

* Balancing

Introduction

when we attached mass to a rotating shaft, in which produce centrifugal force and vibration, in order to prevent the shaft from this effect, we must attached another mass into the opposite side of the shaft, this process is called Balancing.

* Types of balancing

- I, Balancing of rotating masses
- II, Balancing of reciprocating masses

* I/ Balancing of rotating masses

* Cases of of balancing of rotating masses.

* I, Balancing of a single rotating mass by single rotating mass in the same plane.

Case of static balance

In this case the balancing is done by adding (or cutting) mass into the opposite direction, untill the (F_c) be equal.

∴ The condition of static balance :

$$1) - \boxed{\sum F = 0} \rightarrow \text{Force balance} \quad \text{①}$$

$$F_{c1} = F_{c2}$$

$$m_1 r_1 \omega^2 = m_2 r_2 \omega^2$$

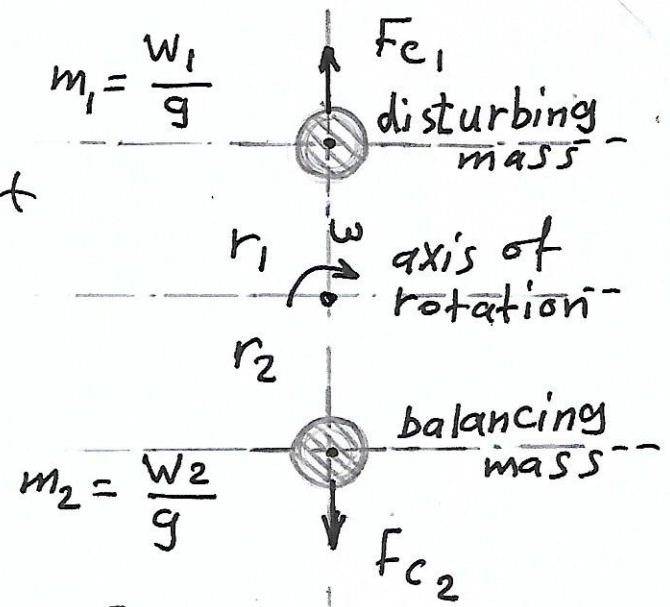
, where F_c - Centrifugal force
 ω - rad/s
 w - weight

$$\frac{W_1}{g} \omega^2 r_1 = \frac{W_2}{g} \omega^2 r_2$$

$$W_1 r_1 = W_2 r_2 \quad \text{--- (2)}$$

$$\Sigma M = 0$$

→ moment balance.
--- (2)



طريقة (أ): شرط التوازن يكون مجموع القوى ومجموع العزوم = 0
طريقة (ب): لفظة تقليل كتلة التوازن، يجب اختيار الدائرتين مع التوازن
طريقة (ج): لفظة تقليل كتلة التوازن، يجب اختيار الدائرتين مع التوازن
II) Balancing of single rotating mass by two masses rotating in different planes.

هذه الحالة تمثل التوازن الديناميكي، حيث عند الحركة بالإضافة إلى القوة يتولد عزم عن المحاور.
In this case we have dynamic balance, in which produce couple which tends to turn the shaft in its bearings, therefore for complete balance, we must place two masses in different planes, parallel to the plane of rotation.

∴ The condition of complete dynamic balance:

