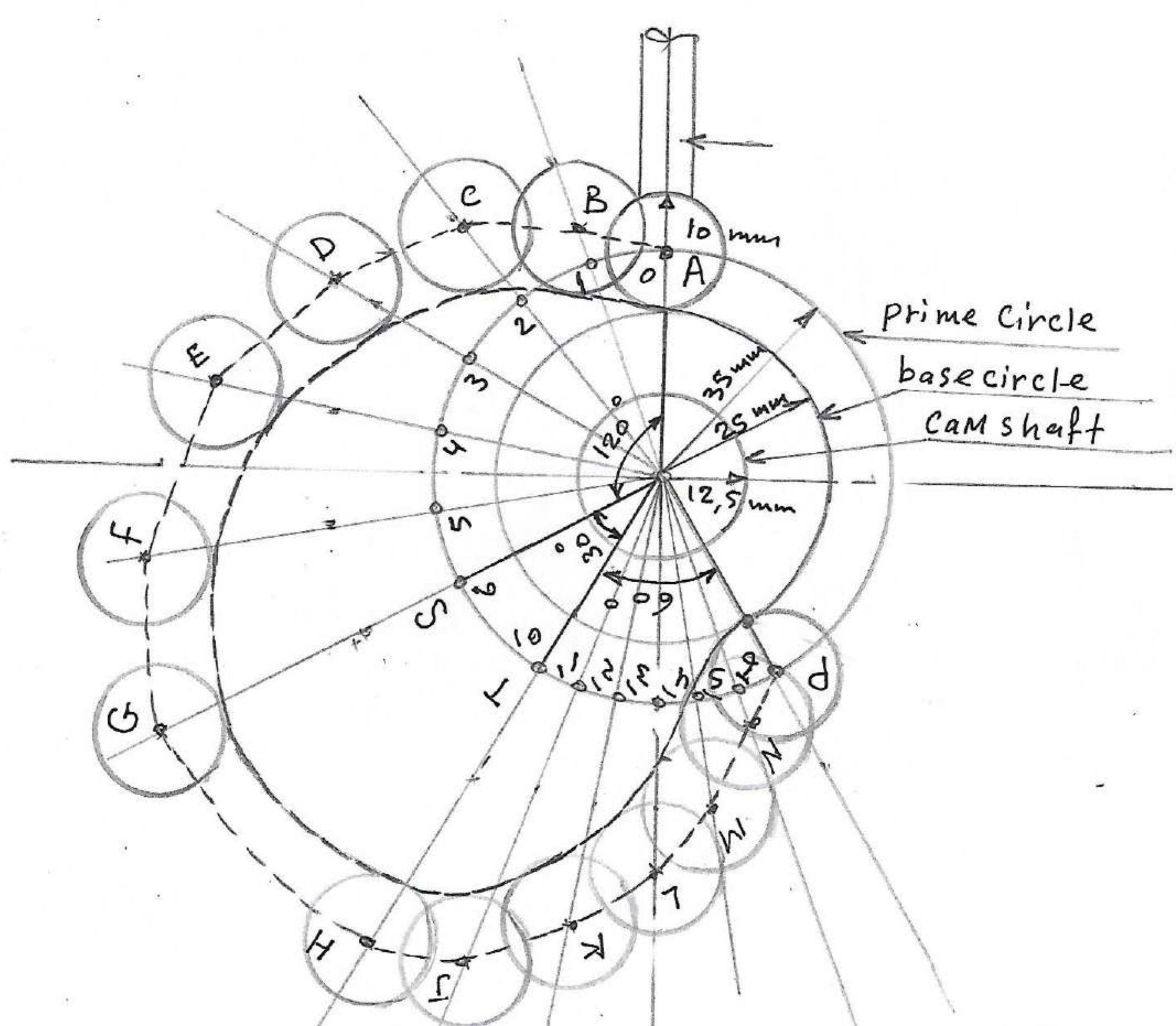
$$\theta_o = 120^\circ * \frac{\pi}{180} = 2,1 \text{ rad}$$

$$\theta_R = 60^\circ * \frac{\pi}{180} = 1,047 \text{ rad.}$$

$$\omega = \frac{2\pi n}{60} = \frac{2 * 3.14 * 100}{60} = 10.47 \text{ rad/s.}$$

III) Draw Cam profile with the follower axis passes through the Center of Cam, with roller follower, as bellow:

1) Draw base circle with Center o, and radius ($r_b = 25$ mm).
2) Draw prime circle with radius $r_p = r_b + \frac{1}{2} D_{roller}$
$\therefore r_p = 25 + 10 = 35$ mm.
3) on prime cercle draw $\theta_o = AOS = 120^\circ$, θ_d $SOT = 30^\circ$, $\theta_R = TOp = 60^\circ$,
4) divide $\theta_o, \theta_d, \theta_R$ - into same equal number, as in S-D → 1, 2, 3 → --Join them with o.
5) Set off 1B, 2C, 3D from S-D → draw circle with r_{roller} from A, B, C --
6) Join A, B, C - to get pitch curve.
7) Draw tangent Curve to the roller from the bottom to get Cam profile, as follow:



IV) Draw Cam profile, when the follower axis passes with offset with Cam axis, as follow:

