* * Velocity and Acceleration

I-Velocity in mechanisms

* Introduction :

The motion of a body is a Combination of a body translation and rotation, in order to show this effect, we Consider a rigid link AB, which moves from its initial position $AB \xrightarrow{to} A_1B_1$, as shown in fig (a)

$$V_B^{T} = V_B^{t} + V_{BA}^{r}$$

اكليه انتقاليه دورانيه

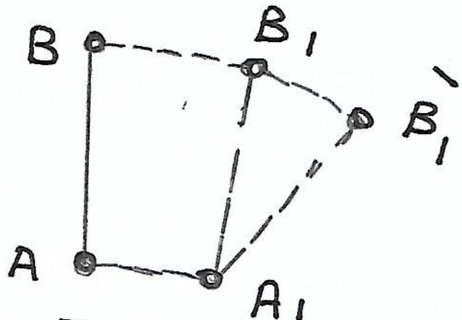


Fig (a)

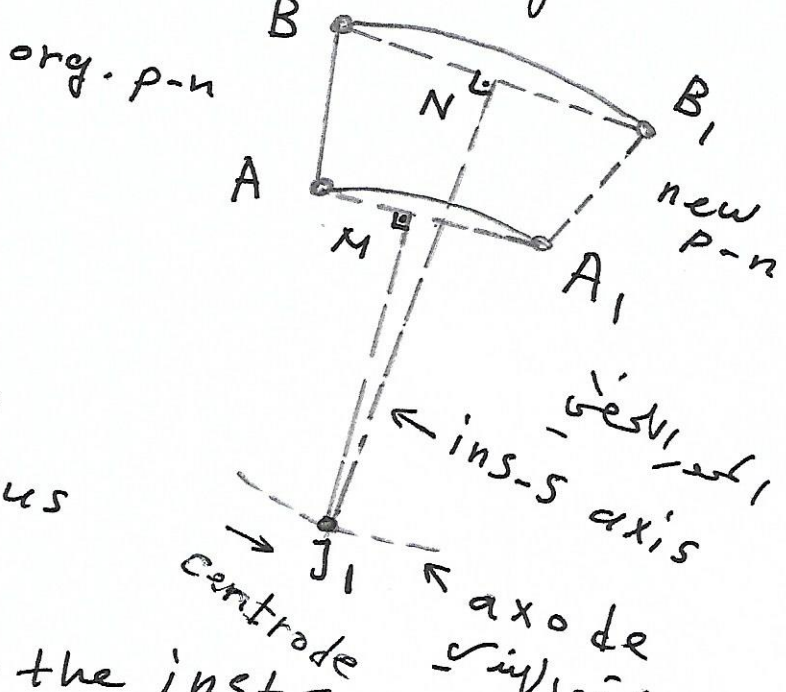
at first the link has a translation motion from $AB \xrightarrow{to} A_1B_1$, and then rotation m-n about (A_1), till it occupies the position A_1B_1. But in the practic is different, because is difficult to see these two separate motions, for this reson we assume the link has a pure rotation about some.

Center known as the instanta-neous Center of rotation (I)

The position of (I) lie on the

عند تقاطع الاعمدة المنصفة

inter section of the right bise-ctors of cords AA_1 and BB_1 at (I_1) as shown in fig (b)



The position of the inst-antaneous centres are changing from one instant to another, so the position of all them is known as Centrode. The line drawn through the J and ⊥ to the plane of motion is called inst-antaneous axis, and the Locus of this axis is known as axode.

1) * Methods for determining the Velocity of a point on a Link

For determining the velocity of any point on a link in M-M, whose direction of m-n is known and, and velocity of other point on the same link is known in dire-ction and magnitude, there are many methods, important of them are:

1- Instantaneous Method: use for simple m-m.
2- Relative method: use for all M-M. so we use it -

طريقة السرعة النسبية

2) II - Relative velocity method:

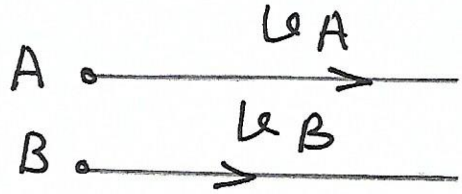
لغرض توضيح طريقة السرعة النسبية يجب توضيح قاعدة المتجهات على جسمان يتحركان على خطوط متوازية ومائلة كما يلي

Here we shall discus the application of vectors for relative velocity of two bodies moving along parallel and inclined lines as shown in fig(c).

