Al.Mustaqbal University

College of engineering and technology

chemical and petroleum industrial Department

class –three   Term-1

Heat transfer – Code No. UOMU0102051

week-2 Steady –state heat conduction in one-diaminsion

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Steady State Heat Conduction in One Dimension

In general the heat conduction equation for one dimension is in three form:

1. In Cartesian coordinates

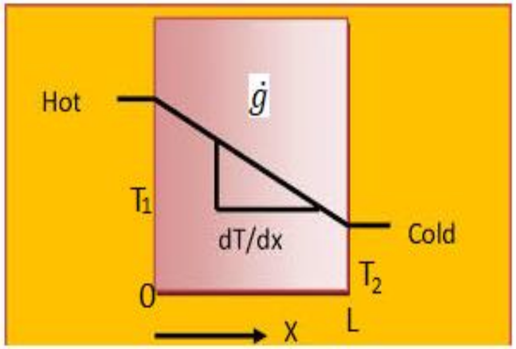
$\frac{\partial}{dx}\left(k\frac{\partial T}{\partial k}\right)+\dot{g}=0$ In this relation k is function of temperature k=k(T)

$\frac{\partial^2 T}{\partial x^2}+\frac{\dot{g}}{k}=0$ In this relation k is constant with change of temperature.

$\frac{d^2 T}{dx^2}=0$ It is with no heat generation.

Steady State Heat Conduction in One Dimension

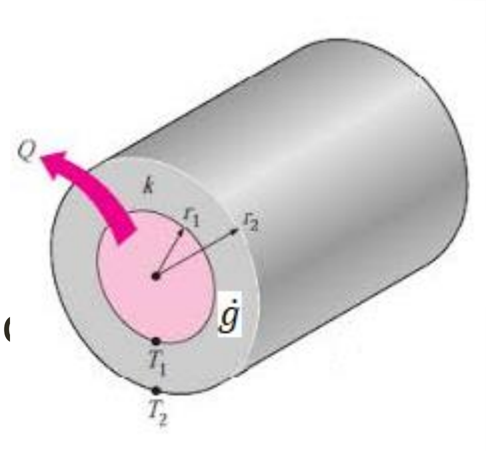
- This simple equation can be used to find the temperature distribution in a wall steadily and with no heat generation if we know the boundary condition.
- The wall in figure is assumed
- that $\dot{g} = 0$



- And $T=T_1$ at $x=0$, $T=T_2$ at $x=L$ as shown
- By integrating $\frac{d^2T}{dx^2} = 0$ two times

Steady State Heat Conduction in One Dimension

- 2- In cylindrical (Polar) coordinates
- $\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\dot{g}=0$
- In this relation k is function of
- temperature k=k(T)
- $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\dot{g}}{k}=0$ In this relation k is functi
  temperature k=k(T)



Steady State Heat Conduction in One Dimension

- $\frac{dT}{dx} = C_1$
- $T = C_1 x + C_2$ C_1, and C_2 are constants of integrations
- $at\ x = 0$ $T=T_1$ then $C_2=T_1$ and at $x = L, T_2 = C_1 L + T_1$
- This relation gives $C_1 = \frac{T_2 - T_1}{L}$, then $T = (T_2 - T_1)\frac{x}{L} + T_1$ or
- $T = T_1\left(1 - \frac{x}{L}\right) + \frac{x}{L}T_2$ It is the temperature distribution in plane wall of steady state with no heat generation.

Steady State Heat Conduction in One Dimension

- $\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$ It is with no heat generation
- To solve this relation with of simple form with B.Cs that at $r=r_1$ $T=T_1$ and at $r=r_2$ $T=T_2$
- And by two integrations we get
- $r\frac{dT}{dr} = C_1 \rightarrow \frac{dT}{dr} = \frac{C_1}{r}$, and $T = C_1 lnr + C_2$
- By substituting the B.Cs we get:
- $T_1 = C_1 lnr_1 + C_2$, and $T_2 = C_1 lnr_2 + C_2$
- $T_2 - T_1 = C_1(lnr_2 - lnr_1) = C_1 ln\left(\frac{r_2}{r_1}\right) \rightarrow C_1 = \frac{T_2 - T_1}{ln\left(\frac{r_2}{r_1}\right)}$