1) Draw base circle with radius $r_b = 25$ mm,
2) Draw prime circle with with radius $r_p = r_b + r_{Roller}$
$\therefore r_p = 25 + 10 = 35$ mm
3) Draw off-set circle with center o and radius $r_{off\text{-}set} = 15$ mm.
4) Join OA and draw the angular displacement for $\theta_o = AoS = 120°$, $\theta_d = SoT = 30°$, and $\theta_R = Top = 60°$
$\theta_R = 150°$.
5) Divide the angular displacement for out and retern stroke in 8 Same equal parts, as in S-D.
6) from point 1, 2, 3 -- and 1̀, 2̀, 3̀ draw tangent to the offset circle.
7) Set off 1B, 2C, -- from the displacement diagram
8) Join the point A, B, C - to get → pitch curve
9) From A, B, C, draw circle with r_{roller}.
10) Bottom tangent to roller → give Cam Curve. as follow;

The data:

$r_{cam\,shaft} = 12,5$ mm

$r_{base} = 25$ mm

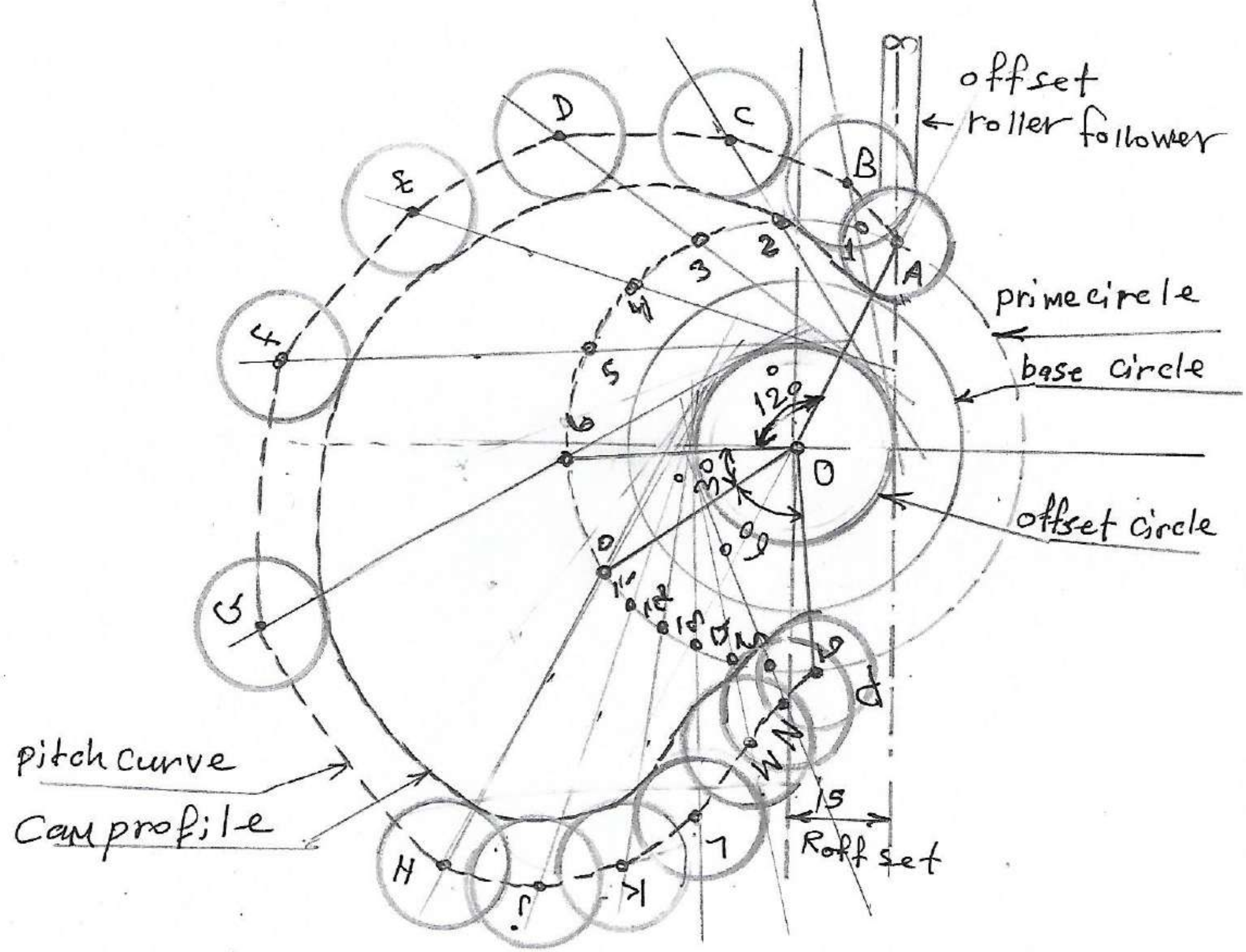
$r_{prime} = 35$ mm

$r_{offset} = 15$ mm

$\theta_o = 120°$

$\theta_R = 60°$

$\theta_d = 30°$



fig(a) Cam profile with offset.