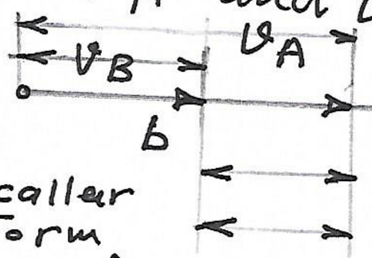
Let A and B two bodies moving along parallel Lines in the same direction, with absolute Veloci--ties v_A, v_B, and

$v_A > v_B$.

the relative velocities of A with respect to B For two // Lines are



Vector difference of v_A and v_B are be :



center Pole

$$v_{AB} = \bar{v}_A - \bar{v}_B \quad \text{----} (1)$$

Scallar Form

The relative velocity of A with respect to B in Vector form

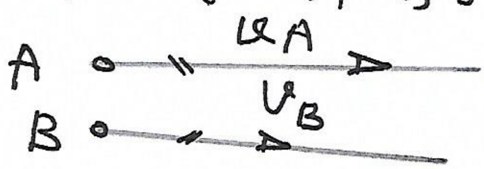
$$\bar{ab} = \bar{oa} - \bar{ob} \quad \text{---} (2)$$

Similarly

$$v_{BA} = \bar{v}_B - \bar{v}_A$$

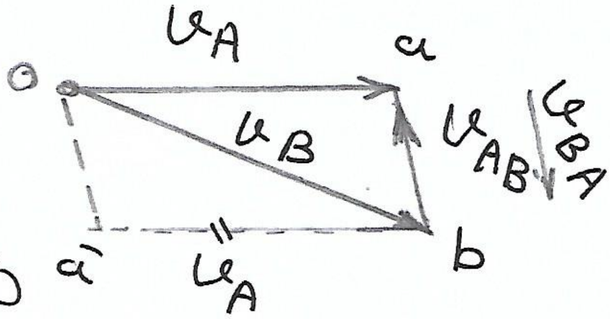
$$\bar{ab} = \bar{ob} - \bar{oa}$$

in an inclined direction as shown or for inclined Lines, the relative



velocity of A with respect of B may be obtained by the law of parallelogram (triangle law) of velocities.

Vector difference of v_A and v_B



$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ --- (1)

$\bar{ba} = \bar{oa} - \bar{ob}$

similarly,

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$ --- (2)

$\bar{ab} = \bar{ob} - \bar{oa}$

For above we conclude that

$v_{AB} = -v_{BA}$

$\bar{ba} = -\bar{ab}$

therefor From (1) and (2)

$\overset{a}{\vec{v}_A} = \overset{T}{\vec{v}_B} + \overset{r}{\vec{v}_{AB}}$

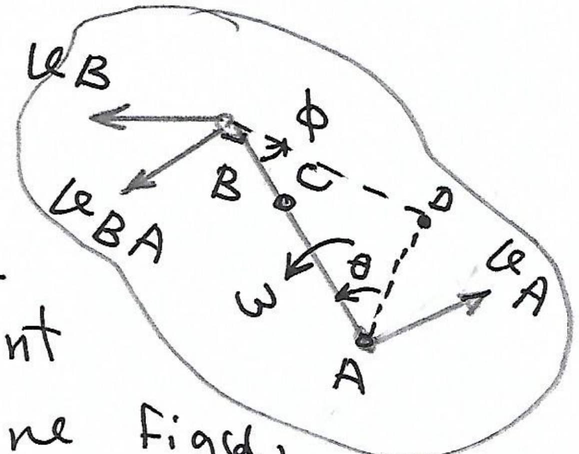
$\overset{a}{\vec{v}_B} = \overset{T}{\vec{v}_A} + \overset{r}{\vec{v}_{BA}}$

مطلقة	a- absolute
انتقالية	T- translation
دورانية	r- rotation.

* Relative velocity of point on link.

