

## \* Examples on belts

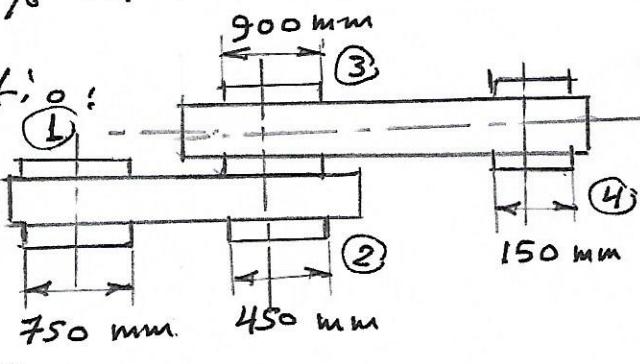
as per I.I.T. examination.

### \* Example (1) :

An engine running at (150 r.p.m), drives a line shaft by means of a belt. The engine pulley is (750 mm) diameter, and the pulley on the line shaft being (450 mm). A (900 mm) diameter pulley on the line shaft drives a (150 mm) diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft, when there is a slip of 2% at each drive.

### \* Solution : Speed ratio:

$$R_V = \frac{N_4}{N_1} = \frac{d_1 * d_3}{d_2 * d_4} \left(1 - \frac{s_1}{100}\right) \left(1 - \frac{s_2}{100}\right)$$



$$\therefore R_V = \frac{N_4}{150} = \frac{750 * 900}{450 * 150} \left(1 - \frac{2}{100}\right) \left(1 - \frac{2}{100}\right) = 9.6$$

$$\therefore N_4 = 150 * 9.6 = \underline{\underline{1440}} \text{ r.p.m}$$

### \* Example (2)

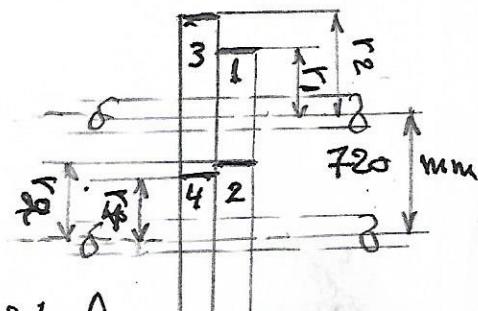
A shaft which rotates at a constant speed of (160) r.p.m. is connected by belt to a parallel shaft (720) mm apart. which has to run at 60, 80 r.p.m. The smallest pulley on the driving shaft is (40) mm in radius. Determine the radius of the stepped pulley by using open belt drive, and neglect thickness and slip.

### Solution:

For open flat belt drive

$$R_{V_1} = \frac{N_2}{N_1} = \frac{r_1}{r_2}$$

$$\text{But } N_1 - N_2 = 160 \text{ r.p.m.} \rightarrow \text{or vice versa}$$



1) for first step

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$$\therefore R_V_1 = \frac{160}{60} = \frac{40}{r_2} \Rightarrow r_2 = \frac{40 * 60}{160} = 106.7 \text{ mm}$$

2) for second step

$$R_V_2 = \frac{N_4}{N_3} = \frac{r_3}{r_4} \Rightarrow r_4 = r_3 * \frac{N_3}{N_4}$$

$$= r_3 * \frac{160}{80} = 2r_3$$

in order to determine  $r_3$  we get to the C-Law.  
by using length of belt for pulley 1, 2

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_2 - r_1)^2}{X}$$

$$= 3,14(40 + 106) + 2 * 720 + \left(\frac{106,7 - 40}{720}\right)^2$$
$$L = 1907 \text{ mm}$$

now the length for pulley 3, 4

$$L = 1907 = \pi(r_3 + r_4) + \frac{(r_4 - r_3)^2}{X} + 2x$$

$$1907 = 3,14(r_3 + 2r_3) + \frac{(2r_3 - r_3)^2}{720} + 2 * 720$$

$$1907 = 9,426r_3 + 0,0014r_3^2 + 1440$$

$$\text{or } 0,0014r_3^2 + 9,426r_3 - 467 = 0$$

$$\therefore r_3 = \frac{-9,426 \pm \sqrt{(9,426)^2 + 4 * 0,0014 * 467}}{2 * 0,0014}$$

$$= \frac{-9,426 \pm 9,564}{0,0028} = 49,3 \text{ mm}$$

$$\therefore r_4 = 2r_3 = 2 * 49,3 = 98,6 \text{ mm}$$

$$0 = \frac{\pm \sqrt{0^2 + 4f^2}}{2P}$$

$$x = \frac{-\frac{\sqrt{0^2 + 4f^2}}{2P} \pm \sqrt{0^2 + 4f^2}}{2P}$$

Final,

\* Example (3) :

Find the power transmitted by a belt running over a pulley of (600)mm diameter at (200)r.p.m. The coefficient of friction between the belt and pulley is ( $\mu = 0.25$ ). angle of lap ( $160^\circ$ ) and maximum tension in the belt is (2500 N)