Steady State Heat Conduction in One Dimension

And by substituting this in one of the upper relations we get:

$T_1 = C_1 lnr_1 + C_2 \rightarrow T_1 = \frac{(T_2 - T_1)}{ln\left(\frac{r_2}{r_1}\right)} lnr_1 + C_2$, this gives

$C_2 = T_1 - \frac{(T_2 - T_1)}{ln\left(\frac{r_2}{r_1}\right)} lnr_1$

Substituting the value of C_1, and C_2 in the following relation

$$T = C_1 lnr + C_2 \rightarrow T = \frac{T_2 - T_1}{ln\left(\frac{r_2}{r_1}\right)} lnr + T_1 - \frac{(T_2 - T_1)}{ln\left(\frac{r_2}{r_1}\right)} lnr_1$$

$$T - T_1 = \frac{T_2 - T_1}{ln\left(\frac{r_2}{r_1}\right)} (lnr - lnr_1)$$

$$\frac{T - T_1}{T_2 - T_1} = \frac{ln\left(\frac{r}{r_1}\right)}{ln\left(\frac{r_2}{r_1}\right)}$$, this is the T.D.E

Or $T = T_1 + (T_2 - T_1) \frac{ln\left(\frac{r}{r_1}\right)}{ln\left(\frac{r_2}{r_1}\right)}$

Steady State Heat Conduction in One Dimension

3- In spherical coordinates

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\dot{g}=0$ In this relation k is function of temperature k=k(T)

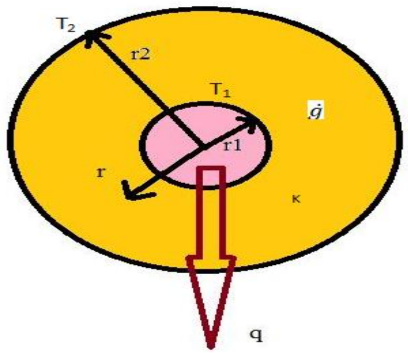
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{\dot{g}}{k}=0$$

In this relation k is function of

temperature k=k(T)

$\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=0$ It is with no heat generation

The solution of this equation gives



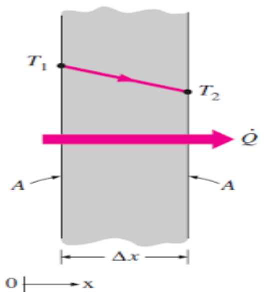
Steady State Heat Conduction in One Dimension

$$\frac{(T-T_1)}{(T_2-T_1)} = \frac{\left(\frac{1}{r_1}-\frac{1}{r}\right)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)} \quad or$$

$T = T_1 + (T_2 - T_1)\frac{\left(\frac{1}{r_1}-\frac{1}{r}\right)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$ It is a T.D.E for spherical shell with no heat generation.

Steady State Heat Conduction in One Dimension

- **<u>Heat conduction through a composite wall</u>**
- At the beginning we toke the relation for heat conduction in a wall of single layer as below
- $\dot{Q}_{cond} = kA\frac{(T_1 - T_2)}{\Delta x}$



Ex.1. find the rate of heat conduction from a wall of thickness 30cm and its area is 4m². The temperature at one surface of the wall is 0°C, and the other surface temperature is 100°C, and the thermal conductivity of the wall material is 17W/m.°C. Find also the heat flux.

Steady State Heat Conduction in One Dimension

- **Solution**: A Plane wall Δx=30cm=0.3m, A=4m^2, T_1=100°C, T_2=0°C, k=17W/m.°C
- **Requirements**: heat transfer by conduction through the wall $\dot{Q}_{cond}$, and the heat flux from the wall $\dot{q}$.
- **Analysis**: the heat transfer by conduction is calculated by Fourier Law
- $$\dot{Q}_{cond} = kA\frac{(T_1 - T_2)}{\Delta x} = 17 \times 4 \times \frac{(100-0)}{0.3} =$$
  22666.667W=22.667kW
- The heat flux is $\dot{q} = \frac{\dot{Q}}{A} = \frac{22666.667}{4} = 5666.667W/m^2$