V) maximum velocity For out and return stroke:

$$V_o = \frac{\pi w S}{2\theta_o} = \frac{3,14 * 10.47 * 0,05}{2 * 2,1} = 0,39\ m/s$$

$$V_R = \frac{\pi w S o}{2\theta_R} = \frac{3,14 * 10,47 * 0,05}{2 * 1,047} = 0,785\ m/s$$

VI) maximum acceleration for out and return stroke.

$$a_o = \frac{\pi^2 w^2 S}{2\theta_o^2} = \frac{(3,14)^2 * (10.47)^2 * 0,05}{2(2,1)^2} = 6.13\ m/s^2$$

$$a_R = \frac{\pi^2 w^2 S}{2\theta_R^2} = \frac{(3,14)^2 * (10.47)^2 * 0,05}{2(1,047)^2} = 24.67\ m/s^2$$

*Example (4)

A Cam, with a minimum radius of (50 mm), rotating clockwise at a uniform speed, is required to give a knife edge follower the motion as described below;

1- To move outwards through (40 mm) during (100°) rotation of the cam.

2, To dwell for next (80°).

3) To return to its starting position during next (90°), and

4, To dwell for the rest period of a revolution (90°).

Draw the profile of the cam.

1) when the line of stroke of the follower passes through the centre of the cam, and

2) When the line of stroke of the follower is offset by (15 mm).

The displacement of the follower is to take place with uniform acceleration, and uniform retardation. Determine the max-m velocity and acceleration of the follower when the cam shaft rotates at (900 r.p.m).

Draw the displacement, velocity and acceleration for one revolution.

Solution: change into radians-

$\theta_o = 100° * \frac{\pi}{180} = 1.745$ rad.

$\theta_R = 90 * \frac{\pi}{180} = 1.571$ rad

$S = 0,04$ m

$\omega = \frac{2\pi n}{60} = \frac{2 * 3,14 * 900}{60} = 94,26$ rad/s

$r_b = 50$ mm.

I) Draw displacement diagram, as follow:

1) Draw horizontal line Ax, and locate $\theta_o = 100^\circ = AS$, $\theta_d = 80^\circ = ST$, $\theta_R = 90^\circ = Tp$, and $\theta_{Re_M} = PQ = 90^\circ$.

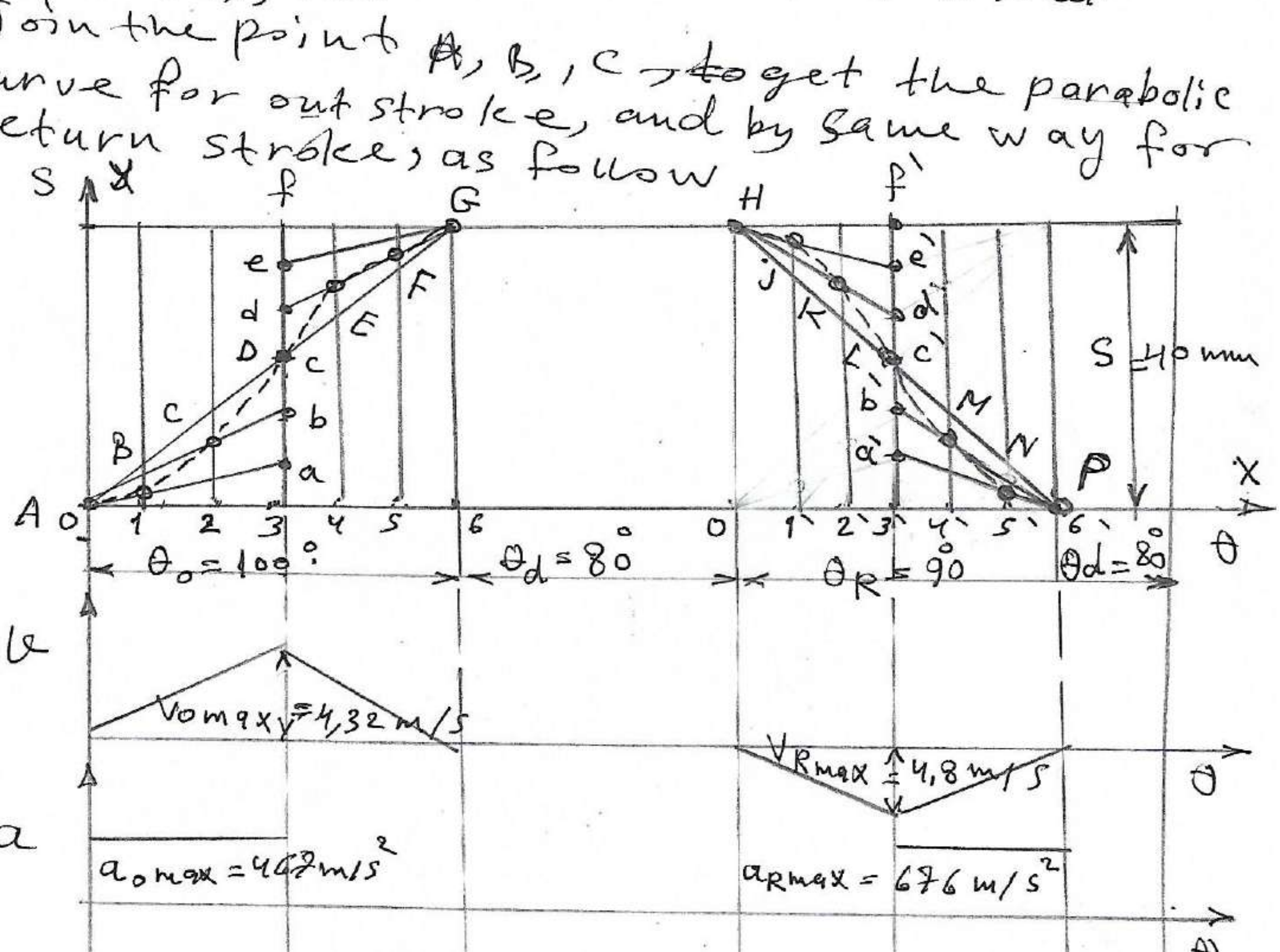
2) Divid AS, $\overline{Tp}$ in to equal number with Scale $2^\circ = 1mm$ Say (6) into 1, 2, 3 -- 6