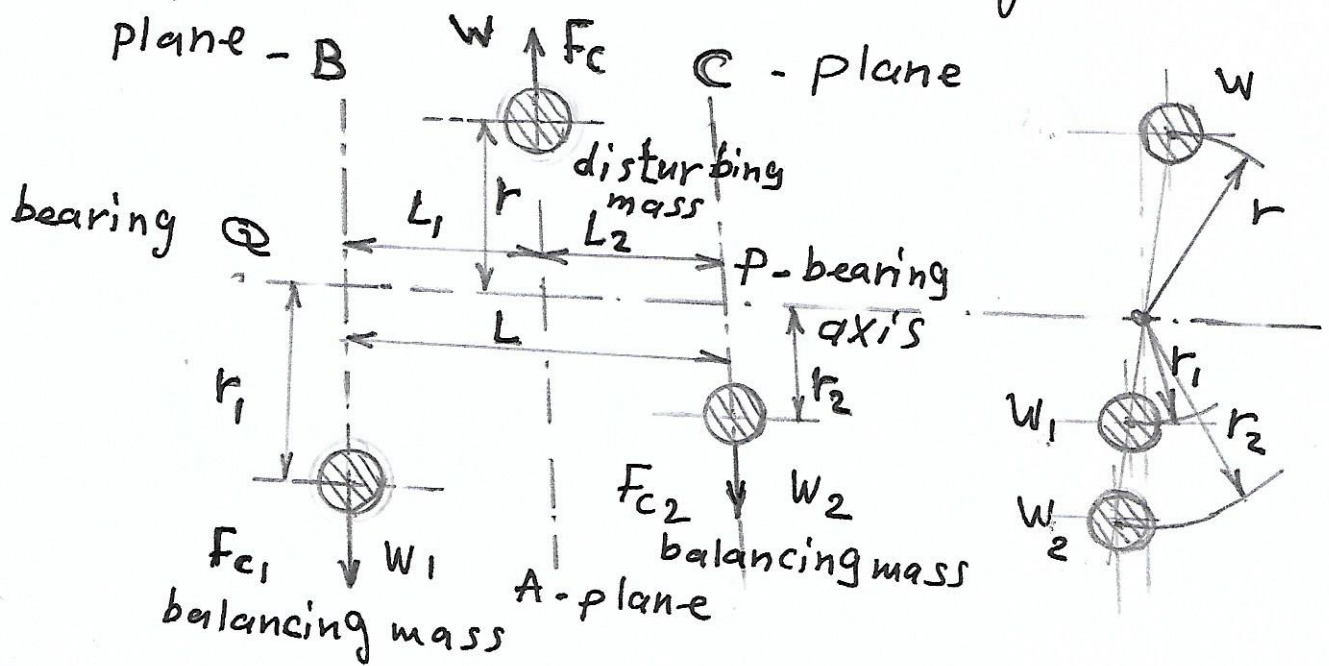
- 1) $\Sigma F = 0$ → static balance.
- 2) $\Sigma M = 0$ → Couple due to force.

* ways of dynamic balance: طرق التوازن الديناميكي
* There are two ways of attaching of two balancing masses

- 1) First way when the plane of the disturbing mass lie between the different

-EN-

planes of the two balancing masses.



∴ The Condition of complete balance

$$1) \sum f = 0$$

$$\therefore F_c = F_{c1} + F_{c2}$$

$$\therefore \frac{W}{g} \omega^2 r = \frac{W_1}{g} \omega^2 r_1 + \frac{W_2}{g} \omega^2 r_2$$

$$\therefore \boxed{Wr = W_1 r_1 + W_2 r_2} \quad \text{--- (1) } \rightarrow \text{static balance.}$$

$$2) \sum M = 0$$

In order to find the magnitude of balancing force (dynamic force in bearing Q) in plane B, we must take:

$$\sum M_P = 0$$

$$\therefore F_{c1} * L + F_c * L_2 = 0$$

$$\therefore F_{c1} * L = F_c * L_2$$

$$\frac{W_1}{g} \omega^2 r_1 L = \frac{W}{g} \omega^2 r L_2$$

$$\therefore \boxed{W_1 r_1 = \frac{W r L_2}{L}} \quad \text{--- (2)}$$

Similar for plane (C).