Steady State Heat Conduction in One Dimension

In heat conduction we can assume

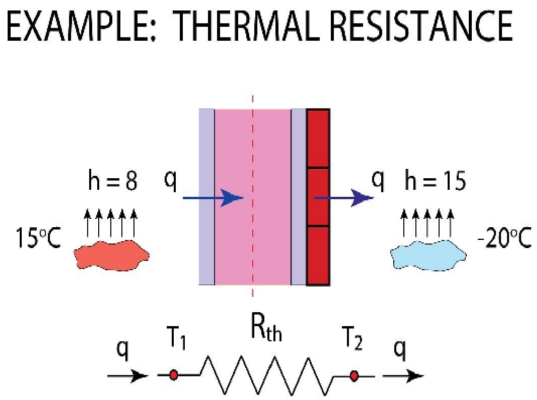
The wall as a wire that pass through it a current due to different in voltage

$$I = \frac{\Delta V}{R}$$ Where:

R is the resistance of the wire.

In this analogy, the current is

the heat transfer;



the voltage difference is the temperature difference. The relation of Fourier law becomes:

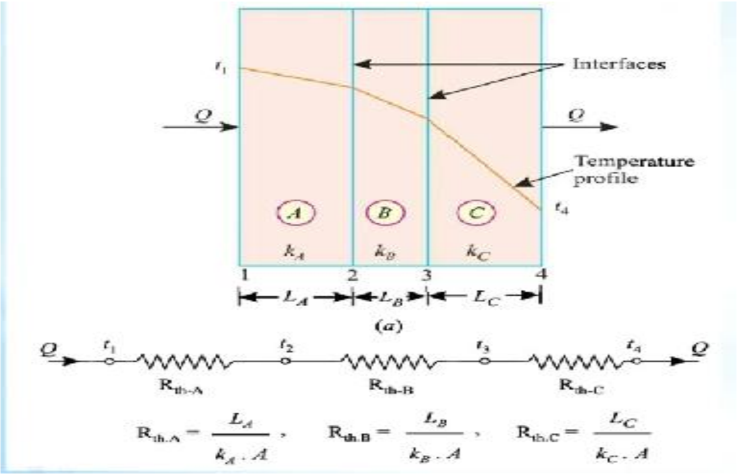
$$\dot{Q}_{cond} = \frac{(T_1 - T_2)}{\frac{\Delta x}{kA}} = \frac{(T_1 - T_2)}{R_{th}}$$

Steady State Heat Conduction in One Dimension

- In general the walls are made of many layers in series and in parallel like that of building walls that are known composite walls.
- To determine the heat conduction from wall of many layers such three layers we can drive the relation for this wall as below.
- The heat flow in each layer is equal.

Steady State Heat Conduction in One Dimension

- $\dot{Q} = \frac{1}{R_{th,A}}(T_1 - T_2) \rightarrow \dot{Q}R_{th,A} = (T_1 - T_2)$
- $\dot{Q} = \frac{1}{R_{th,B}}(T_2 - T_3) \rightarrow \dot{Q}R_{th,B} = (T_2 - T_3)$
- $\dot{Q} = \frac{1}{R_{th,C}}(T_3 - T_4) \rightarrow \dot{Q}R_{th,C} = (T_3 - T_4)$
- Adding these three equations together gives



Steady State Heat Conduction in One Dimension

- $\dot{Q}R_{th,A} + \dot{Q}R_{th,B} + \dot{Q}R_{th,C} = (T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4)$
- $\dot{Q}\left(R_{th,A} + R_{th,B} + R_{th,C}\right) = (T_1 - T_4)$
- $\dot{Q}_{cond} = \frac{T_1 - T_4}{R_{th,A} + R_{th,B} + R_{th,C}}$

Steady State Heat Conduction in One Dimension

Ex.2. An oven wall is constructed of three layers. The internal layer is of fire brick of thickness 15cm and thermal conductivity 0.55W/m.°C. The mid layer is of glass wool of thickness 5cm and thermal conductivity 0.03W/m.°C. The outer layer is of Iron of thickness 0.2cm and thermal conductivity 80W/m.°C. The temperature of internal surface of the fire brick is 1000°C, and the temperature of the external surface of the iron is 50°C. If the surface area of the oven is $2m^2$, find the rate of heat transfer and the interface temperature between fire brick and glass wool and between glass wool and iron.