3) Draw Vertical line Ay for (strok = 40mm) and devide into Same number (6) → from 3f and 3f` into 1, 2, 3 -- 6.

4) Since the follower move with uniform acc-n and retar-ino, then we have double parabolic Curves for out and return s-ice.

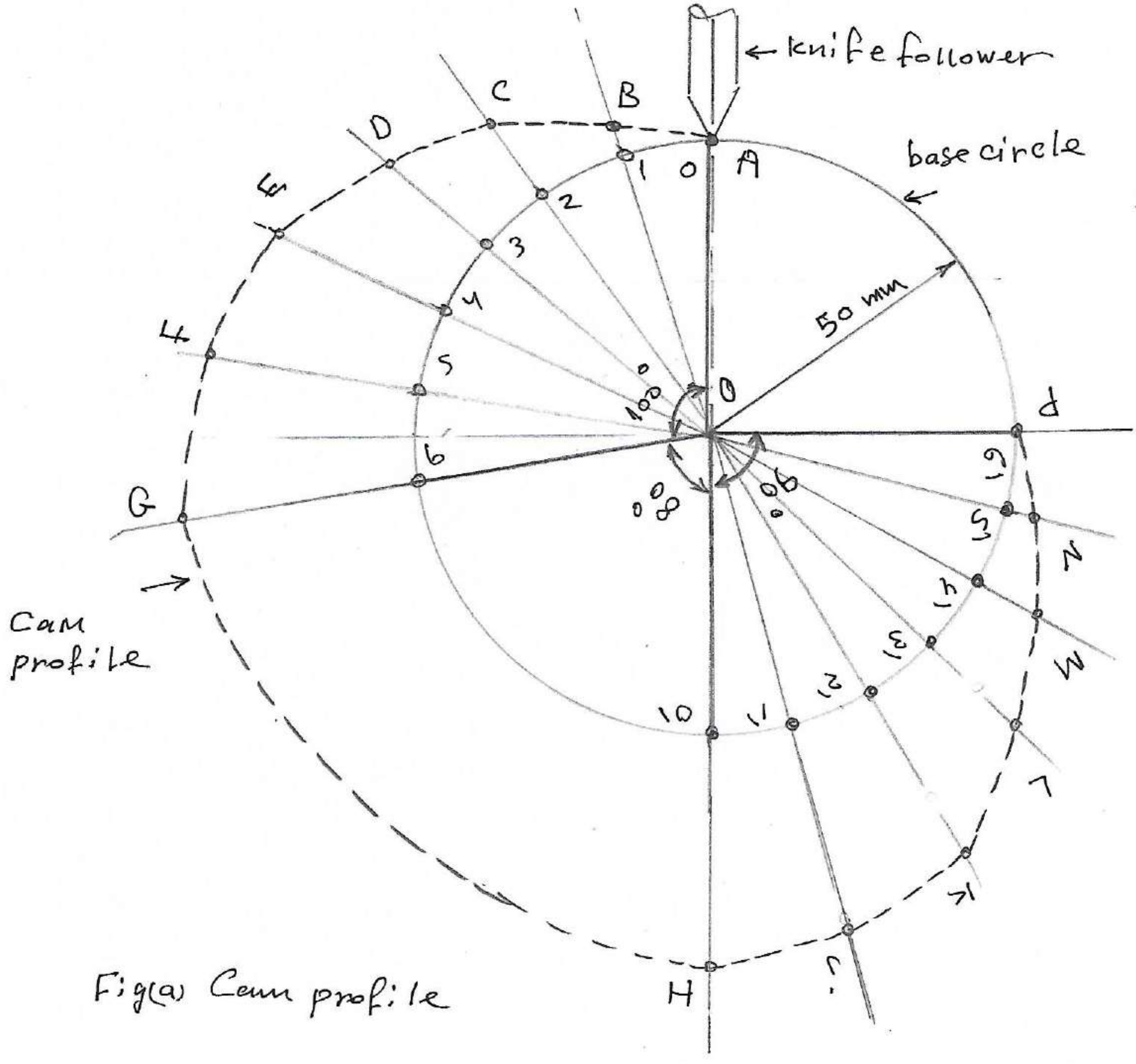
5) Join A with a, b, c -- and locate the inter-secting point A, B, C -- with a vertical lines 1, 2, 3 —

6) Join the point A, B, C → to get the parabolic Curve for out stroke, and by Same way for return stroke, as follow



II) Draw profile of cam when the follower passes through the center of cam, as follow:

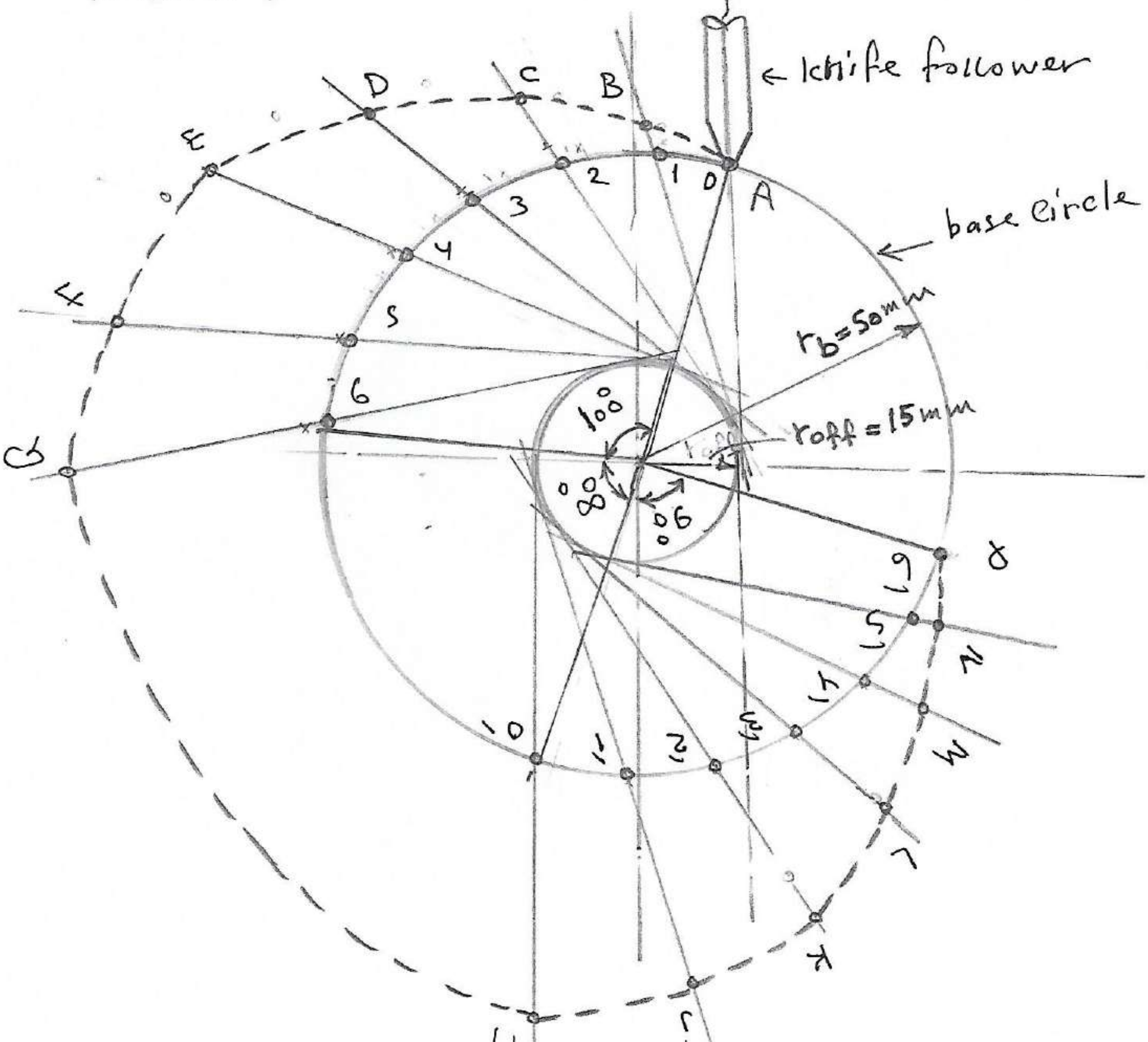
1) Draw base circle with $r_b = 50mm$
2) Divide the base circle into $\theta_o = 100^\circ$, $\theta_d = 80^\circ$, $\theta_R = 90^\circ$, $\theta_{Re} = 80^\circ$, and divide them into same number as in S-D, → 1, 2, → 6.
3) Join the point, 1, 2, 3 with o
4) added the length B_1, C_1 - from S-D
5) Join A, B, C → to get Cam profile, as follow:



Fig(a) Cam profile

III) Draw cam profile with the follower passes offset with the cam center, as follow:

1) Draw base circle with radius (r_b = 50 mm).
2) Draw offset circle with radius (r_{offs} = 15 mm).
3) Divide the base circle into $\theta_o = 100°$, $\theta_d = 80°$, $\theta_R = 90°$, $\theta_{Rem} = 90°$, and divide θ_o, θ_R into same equal parts (1, 2 --6).
4) Draw tangent to the base circle passing through (1, 2, --6),
5) Added the lengths from S-D and locate the points A, B -- P,
6) By joing the point, A, B, --P → we get cam profile. as follow:



IV) Max-m velocity for out and return stroke.

$$V_{o\,max} = \frac{2\omega S}{\theta_o} = \frac{2 * 94.26 \times 0{,}04}{1.743} = 4.32\ m/s$$

$$V_{R\,max} = \frac{2\omega S}{\theta_R} = \frac{2 * 94.26 * 0{,}04}{1.371} = 4.8\ m/s$$

IV) max-m acceleration for out and return stroke.