$$\sum M_Q = 0$$

$$\rightarrow +F_{C_2} \cdot k_2 + F_C \cdot k_1 = 0$$

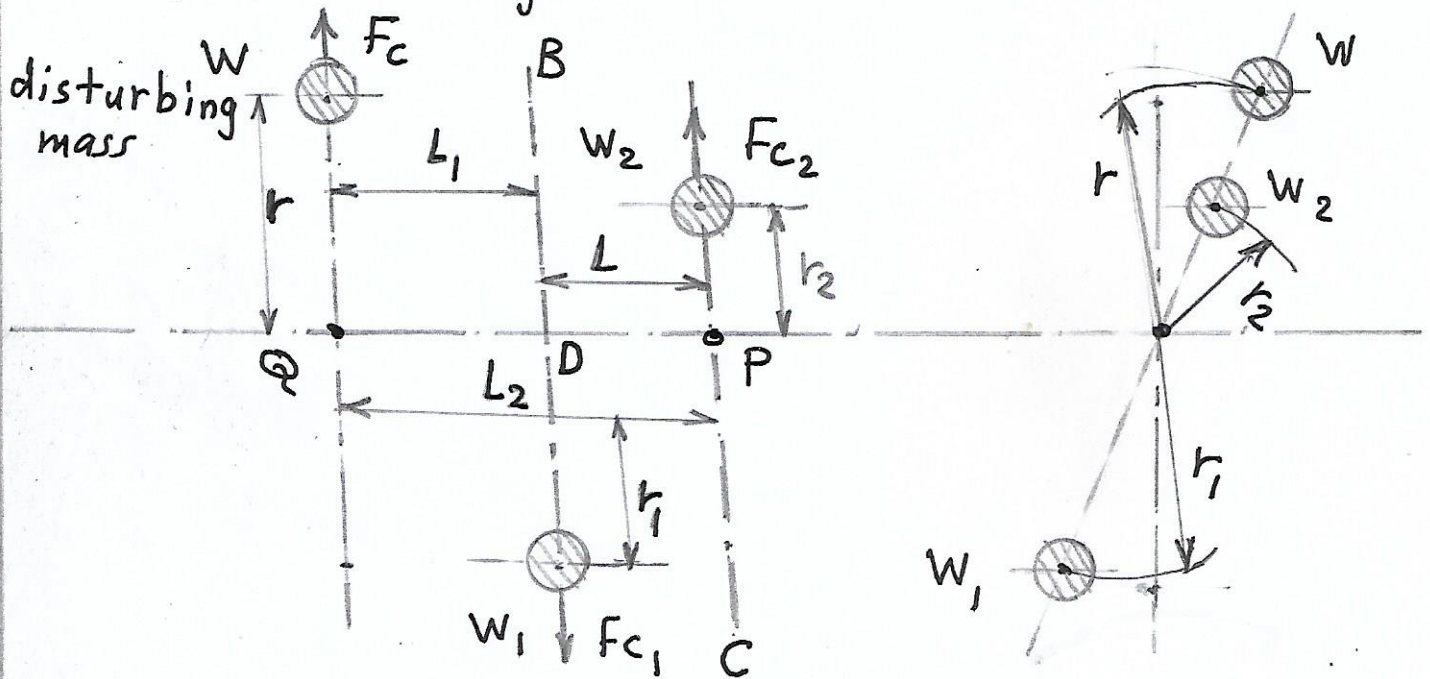
$$\therefore F_{C_2} \cdot L = F_C \cdot L_1$$

$$\frac{W_2}{g} \omega^2 r_2 L = \frac{W}{g} \omega^2 r L_1$$

$$W_2 r_2 = \frac{W r L_1}{L} \quad \text{--- (3)}$$

$\therefore$ equations (1), (2), (3) $\rightarrow$ represent condition of Complete dynamic balance.

2) The second way when the plane of the disturbing mass lies on one end of the planes of balancing masses



$\therefore$ The condition of the Complete balance be:

$$1) \sum f = 0$$

$$\therefore F_C + F_{C_2} = F_{C_1}$$

$$\therefore W r + W_2 r_2 = W_1 r_1 \quad \text{--- (1)} \rightarrow \text{static balance.}$$