Solution: An oven Wall , Three layers Δx_A=15cm=0.15m, k_A=0.55W/m°C, Δx_B=5cm=0.05cm, k_B=0.03/m.°C, Δx_C=0.2cm=0.002m, k_B=80W/m.°C, A=$2m^2$, T_1=1000°C, T_4=50°C.

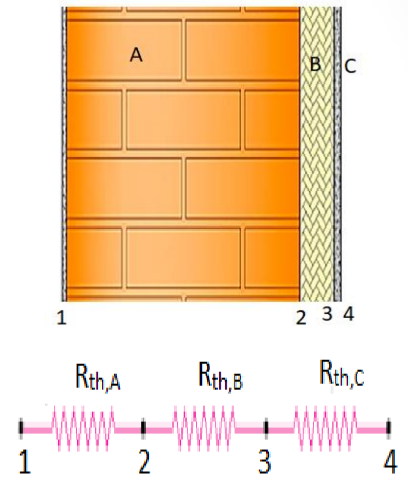
Requirements: heat transfer rate $\dot{Q}$, T_2, and T_3.

Steady State Heat Conduction in One Dimension

$$R_{th,A} = \frac{\Delta x_A}{k_A A} = \frac{0.15}{0.55\times2} = 0.136^{o}C/W,$$

$$R_{th,B} = \frac{\Delta x_B}{k_B A} = \frac{0.05}{0.03\times2} = 0.833^{o}C/W,$$

$$R_{th,C} = \frac{\Delta x_C}{k_C A} = \frac{0.002}{80\times2} = 0.0000125^{o}C/W,$$



$$\dot{Q} = \frac{T_1 - T_4}{R_{th,A}+R_{th,B}+R_{th,C}} = \frac{1000-50}{0.136+0.833+0.0000125} = 980.38W$$

No to find T_2, and T_3

$$\dot{Q} = \frac{1}{R_{th,A}}(T_1 - T_2) \rightarrow 980.38 = \frac{1}{0.136}(1000 - T_2)$$

Steady State Heat Conduction in One Dimension

$T_2 = 1000 - 980.38 \times 0.136$=866.668°C

$\dot{Q} = \frac{T_1 - T_3}{R_{th,A} + R_{th,B}} \rightarrow 980.38 = \frac{1000 - T_3}{0.136 + 0.833} \rightarrow T_3 = 1000 - 980.38 \times 0.969$

T_3=5.012°C

Or $\dot{Q} = \frac{1}{R_{th,B}}(T_2 - T_3) \rightarrow 980.38 = \frac{1}{0.833}(866.668 - T_3)$

$T_3 = 866.668 - 980.38 \times 0.833 = 50.012$°C

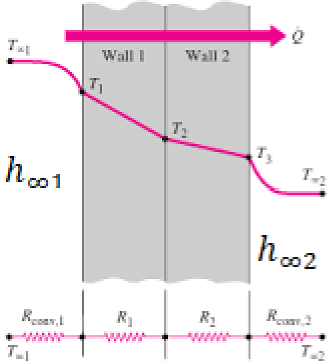
Or $\dot{Q} = \frac{1}{R_{th,C}}(T_3 - T_4) \rightarrow 980.38 = \frac{(T_3 - 50)}{0.0000125} \rightarrow T_3 = 50 + 980.38 \times 0.0000125 = 50.012$

Steady State Heat Conduction in One Dimension

- **Conduction-Convection Systems**

In general the outer surfaces of any wall from the two sides are exposed to convection. So these convection effects shell effect the heat transfer through the wall.

Consider a wall is made of two layers and exposed to convec
two outer surfaces.



The rate of heat transfer in convection $T_{\infty1}$ is

$$\dot{Q} = h_{\infty1}A(T_{\infty1} - T_1) = \frac{(T_{\infty1} - T_1)}{\frac{1}{h_{\infty1}A}} = \frac{(T_{\infty1} - T_1)}{R_{conv,1}}$$

$$\dot{Q} = h_{\infty2}A(T_3 - T_{\infty2}) = \frac{(T_3 - T_{\infty2})}{\frac{1}{h_{\infty2}A}} = \frac{(T_3 - T_{\infty2})}{R_{conv,2}}$$

$R_{conv,1} = \frac{1}{h_{\infty1}A}$, *and* $R_{conv,2} = \frac{1}{h_{\infty2}A}$ are convection thermal resistances.