$$a_{o\,max} = \frac{4\omega^2 S}{(\theta_o)^2} = \frac{4 * (94.26)^2 * 0{,}04}{(1.743)^2} = 467\ m/s^2$$

$$a_{R\,max} = \frac{4\omega^2 S}{\theta_R^2} = \frac{4 * (94.26)^2 * 0{,}04}{(1.571)^2} = 576\ m/s^2$$

Example (5):

Design a Cam for operating the exhaust valve of an oil engine. It is required to give equal uniform acceleration and retardation during opening and closing of the valve each, of which corres-ponds with 60° of cam rotation. The valve must remain in the fully open position for 20° of cam rotation. The lift of the valve is (40 mm), and the least radius of the cam is (40 mm). The follower is provided with a roller of radius (10 mm), Draw cam profile, if: 1, line of stroke passes through the axis of the cam, 2) line of stroke is off set by (20 mm) from the center of the cam.
3 = Determine the max-m velocity and acceleration of the follower, when the cam shaft rotates at 900 r.p.m, 4 - Draw displacement velocity and acceleration for complete revolution.

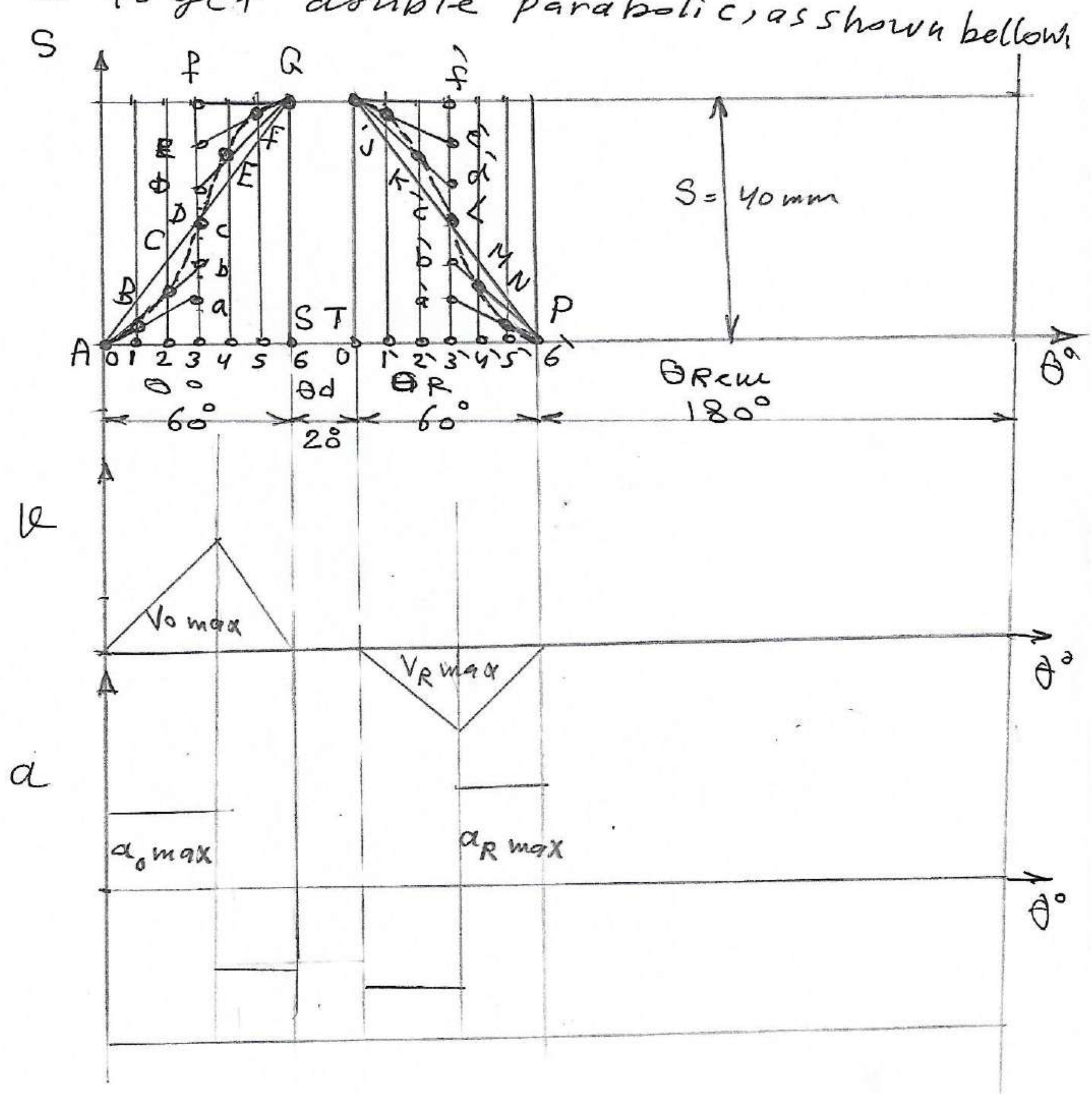
I) Draw: S-D, V-D, A-D, as follow:

1, Draw horizontal line ASTP represent ~~the~~ angular displacement, with scale (2C°=1m), then mark, $\theta_o = 60°$, $\theta_d = 20°$, $\theta_R = 60°$, $\theta_{Rem} = 180°$.