Consider two point A and B on a link as shown in fig (d)

7) The relative velo-
- city v_{BA} of the
point B with res-
- pect to the point
A is ⊥ to the line fig(d),
joining these two points,



∴ $v_{BA} \perp BA$ at B in direction of rotation. A) when angular velocity is known then:

$$v_{BA} = \omega \cdot \overset{(r)}{AB} = \bar{ab} \quad \ldots (1)$$

similarly, the velocity of any point (C) on AB with respect to A.

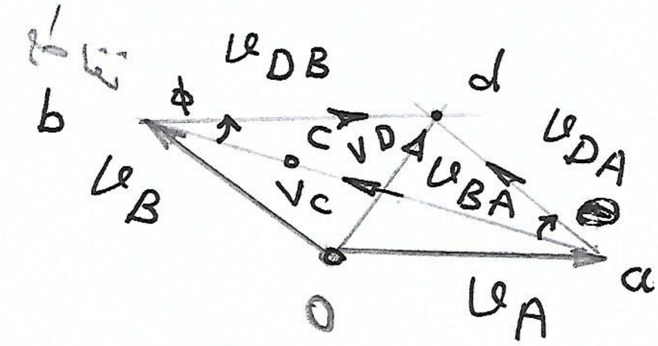
$$v_{CA} = \omega \cdot AC = \bar{ac} \quad \ldots (2)$$

by dividig (2) on (1)
and
$$\frac{v_{CA}}{v_{BA}} = \frac{\omega \cdot AC}{\omega \cdot AB} = \frac{AC}{AB} = \frac{\bar{ac}}{ab} \quad \ldots (3)$$

II
B) By using the velocity diagram.

This method is used when absolute velocity of the point A is known in magnitude and direction, and the absolute velocity of the point B is known only in direction . there for the magnitude of v_B may be find by drawing velocity diagram as follow

by using a sutable scale :



$$K_V = \frac{V_A}{\bar{V_A}}$$

1) take pole o
2) throw o draw oa // and = to V_A .
3) through a draw line $\perp$ to AB → represent v_{BA}
4) through o draw line // to V_B meeting v_{BA} at b .
5) ob is the required magnitude of v_B .

The absolute velocity of any point like C on AB may be determined by dividing vector ab at c into the same ratio .

$$\frac{ac}{ab} = \frac{Ac}{AB}$$

and $c\bar{o}$ represent V_c .

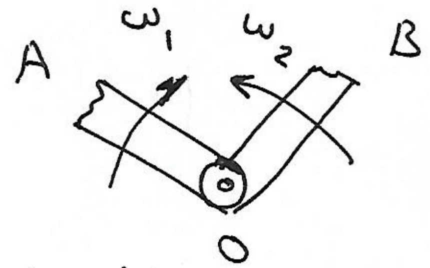
ملاحظ: السرعة المطلقة لاي نقطة مثل D خارج (AB)؟ يمكن الحصول عليها باكمال المثلث abd المشابه للمثلث ABD .

Note :
The absolute velocity of any other point D, out side AB may be obt-aned by completing the velocity traingle abd similar to traingle ABD.

* Rubbing velocity at a pin joint

For the M-M which connected by pin joint, define as the algebraic sum between the angular veloci-ties multiplied by the redius of the pin.

let: oA and oB, as shown two links with pin joint.



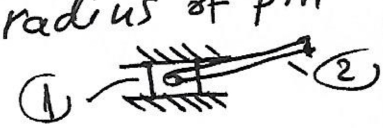
∴ Rubbing velocity at pin joint o

$(\omega_1 - \omega_2) * r$ —— (1) when the two links move in the same direction.

and,

$(\omega_1 + \omega_2) * r$ —— (2) when the two links moves in the oppo-site direction.

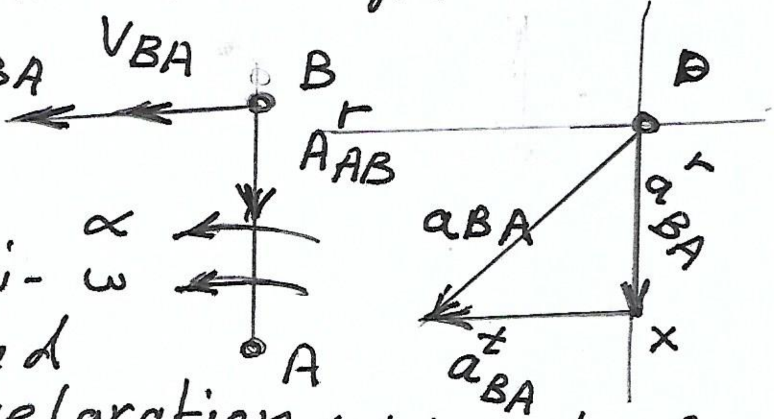
Note:
when the pin connect one sliding link and other turn
∴ $\omega = 0$ for slide link
and Rub $v = \omega * r$; where | ω - for turn link, r - radius of pin