Solution : given data are

$$d = 600 \text{ mm} = 0.6 \text{ m}, N = 200 \text{ r.p.m}, \mu = 0.25,$$

$$\theta = 160^\circ = 160 * \frac{\pi}{180} = 2.793 \text{ rad}, T_1 = 2500 \text{ N}$$

$$\therefore P = (T_1 - T_2) * V, \text{ W}$$

$$V = \frac{\pi d_1 n_1}{60} = \frac{3.14 * 0.6 * 200}{60} = 6.284 \text{ m/s}$$

$$2.3 \log \left( \frac{T_1}{T_2} \right) = M\theta = 0.25 * 2.793 = 0.6982$$

$$\therefore \log \left( \frac{T_1}{T_2} \right) = \frac{0.6982}{2.3} = 0.3036$$

$$\therefore \frac{T_1}{T_2} = 2.01 \rightarrow \text{anti log}(0.3036)$$

$$\therefore T_2 = \frac{T_1}{2.01} = \frac{2500}{2.01} = 144 \text{ N}$$

$$\therefore P = (2500 - 144) * 6.284 = 7.89 \text{ W}$$

\* Example (4) : and the other 200mm diameter

Two pulleys, one (450) mm diameter are on parallel shafts (1.95)m apart and are connected by a crossed belt, find the length of the belt required, and the

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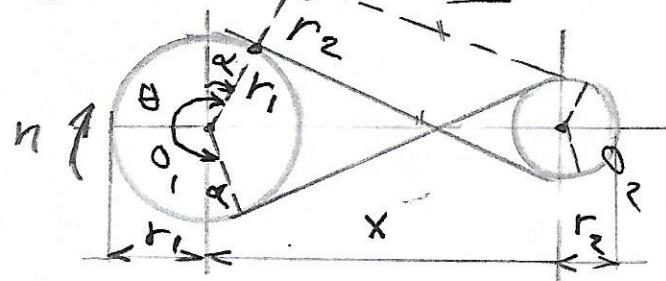
angle of contact between the belt and each pulley, and what power can be transmitted by the belt, when the larger pulley rotates at (200) r.p.m., If the maximum permissible tension in the belt is (1kN), and the coefficient of friction between the belt and pulley is ( $\mu = 0,25$ ),  $d_2 = 200\text{ mm}$

Solution: Given data are

$d_1 = 450\text{ mm} = 0,45\text{ m}$ ,  $r_1 = 0,225\text{ m}$ ,  $d_2 = 200\text{ mm}$   
 $r_2 = 0,1\text{ m}$ ,  $X = 1.95\text{ m}$ ,  $N_1 = 200\text{ r.p.m}$ ,  $T_x = 1000\text{ N}$   
 $\mu = 0,25$ , cross belt drive.

1)  $P = (T_1 - T_2) * V \rightarrow$  transmitted power.

$$V = \frac{\pi d_1 N_1}{60} = \frac{3,14 * 0,45 * 200}{60} = 4,714 \text{ m/s}$$



2) length of cross belt drive

$$\begin{aligned} l &= \pi(r_1 + r_2) + 2X + \frac{(r_1 + r_2)^2}{X} \\ &= 3,14(0,225 + 0,1) + 2 * 1.95 + \frac{(0,225 + 0,1)^2}{1.95} = 4.975 \text{ m} \end{aligned}$$

3) Angle of contact  $\rightarrow \theta$  between belt and pulley

$$\theta = 180 + 2\alpha$$

$$\sin \alpha = \frac{r_1 + r_2}{X} \rightarrow \text{from right } \triangle O_1 O_2 M$$

$$= \frac{0,225 + 0,1}{1.95} = 0,1667$$

$$\therefore \alpha = 9,6^\circ$$

$$\therefore \theta = 180^\circ + 2\alpha = 180^\circ + 2 \times 9.6^\circ = 199.2^\circ$$

$$\therefore \theta = 199.2^\circ * \frac{\pi}{180} = 3.477 \text{ rad. راديان}$$

\* for determining  $\frac{T_1}{T_2}$  in slack side.

$$2.3 \log \left( \frac{T_1}{T_2} \right) = \mu \theta = 0.25 * 3.477 = 0.8692$$

$$\therefore \log \frac{T_1}{T_2} = \frac{0.8692}{2.3} = 0.378$$

$$\therefore \frac{T_1}{T_2} = 2.387 \quad (\text{anti log } 0.378)$$

$$\therefore \frac{T_1}{T_2} = 2.387 = \frac{1000}{2.387} = 419 \text{ N}$$

$$\therefore P = (T_1 - T_2) * V = (1000 - 419) * 4.714 = \underline{\underline{2740 \text{ W}}}$$

### Example (5) :

A shaft rotating at (200)r.p.m drives another shaft at (300) r.p.m and transmits (6)kw power through a belt. The belt is (100) mm wide and (10) mm thick. The distance between the shafts is (4 m). The smaller pulley is (0.5) m in diameter. Calculate the stress in the belt, if it is an open belt drive, and coefficient of friction between belt and pulley (0.3).

