Transfer Function: The transfer function of a linear, time-invariant Differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero. Consider the linear time invariant system defined by the following differential equation:

$$a_0 \overset{(n)}{y} + a_1 \overset{(n-1)}{y} + \cdots + a_{n-1}\dot{y} + a_n y = b_0 \overset{(m)}{x} + b_1 \overset{(m-1)}{x} + \cdots + b_{m-1}\dot{x} + b_m x \qquad (n \geq m)$$

where y is the output of the system and x is the input. The transfer function of this system is the ratio of the Laplace transformed output to the Laplace transformed input when all initial conditions are zero, or

$$\text{Transfer function} = G(s) = \left. \frac{\mathcal{L}[\text{output}]}{\mathcal{L}[\text{input}]} \right|_{\text{zero initial conditions}} = \frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$$

Block Diagram: A block diagram is a simple visual tool that uses rectangular blocks and connecting arrows or lines to represent the components, functions, or operations of a system and the relationships or data flow between them. They provide a high-level, functional overview of complex systems, making them easier to understand, design, and analyze in fields like engineering, software development, and process management.

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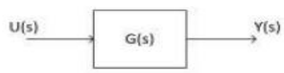
- Block diagrams consist of
  - ***Blocks*** – these represent subsystems – typically modeled by, and labeled with, a transfer function
  - ***Signals*** – inputs and outputs of blocks – signal direction indicated by arrows – could be voltage, velocity, force, etc.
  - ***Summing junctions*** – points were signals are algebraically summed – subtraction indicated by a negative sign near where the signal joins the summing junction
- The basic input/output relationship for a single block is:



$$Y(s) = U(s) \cdot G(s)$$

- Block diagram blocks can be connected in three basic forms:
  - ***Cascade***
  - ***Parallel***
  - ***Feedback***

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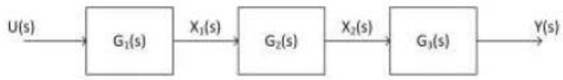
- Blocks connected in ***cascade***:



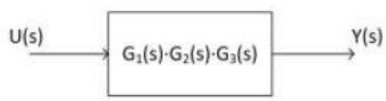
$X_1(s) = U(s) \cdot G_1(s), \quad X_2(s) = X_1(s) \cdot G_2(s)$

$Y(s) = X_2(s) \cdot G_3(s) = X_1(s) \cdot G_2(s) \cdot G_3(s)$

$Y(s) = U(s) \cdot G_1(s) \cdot G_2(s) \cdot G_3(s) = U(s) \cdot G_{eq}(s)$

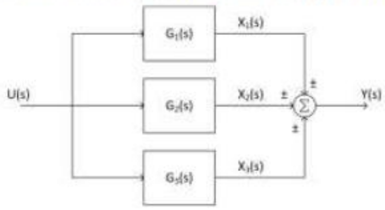
$G_{eq}(s) = G_1(s) \cdot G_2(s) \cdot G_3(s)$

- The equivalent transfer function of cascaded blocks is the ***product*** of the individual transfer functions



- **Parallel Form**

- Blocks connected in parallel:



$X_1(s) = U(s) \cdot G_1(s)$

$X_2(s) = U(s) \cdot G_2(s)$

$X_3(s) = U(s) \cdot G_3(s)$

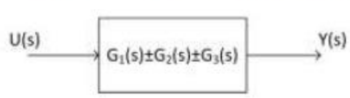
$Y(s) = X_1(s) \pm X_2(s) \pm X_3(s)$

$Y(s) = U(s) \cdot G_1(s) \pm U(s) \cdot G_2(s) \pm U(s) \cdot G_3(s)$

$Y(s) = U(s)[G_1(s) \pm G_2(s) \pm G_3(s)] = U(s) \cdot G_{eq}(s)$

$G_{eq}(s) = G_1(s) \pm G_2(s) \pm G_3(s)$

- The equivalent transfer function is the ***sum*** of the individual transfer functions:



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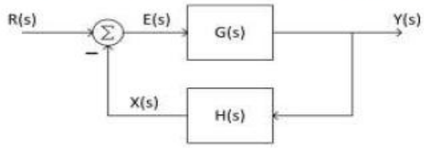
- **Feedback Form**

- Of obvious interest to us, is the ***feedback form***:

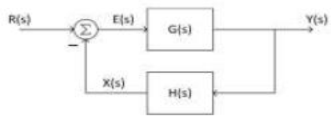


$$Y(s) = E(s)G(s)$$
$$Y(s) = [R(s) - X(s)]G(s)$$
$$Y(s) = [R(s) - Y(s)H(s)]G(s)$$

$$Y(s)[1 + G(s)H(s)] = R(s)G(s)$$
$$Y(s) = R(s) \cdot \frac{G(s)}{1 + G(s)H(s)}$$

- The ***closed-loop transfer function***, $T(s)$, is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$