* II. Acceleration in Mechanism

* Acceleration diagram of a link:

let we have two points A and B on a rigid link, as shown in fig (a).



The point B moves with respect to A with - angular veloci- -ty (ω) rad/s, and with angular Accelaration (α) rad/s²

∴ The total acceleration (a_{AB}) of a partical whose velocity change in magnitude and direction at any instant has two Components: as Follow:

1- Radial Component which is // to AB or ⊥ v_{BA} from B→A indirection of ω.

$$a^r_{BA} = \omega^2 \cdot \overset{r}{\hat{BA}} = \left(\frac{v_{BA}}{BA}\right)^2 * BA = \frac{v_{BA}^2}{BA} \quad \text{--- (1)}$$

2- tangential Component which is ⊥ AB → or // to v_{BA}

$$\therefore a^t_{BA} = \alpha * BA \quad \text{---- (2)}$$

∴ Total acceleration of B with respet to A:

$$a_{BA} = a^r_{BA} + a^t_{BA} \quad \text{--- (3)} \quad \text{where:}$$

$$= \omega^2 * BA + \alpha * BA = \frac{v_{BA}^2}{BA} + \alpha * BA \quad \text{--- (3)}$$

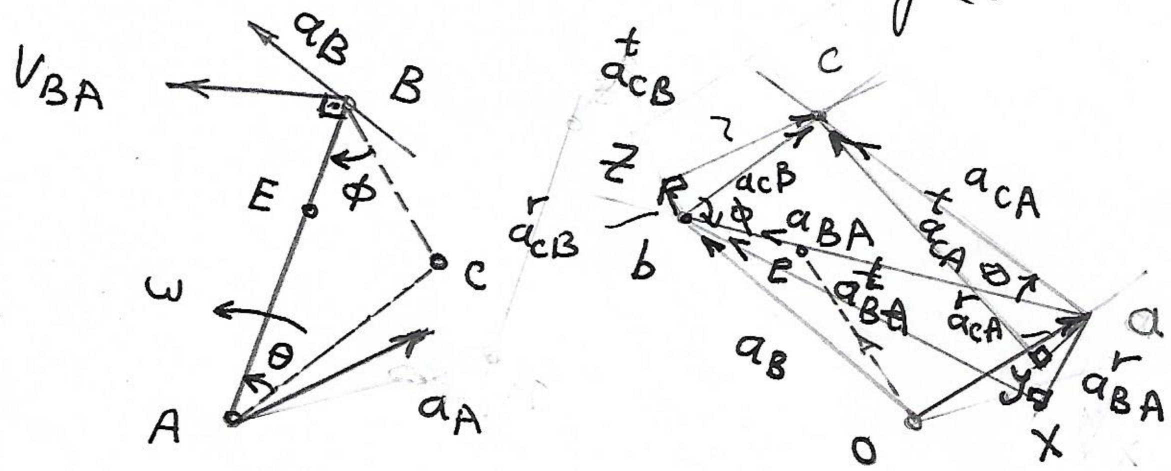
where angular acceleration of L-BA

$$\boxed{\alpha_{BA} = \frac{a^{t}_{BA}}{BA}} \quad ---- (4)$$

and the acceleration diagram as shown in fig (b), by taking pol (b), from b // to V_{BA} draw a^{r}_{BA} and $\perp$ to a^{r}_{BA} draw a^{t}_{BA}.

* Acceleration of point on a link

let we have two points A, B on a rigid link AB, as shown in fig (a).



→ in which the absolute acceleration of p-(A) - a_A - is known in direction and magnitude, and

a_B - absolute acceleration of p-B is known only in direction.

∴ The magnitude of a_B may be determine by drawing the acceleration diagram as follow:

1- Take pol o.