2) Divide AS, TP into same equal parts (as six), and draw vertical lines, 1, 2 -- 6 throw them.

3- Dived the line of strok 3f, 3f`, into same equal parts, a, b, -- f, and complete the S-D.

4- Locate the cross points, A, B -- G, and join them to get double parabolic, as shown bellow,



II) Draw Cam profile, when when line of stroke passes through the center of Cam, as follow.

1) Draw base circle with (r_b = 40 mm).

2) Draw pime circle wit $r_p = r_b + \frac{1}{2} r_R) = 40+10 = 50$ mm).

3) Locate the angles on the prime circle $\theta_o = 60°$, $\theta_d = 20$, $\theta_R = 60$, and divide them into same equal parts 1, 2, --6 as in S-D; draw lines from O throw 1, 2, --6,

4) Add the length 1B, 2C --

5) Join the points, A, B -- to get the Cam profiles as follow:

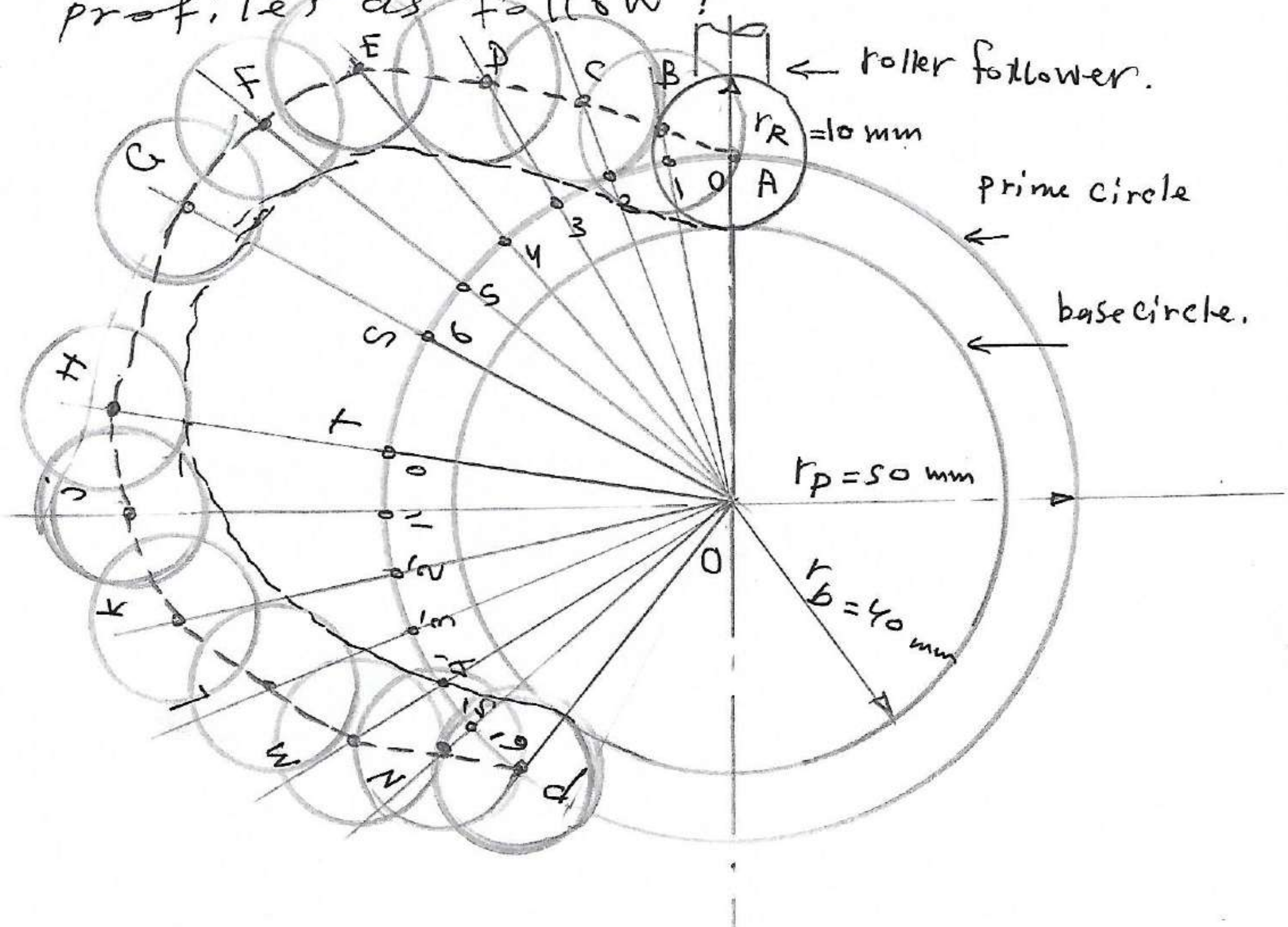


fig a) Cam profile with axis passes through the center of the Cam.

III) Draw Cam profile, when the line of stroke is off-set by 20 mm from the axis of the Cam, as follow:

1- Draw base circle with radius ($r_b = 40$ mm) and center o.

2) Draw prime circle with radius ($r_p = r_b + r_R = 40+10 =$ $= 50$ mm and center o.

3) Draw offset circle with radius $r_{off} = 20$ mm) and center o.