$$2) \sum M_P = 0$$

$$\therefore W_1 r_1 L = W_2 r_2 L$$

$$\therefore \boxed{W_1 r_1 = \frac{W_2 r_2 L}{L}} \quad \dots (2)$$

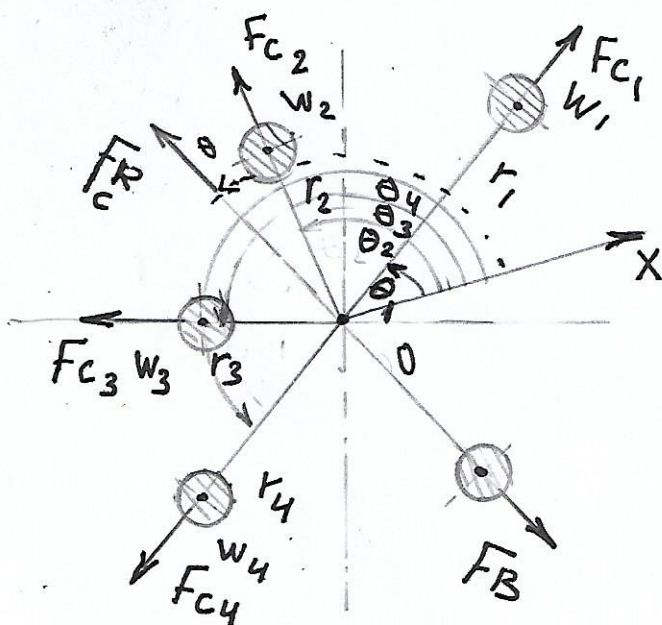
$$\sum M_D = 0$$

$$\therefore W_2 r_2 L = W_1 r_1 L$$

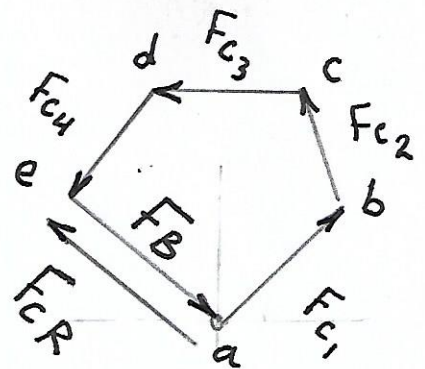
$$\therefore \boxed{W_2 r_2 = \frac{W_1 r_1 L}{L}} \quad \dots (3)$$

III, Balancing of several masses rotating in the same plane : توازن مجموعه کتل در وقتی متوازن باشند

Let we have any number of masses for example four masses, with weights (W_1, W_2, W_3, W_4), angles ($\theta_1, \theta_2, \theta_3, \theta_4$), radius (r_1, r_2, r_3, r_4) rotates in one plane about axis $OX \perp$ to the plane at O .



Fig(a)



Fig(b)

where $\begin{cases} ea = F_B \\ ae = F_{CR} \end{cases}$

* The magnitude and position of the balancing mass ~~may~~ may be found by two methods:

* 1) Analytical Method: الطريقة التحليلية

Is done by resolving the centrifugal forces horizontally and vertically, then we determine the resultant:

$$\sum R_H = W_1 r_1 \cos \theta_1 + W_2 r_2 \cos \theta_2 + \dots$$

$$\sum R_V = W_1 r_1 \sin \theta_1 + W_2 r_2 \sin \theta_2 + \dots$$

→ ∴ The magnitude of the resultant Centrifugal Force make angle θ with horizontal.

$$F_{CR} = \sqrt{(\sum R_H)^2 + (\sum R_V)^2} \quad \dots \dots \textcircled{1}$$

$$\tan \theta = \frac{\sum R_V}{\sum R_H} \quad \dots \dots \textcircled{2}$$

∴ The balancing force = Resultant C-force and in opposite direction, and the weight of the balancing force be:

$$F_{CR} = F_B = \frac{W}{g} \omega^2 r \quad \dots \dots \textcircled{3}$$

* 2) Graphical Method:

الطريقة البيانية
من الممكن الحصول على قيمة واتجاه كتلة التوازن
بتتبع الخطوات التالية:

The magnitude and position of balancing mass may be obtained as follows:

- 1) Draw space diagram with ^{the} positions of all masses Fig (a).
 ١- ادرم مخطط الزمان لموقع كل كتلة.
- 2- Find C-force for each mass (F_C) or their torque (Wr) (product of mass * radius).
 ٢- اوجد قوة الطرد المركزي او العزم لكل كتلة.

with

$\left\{ \begin{array}{l} \text{weights} \\ \text{radius} \\ \text{angles} \end{array} \right.$	w_1, w_2, w_3, w_4
	r_1, r_2, r_3, r_4
	$\theta_1, \theta_2, \theta_3, \theta_4$

and let the system (shaft) be balanced by two masses rotating in the planes L, M with

$\left\{ \begin{array}{l} \text{weight} \\ \text{radius} \\ \text{angles} \end{array} \right.$	w_L, w_M
	r_L, r_M
	θ_L, θ_M

$\therefore$ For Complete balance, we must determine the magnitude and direction of the balancing masses as follow:

- 1- Draw the linear and angular position of all the masses, as in fig(a), Fig(b), and transverse all the planes into reference plane, which is $\perp$ to the axis of rotation like (L), so the distance of all planes at left of R.P (-) and at right (+)