2 - Take suitable scale, and find the acceleration scale module.

$$\boxed{k_a = \frac{a_A}{\bar{a}_A}} \quad --- (1)$$

3 - $a_A * k_a = \bar{oa}$ change into scale,

4 - For drawing a_A, from o draw $oa \parallel =$ to a_A.

5, $a_{BA} = a^r_{BA} + a^t_{BA}$ → the acceleration of B w.r.t. the A, where

→ $a^r_{BA} = \frac{V^2_{BA}}{BA}$, and $V_{BA} = W * BA$ → from V-D, and من تعويض السرعة

$a^r_{BA} * K_A = ax$ → into scale. تحويل

∴ from a draw vector $ax = \parallel a^r_{BA}$, with direction from B to a.

→ a^t_{BA} - from x draw $\overset{V-r}{\perp}$ ax, with directoin of V_{BA}, and

→ From o draw $\overset{vector}{V\text{-}r}$ $\parallel \dot{a}_B$ and in same direction to intersect at قطع p. b, then join ba,

and take | Vector $ob = a_B$
= $bx = a^t_{BA}$
= $ba = a_{BA}$ total ac of B w.r.t. A.

∴ For angular accelaration $\alpha = ?$

$$\bar{Xb} * k_a = a^t_{BA} = \alpha * BA$$

$$\therefore \boxed{\alpha = \frac{a^t_{BA}}{BA}} \quad --- (2)$$

* The acceleration diagram for any other point on the link AB, like C.
→ draw triangle △ abc ≅ similar to triangle ABC. where the acceleration:

a_{CA} → of C with r. to A
a_{CB} → of C with r. to B

* and $a_{CA} = a^{r}_{CA} + a^{t}_{CA}$, by same way

$a^{r}_{CA} = ay$ // CA → from C → A باتجاه
$a^{t}_{CA} \perp ay$ From y
a_{CA} → form angle θ with a_{BA} intersect at c.

* also : $a_{CB} = a^{r}_{CB} + a^{t}_{CB}$, where

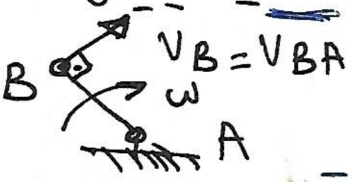
$a^{r}_{CB} = bz$ // Bc from c → B
$a^{t}_{CB} = zc \perp bz$
a_{CB} Form angle ϕ with a_{BA} → c.

Notes

ملاحظات

ملاحظة ①: السرعة النسبية لنقطة متحركة بالنسبة إلى نقطة ثابتة تساوي السرعة المطلقة للنقطة المتحركة. كذلك المركبة القطرية للتعجيل لتلك النقطة تساوي التعجيل المطلق لتلك النقطة وليس لا مركبة مماسية للتعجيل

$V_B = V_{BA} = \omega_{BA} * BA$



R - pair

$$\begin{cases} a_B = a^r_{BA} = \omega^2_{BA} * BA = \dfrac{V^2_{BA}}{BA} \parallel BA \\ a^t_{BA} = 0 \end{cases}$$

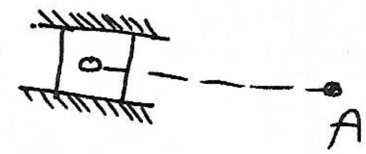
ملاحظة (٢) :

لغرض حساب المركبة القطرية للتعجيل اذا كانت السرعة الزاوية الثابتة غير معلومة ، يجب رسم مثلث السرعة لغرض الحصول على السرعة النسبية :

ملاحظة (٣) :

النقطة التي تتحرك على خط مستقيم ليس لها مركبة قطرية للتعجيل وانما فقط مركبة مماسية وتساوي التعجيل المطلق لتلك النقطة وتكون موازية لاتجاه السرعة لتلك النقطة

$a_B = ab$ - absolute acceleration only $\parallel V_B$



$a^r_B = 0$

ملاحظة (٤)

لغرض الحصول على تعجيل اي نقطة مثل E على العتلة AB نقسم المخطط ba عند النقطة (e) بنفس نسبة العتلة AB عند النقطة E وإذا كانت في المنتصف ايضا تكون في منتصف المخطط

$$\frac{AE}{AB} = \frac{ae}{ab}$$