4) Divide the prime circle into $\theta_o = 60°$, $\theta_d = 20°$ $\theta_R = 60$, into equal number as in S-D 1, 2, --6,. ~~added~~

5) Draw tangent from the offset circle. passing through 1, 2 -- 6., and add the lengths to them from S-D, 1B, 1C --

6) Join point A, B, -- to get Cam profile as follow

IV) max-m velocity:

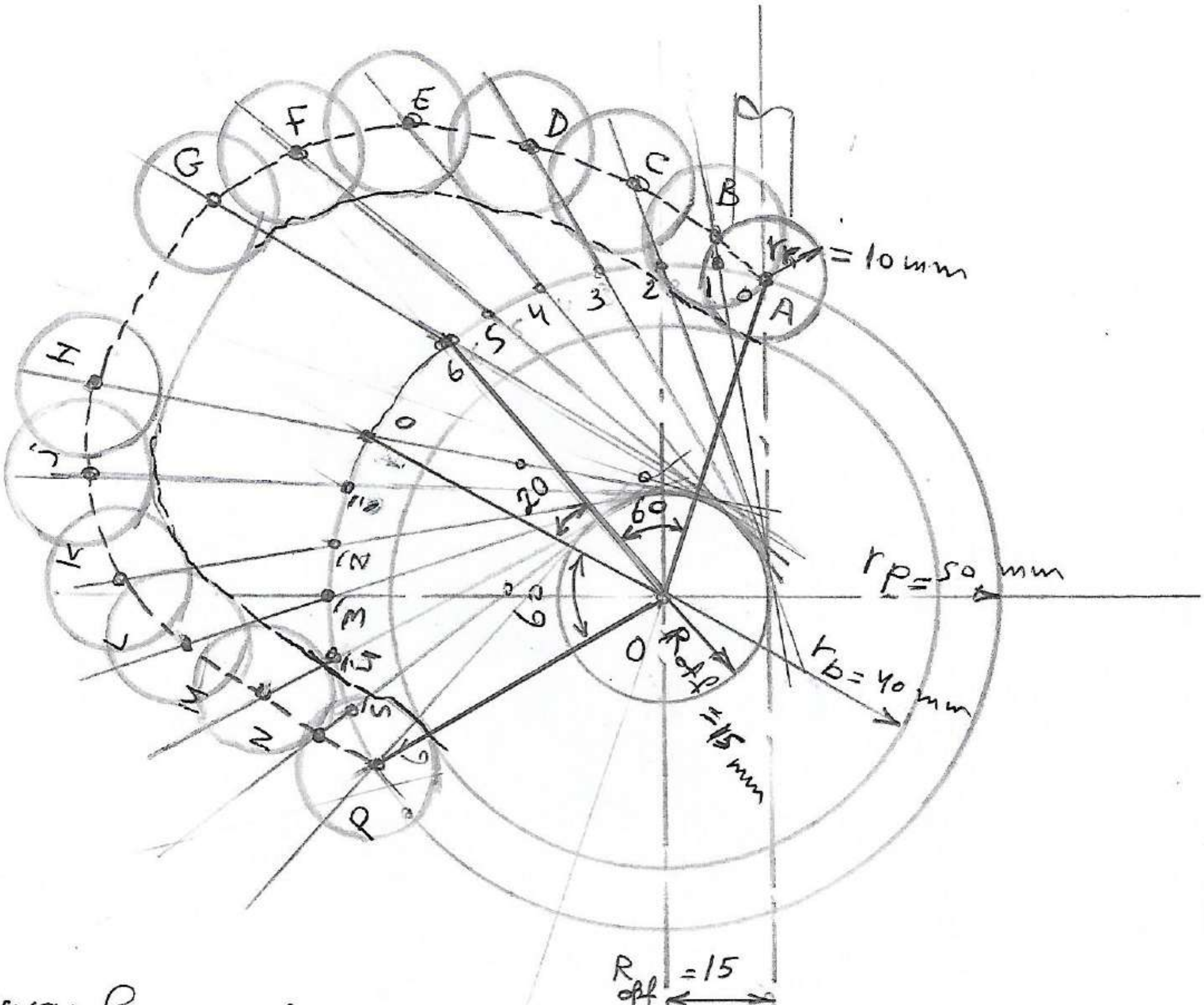
$$V_{o\,max} = \frac{2 w S}{\theta_o} = \frac{2 * 942 * 0,04}{1.047} = 71.977 \text{ m/s}$$

$$V_{R\,max} = \frac{2 w S}{\theta_R} = \frac{2 * 942 * 0,04}{1.047} = 71.977 \text{ m/s}$$

V) max-m acceleration.

$$a_{o\,max} = \frac{4 w^2 S}{(\theta_o)^2} = \frac{4 * (942)^2 * 0,04}{(1.047)^2} = 141978 \text{ m/s}^2$$

$$a_{R\,max} = \frac{4 w^2 S}{(\theta_R)^2} = \frac{4 * (942)^2 * 0,04}{(1.047)^2} = 141978 \text{ m/s}^2$$



Fig(a) Cam profile with off-set

$$\omega = \frac{2\pi n}{60} = \frac{2 * 3.14 * 900}{60} = 942 \text{ rad/s}.$$

$$\theta_o = \frac{60 * \pi}{180} = 1.047 \text{ rad}$$

$$\theta_R = 1.047 \text{ rad}.$$