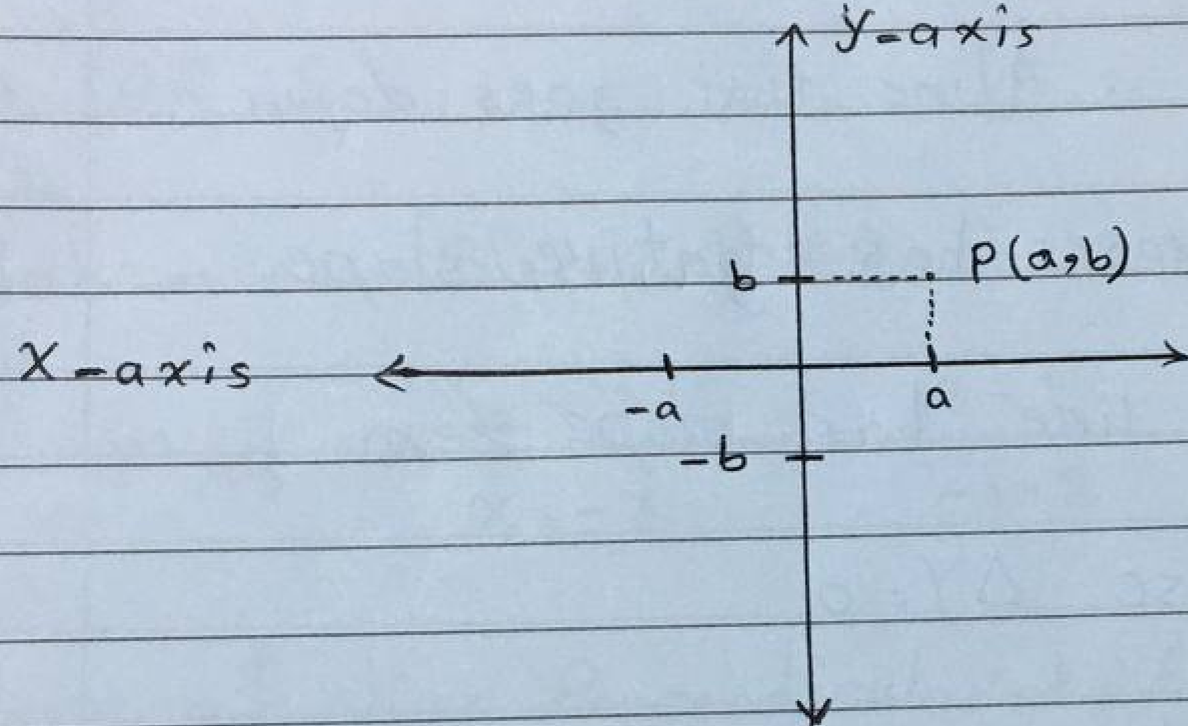
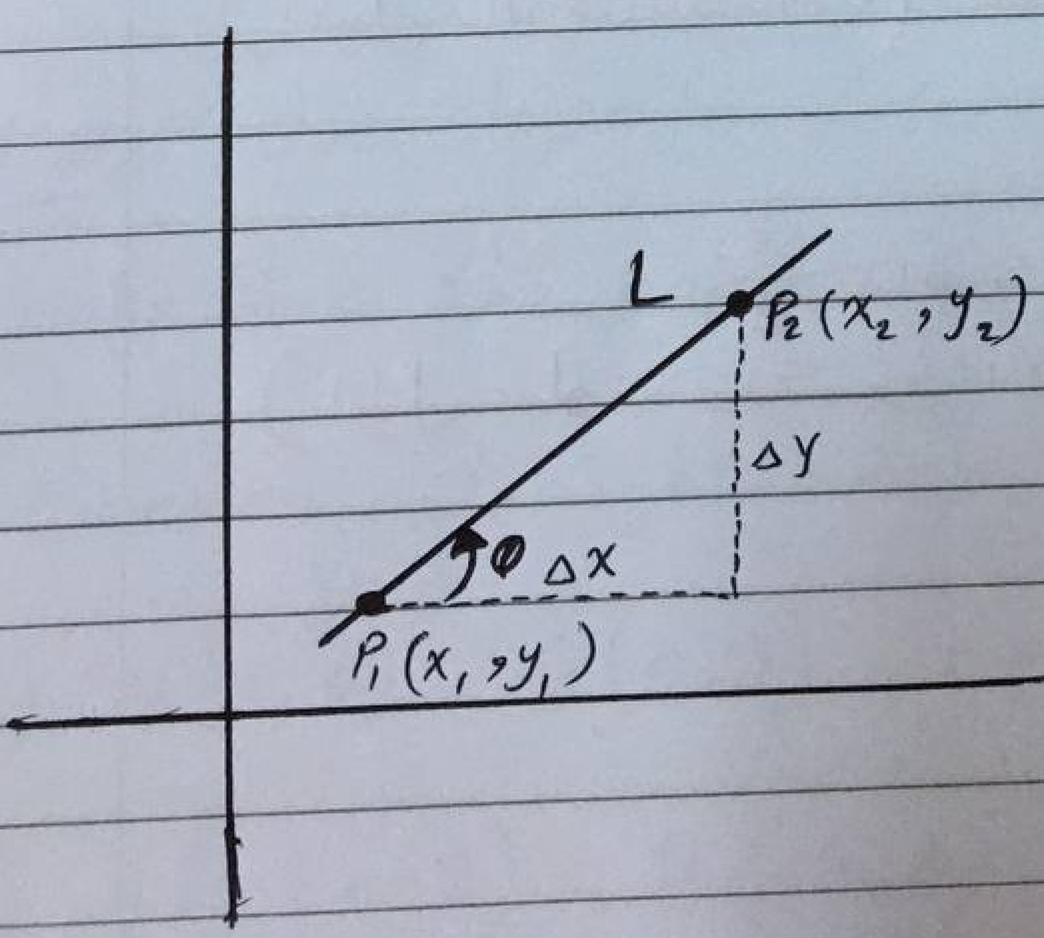
Ch 1