$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

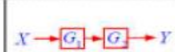
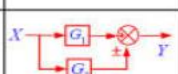
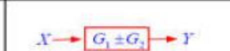
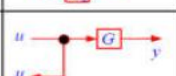
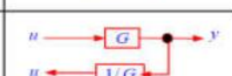
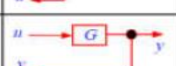
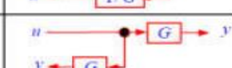
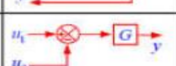
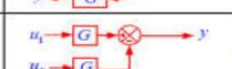
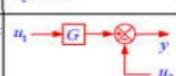
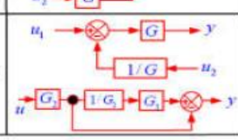
- Note that this is ***negative feedback***, for ***positive feedback***:

$$T(s) = \frac{G(s)}{1 - G(s)H(s)}$$

- The $G(s)H(s)$ factor in the denominator is the ***loop gain*** or ***open-loop transfer function***
- The gain from input to output with the feedback path broken is the ***forward path gain*** – here, $G(s)$
- In general:

$$T(s) = \frac{\text{forward path gain}}{1 - \text{loop gain}}$$

Basic rules with block diagram transformation

	Manipulation	Original Block Diagram	Equivalent Block Diagram	Equation
1	Combining Blocks in Cascade			$Y = (G_1G_2)X$
2	Combining Blocks in Parallel; or Eliminating a Forward Loop			$Y = (G_1 \pm G_2)X$
3	Moving a pickoff point behind a block			$y = Gu$ $u = \frac{1}{G}y$
4	Moving a pickoff point ahead of a block			$y = Gu$
5	Moving a summing point behind a block			$e_2 = G(u_1 - u_2)$
6	Moving a summing point ahead of a block			$y = Gu_1 - u_2$ $y = (G_1 - G_2)u$

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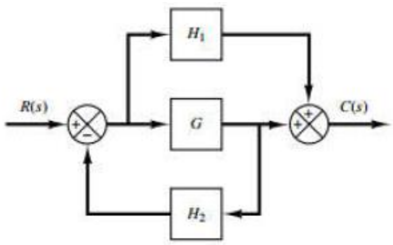
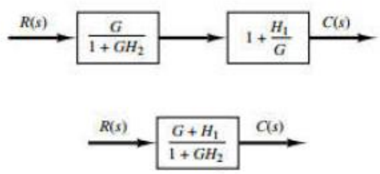
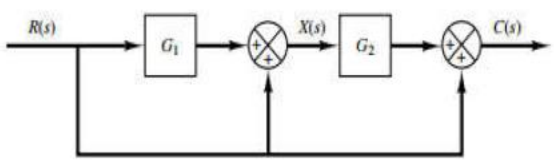
Example1: Simplify the block diagram shown in the figure below, Obtain the transfer function relating C(s) and R(s)?



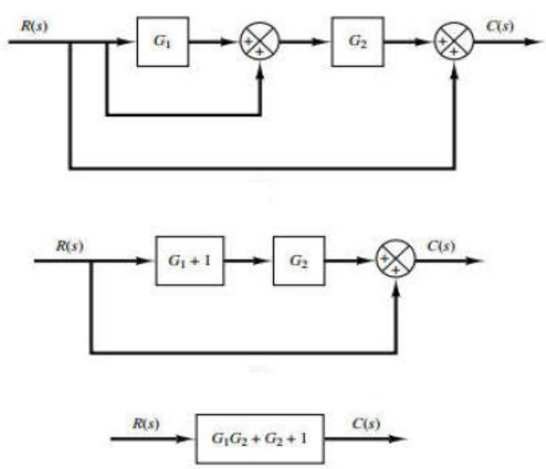
Solution:



Example2: Simplify the block diagram shown in the figure below, Obtain the transfer function relating C(s) and R(s)?



Solution:



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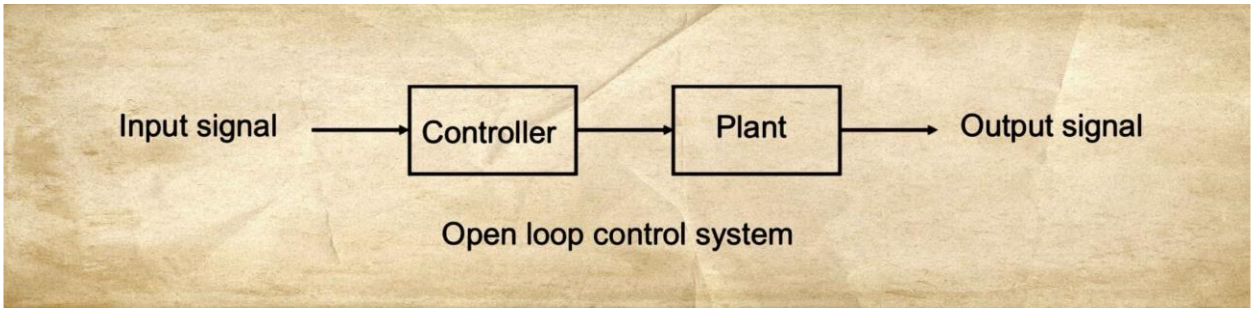
Open loop control system: A control system in which the o/p signal has no effect upon the control action. Ex. heater, light, washing machine.



Close loop control system: A control system in which the o/p signal has a direct effect upon the control action. Ex. A/C systems, Car cruise control.