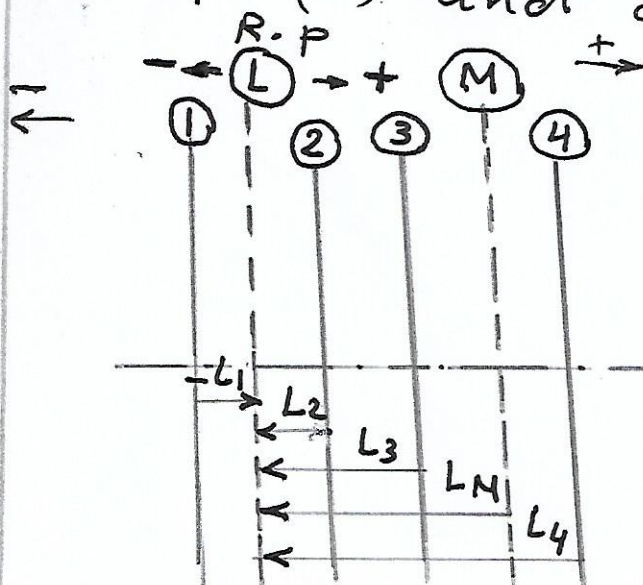


Fig (a) Linear position

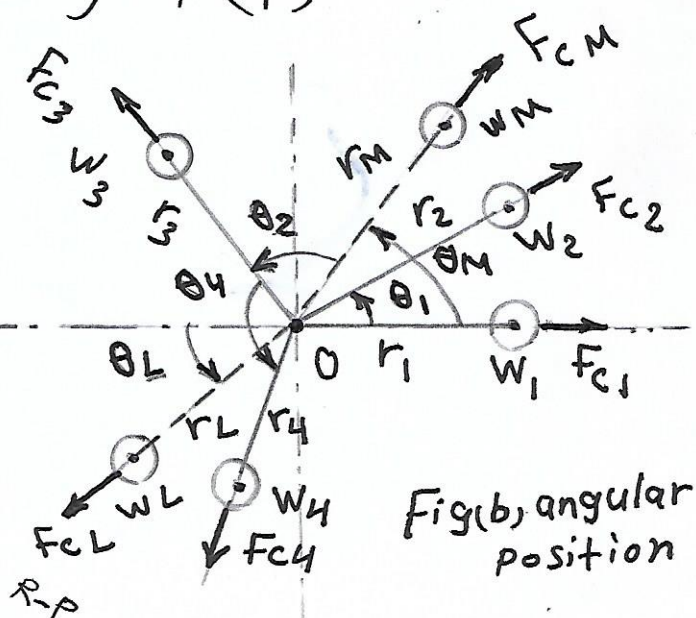


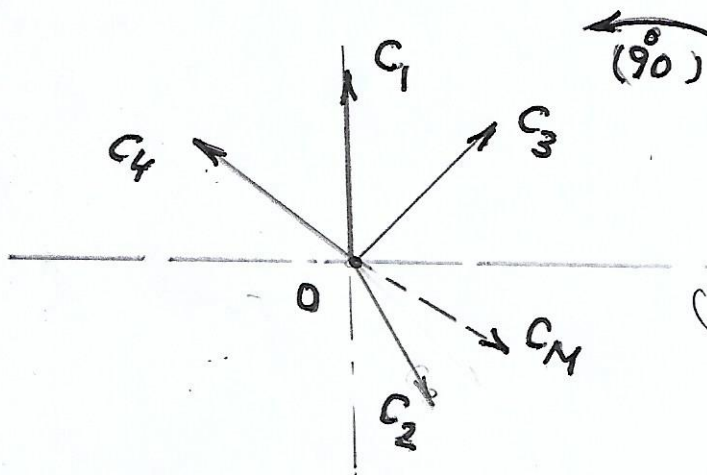
Fig (b) angular position

2- Tabulate the data, as in table (2)