Coordinates for the plane.

Cartesian Coordinates :-



The slope of line :-



$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{where } \Delta x \neq 0$$

- The line goes uphill as x increases has apostive slope. Aline that goes down hill as x increases has negative slope.
- A horizontal line has slope zero

because $\Delta y = 0$

- Parallel line have same slop.

العمودي

* poreld (الميل مالتهن متساوي)

* ~~المتعامدين~~　　$m = \frac{-2}{3}$, $m = \frac{3}{2}$

(الميل مقلوب وعكس الاشارة)

Ex1// Find The slope of the line determined by two point A(2,1) and B(-1,3) and Find the common slope of the line Perendicular متعامد to AB ?

Sol:- Slope of AB

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{-1-2} = \frac{-2}{3}$$

Slope of line Perpendicular to AB is

$$= -\frac{1}{m_{AB}} = \frac{3}{2}$$

H.w// Use slope to determine in each case whe thor the poient are colliner (lie or Common straight line) نشوفهن اذا جانن على خط واحد يعني انطلع الميل مالتهن

a) A(1,0), B(0,1), C(2,1)

b) A(-3,-2), B(-2,0), C(-1,2), D(1,6)

Sol :-

a) $m_{AB} = \frac{1-0}{0-1} = \boxed{-1}$

$m_{BC} = \frac{1-1}{2-0} = \boxed{0} \neq m_{AB}$

The points A, B, C aren't lie on a common straight line.

b) $m_{AB} = \frac{0-(-2)}{-2-(-3)} = \boxed{2}$

$m_{BC} = \frac{2-0}{-1-(-2)} = \boxed{2}$

$m_{CD} = \frac{6-2}{1-(-1)} = \boxed{2}$

Since $m_{AB} = m_{BC} = m_{CD}$

Hence the points A, B, C, D are collinear.

Equation of line :-

Non vertical lines :-

The point-slope equation of the line throught the point (x_1, y_1) with slope m is :-

$$\boxed{y - y_1 = m(x - x_1)}$$

Ex // write an equation for the line that passes through point :-

a) $P(-1, 3)$ with slope $m = -2$

b) $P_1(-2, 0)$ and $P_2(2, -2)$

Sol :- a) $y - y_1 = m(x - x_1) \Rightarrow y - 3 = -2(x - (-1))$

$$1y + 2x = 1 \qquad \boxed{Ay + Bx = C}$$

b) $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2 - 0}{2 + 2} = \dfrac{-2}{4} = \dfrac{-1}{2}$

$y - y_1 = m(x - x_1) \Rightarrow \left[y - 0 = \frac{-1}{2}(x + 2)\right] * 2$

$$2y + x = -2$$

H.w// Find the slop of the line

$$3x+4y=12$$

Sol:- $[3x+4y=12]\div 4$

$$\frac{3}{4}x+y=3$$

$$y=\frac{-3}{4}x+3$$

$\therefore$ the slope is $m=\frac{-3}{4}$

Matrices

A matrix is a rectangular array of elements from a field The order or size of matrix is specified by the number of rows and the number of column, A an "m" by "n" matrix has m rows and n Columns and The element in the (ith) row and (j th) Column is often denoted by (a_{ij}).