Solution: given data  
 $N_1 = 200 \text{ r.p.m.}$ ,  $N_2 = 300 \text{ r.p.m.}$ ,  $P = 6 \text{ kw}$  for ②  
 $b = 100 \text{ mm}$ ,  $t = 10 \text{ mm}$ ,  $X = 4 \text{ m}$ ,  $d_e = 0.5 \text{ m}$ ,  $\mu = 0.3$ .

$$\sigma = \frac{T_1}{A} = \frac{T_1}{b * t}$$

$$P = (T_1 - T_2) * V$$

$P_1 = P_2 = P$  on both pulleys

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$$V_2 = \frac{\pi d_2 N_2}{60} = \frac{3,14 \times 0,5 \times 300}{60} = 7,855 \text{ m/s}$$

where:

$$R_V = \frac{N_2}{N_1} = \frac{d_1}{d_2} \Rightarrow d_1 = \frac{N_2 d_2}{N_1} = \frac{300 \times 0,5}{200} = 0,75 \text{ m}$$

angle of contact on small pulley  $\rightarrow$  for open b-drive.

$$\theta = 180 - 2\alpha$$

$$\tan \alpha = \frac{r_1 - r_2}{d} = \frac{0,37 - 0,5}{0,75} = 0,03125^\circ$$

$$\therefore \alpha = 1.8^\circ \quad \therefore \theta = 180 - 2 \times 1.8^\circ = 176.4^\circ$$

$$\therefore \theta = 176.4^\circ \times \frac{\pi}{180} = 3.08 \text{ rad}$$

$$2,3 \log\left(\frac{T_1}{T_2}\right) = \mu \theta = 2,3 \times 3,08 = 0,924$$

$$\log \frac{T_1}{T_2} = \frac{0,924}{2,3} = 0,4017 \quad \text{By (Anti log)}$$

$$\therefore \frac{T_1}{T_2} = 2,52 \Rightarrow T_1 = 2,52 T_2 \quad \text{--- (1)}$$

From Power  $P = (T_1 - T_2) \times V$

$$6 \times 10^3 = (T_1 - T_2) \times 7.825$$

$$\therefore T_1 - T_2 = \frac{6 \times 10^3}{7.825} = 764 \text{ N} \quad \text{--- (2)}$$

by sub (1) in (2) we get

$$2,52 T_2 - T_2 = 764$$

$$\therefore T_2 = 503 \text{ N}, T_1 = 2,52 \times 503 = 1267 \text{ N}$$

For the stress:

$$\sigma = \frac{T_1}{A} = \frac{T_1}{b \times t} = \frac{1267}{10 \times 100} = 1267 \text{ N/mm}^2$$

$$\theta = 180 + 2\alpha$$

$$A = 180 - 2\alpha$$



$$= 1267 \text{ N/mm}^2$$

\* Example (6) :

A leather belt is required to transmit (7.5) kW from a pulley (1.2) m in diameter running at (250) r.p.m. The angle embraced is ( $\theta = 165^\circ$ ) and the coefficient of friction between the belt and the pulley is ( $\mu = 0.3$ ). If the safe working stress for the leather belt is ( $\sigma = 1.5 \text{ MPa}$ ), density of leather ( $\rho = 1 \text{ Mg/m}^3$ ), and thickness of belt ( $t = 10 \text{ mm}$ ), determine the width of the belt taking centrifugal tension into account.

Answer =  $b = 65.8 \text{ mm}$ .

Home work  $\rightarrow b = ?$

$$\sigma = \frac{T}{A} = \frac{T}{bxt} \quad \text{Stress.} \rightarrow \sigma \propto 1/b$$

$$A = b \times t \quad \text{Area} \rightarrow A \propto b$$

$$m = \text{Volume} \times \rho = A \times l \times \rho \quad \text{Volume} \rightarrow m \propto b$$

$$T_c = mr\omega^2 = m \cdot V^2$$

$$V = \frac{\pi d n}{60} = \frac{3.14 \times 1.2 \times 250}{60} = 15.71 \text{ m/s} \quad \text{Circular motion}$$

$$P = (T_1 - T_2) \times V \Rightarrow 7500 = (T_1 - T_2) \times 15.71$$

$$\therefore \frac{T_1 - T_2}{15.71} = \frac{7500}{15.71} = 477.4 \text{ N} \quad \text{--- (1)}$$

$$2,3 \log \left( \frac{T_1}{T_2} \right) = \mu \cdot \theta = 0.3 \times \frac{11 \times 165}{180} = 0.864$$

$$\therefore \log \left( \frac{T_1}{T_2} \right) = \frac{0.864}{2,3} = 0.3756 \Rightarrow \frac{T_1}{T_2} = 2375 \quad \text{Antilog.} \quad \text{--- (2)}$$

$$\text{from (1) and (2)} \Rightarrow T_1 = 824.6 \text{ N}, \quad T_2 = 347.2 \text{ N}$$

$$m = V \times \rho = A \times L \times \rho = b \times t \times L \times \rho = 10 b \times 15.71 \times 1000 = 15000 b$$

$$T_c = m \times V^2 = 10 b \times (15.71)^2 \times 1000 = 2468 b \text{ N} = 10 b \lg g$$