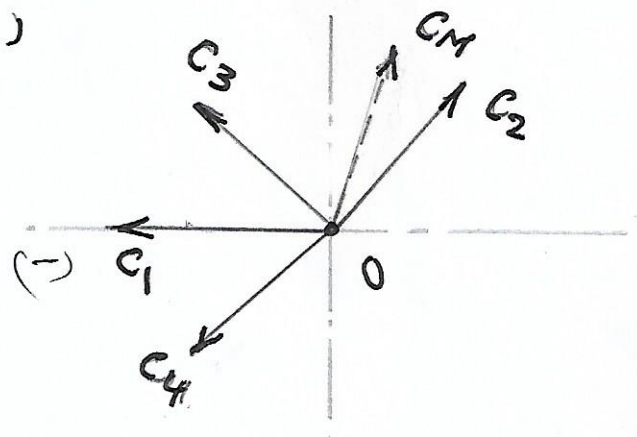
1	2	3	4	5	6
Plans	Weights $W(N)$	radius $r(mm)$	Centrifugal Force $\frac{W \cdot r}{g}$ $W \cdot r (N \cdot m)$	distance from R.P $L(m)$	Couple $\frac{W \cdot r^2}{g}$ $W \cdot r \cdot L (N \cdot m^2)$
1	W_1	r_1	$W_1 r_1$	$-L_1$	$-W_1 r_1 L_1$
L(R.P)	W_L	r_L	$W_L r_L$	0	0
2	W_2	r_2	$W_2 r_2$	L_2	$W_2 r_2 L_2$
3	W_3	r_3	$W_3 r_3$	L_3	$W_3 r_3 L_3$
M	W_M	r_M	$W_M r_M$	L_M	$W_M r_M L_M$
4	W_4	r_4	$W_4 r_4$	L_4	$W_4 r_4 L_4$

3- representation of Couple Vectors, as Follow.

Vector Couple of W_1 is C_1 , which is $\perp$ to W_1 , From 0 to the paper. by same way draw with magnitude $|C_1, C_2, C_3, C_4|$ as in fig (c).
 $-W_1 r_1 L_1, W_2 r_2 L_2, \dots$



Fig(c) Vector Couple



Fig(d) vector couple direction

by turning the Vector Couple 90 Counter clock wise became the Vector Couple and the Centrifugal force in the Same direction as in Fig (d). unless the vector C_1 in opposite direction as in table (2).

4- Drawing of Couple polygon, From column (6), table (2).

by taking pol and suitable scale, and draw all the Couple Vectors with same magnitude, direction and sense fig(e).

assume $1 \text{ N.m}^2 = 1 \text{ mm}$ Scale.

where:

$$C_M = d_o = W_M r_M L_M$$

∴ from Couple polygon we can determine the magnitude and direction of mass W_M as

Follow:

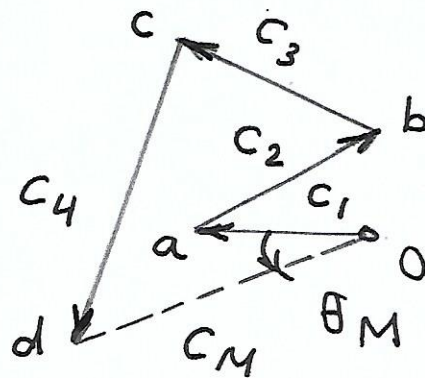
$$W_M = \frac{d_o (\text{N.m}^2)}{r_M L_M (\text{m}^2)} \quad \text{--- (1) where } \boxed{C_L = 0}$$

$\theta_M \Rightarrow$ angle of inclination

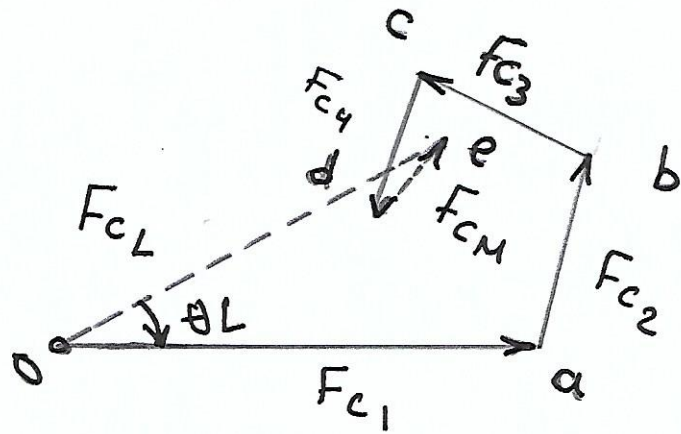
From o draw line // and = to d_o and inclined with C_1 at angle θ_M

5) Drawing of Centrifugal force polygon: From column (4) table (2) as in fig (f).

By taking pol and suitable scale draw all F_c with same magnitude and direction.



Fig(e) Couple polygon



Fig(f) Force polygon.

Where vector $eo = F_{cL} = W_L r_L$

∴ From C-force polygon we can determine the magnitude and direction of mass M_L as follows:

- mag- d $W_L = \frac{eo \text{ (N.m) from scale}}{r_L \text{ (m)}} \quad \text{--- (2)}$

- direction $\theta_L =$ measure between F_{cL} and Line F_c in the same direction.