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

A vector is matrix with single row or Column of n element.

$$A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \qquad A = [a_1, a_2, \ldots\ldots, a_n]$$

The Identity matrix

$$I = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}$$ is square matrix

The diagonal matrix

$$D = \begin{bmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_n \end{bmatrix}$$

المصفوفات

1- Equality, two $m \times n$ matrices and A and B are said tobe equal if

$$\boxed{a_{ij} = b_{ij}}$$

Ex 1 // Find the value of x, y for the following matrix equation :-

$$\begin{bmatrix} x-2y & 0 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -2 & x+y \end{bmatrix}$$

Sol :- $x - 2y = 3$ ...... ①

$2 * [x + y = 6]$ ...... ②

$x - \cancel{2y} = 3$

$2x + \cancel{2y} = 12$

بالجمع

$3x = 15$

$$\boxed{x = 5}$$

$5 + y = 6 \Rightarrow \boxed{y = 1}$

2- Addition الجمع

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

$$A \mp B = \begin{bmatrix} a_{11} \mp b_{11} & a_{12} \mp b_{12} & \cdots & a_{1n} \mp b_{1n} \\ a_{21} \mp b_{21} & a_{22} \mp b_{22} & \cdots & a_{2n} \mp b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} \mp b_{m1} & a_{m2} \mp b_{m2} & \cdots & a_{mn} \mp b_{mn} \end{bmatrix}$$

Thus

1- $A + B = B + A$

2- $A + (B + C) = (A + B) + C$

3- $A - (B - C) = A - B + C$

Example :-

Find A+B and A-B

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 & 2 \\ 2 & 3 & 1 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 3 & -1 & 5 \\ 3 & 3 & -3 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 1 & 3 & 1 \\ -1 & -3 & -1 \end{bmatrix}$$

3- Multiplication by ascalar.

Ex: Assume $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 5 & -1 \end{bmatrix}$

Find 3A

$$3A = \begin{bmatrix} 9 & 6 & 3 \\ 0 & 15 & -3 \end{bmatrix}$$

4- Multiplication :-

its necessary that the number of column A be equal to the number of rows of B, The dimension of such malrices are said be conformable, if A is of dimension $m \times p$ and B is $p \times n$ than the ijth element of product $C = AB$ is computed as

$$C_{ij} = \sum_{K=1}^{p} a_{iK} b_{Kj}$$

This is the sum of the products of correspondrray elements in the ith row of A and jth Column of B, The dimension of AB are of Course $m \times n$.

Ex/- Assume

$$A = \begin{array}{c} \begin{array}{ccc} ① & ② & ③ \end{array} \\ \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix} \end{array}$$

and

$$B = \begin{array}{c} ① \\ ② \\ ③ \end{array} \begin{bmatrix} 6 & 5 & 4 \\ -1 & 1 & -1 \\ 0 & 2 & 0 \end{bmatrix}.$$

Sol:- $A \times B$

$$= \begin{bmatrix} 1*6+2(-1)+3*0 & 1*5+2*1+3*2 & 1*4+2(-1)+3*0 \\ -1*6+0(-1)+1*0 & -1*5+0*1+1*2 & -1*4+0(-1)+1*0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 13 & 2 \\ -6 & -3 & -4 \end{bmatrix}$$

a) $A(B+C) = AB + AC$

b) $A(BC) = (AB)C$

c) $AB \neq BA$

d) $AI = IA = A$

Ex // Assume $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$

verify that $AB \neq BA$

$$AB = \begin{bmatrix} 1\times3+2\times2 & 1\times-1+1\times1 \\ 0\times3+3\times2 & 0\times-1+3\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 3+4 & -1+1 \\ 0+6 & 0+3 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 6 & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3\times1+(-1)(0) & 3\times2+(-1)(3) \\ 2\times1+1\times0 & 2\times2+1\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 \\ 2 & 7 \end{bmatrix}$$

$\therefore AB \neq BA$