Then

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{conv,1} + R_1 + R_2 + R_{conv,2}}$$

Steady State Heat Conduction in One Dimension

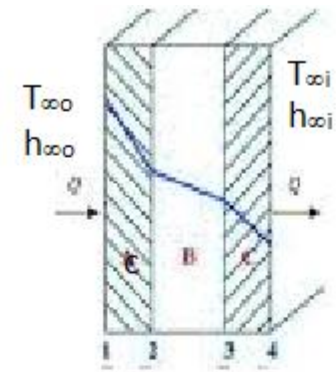
- **Ex.3. A room wall of dimension 3mx4m is constructed of brick with thickness 20cm, and thermal conductivity of 0.8W/m.°C. the wall is covered with concrete layer of thickness 2cm, with thermal conductivity of 2.0W/m.°C on both sides of the brick. The outside surface is exposed to air temperature of 50°C and convection heat transfer coefficient of 25W/m^2.°C. The inner surface of the wall is exposed to inner air of temperature 20°C with coefficient of convection heat transfer of 15W/m^2.°C. Find the rate of heat transfer to the room through the wall and find inner and outer surface temperature.**

Steady State Heat Conduction in One Dimension

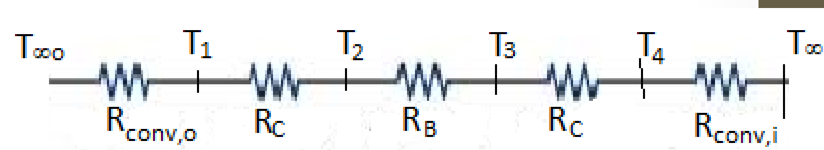
- Solution: a room wall of L=3m, W=4m, A=$L \times W = 3 \times 4 = 12m^2$
- Wall construction: Brick Δx_B=20cm=0.2m, k_B=0.8W/m.^{o}C,
- Concrete on inside and outside Δx_{c1}=Δx_{c2}=2cm=0.02m, k_{c1}=k_{c2}=2W/m.^{o}C
- Outside surface is exposed to air $T_{\infty o}$=50^{o}C, $h_{\infty o}$=25W/m^2.^{o}C
- Inside surface is exposed to air $T_{\infty i}$=20^{o}C, $h_{\infty i}$=15W/m^2.^{o}C
- Requirements: heat transfer rate $\dot{Q}$, inside surface temp. T_1, and outside surface temp. T_4.

Steady State Heat Conduction in One Dimension

- <u>Analysis:</u>
- $R_{conv,o} = \frac{1}{h_{\infty o}A} = \frac{1}{25\times12} = 0.003333$°C/W
- $R_{conv,i} = \frac{1}{h_{\infty i}A} = \frac{1}{15\times12} = 0.005555$°C/W
- $R_{c1} = R_{c2} = \frac{\Delta x_C}{k_C A} = \frac{0.02}{2\times12} = 0.0008333$°C/W
- $R_B = \frac{\Delta x_B}{k_B A} = \frac{0.20}{0.8\times12} = 0.020833$°C/W



- $\dot{Q} = \frac{T_{\infty o}-T_{\infty i}}{R_{conv,o}+R_{c1}+R_B+R_{c2}+R_{\infty i}} = \frac{50-20}{0.003333+0.0008333+0.020833+0.0008333+0.005555} = 1136.867W$



- To find T_1 the temperature of outer surface
- $\dot{Q} = \frac{T_{\infty o}-T_1}{R_{conv,o}} \rightarrow 1136.867 = \frac{50-T_1}{0.00333} \rightarrow T_1 = 50 - 3.79 = 46.21$°C
- To find T_4 the temperature of inner surface
- $\dot{Q} = \frac{T_4-T_{\infty i}}{R_{conv,i}} \rightarrow 1136.867 = \frac{T_4-20}{0.005555} \rightarrow T_4 = 20 + 26.315$°C

Steady State Heat Conduction in One Dimension

- **The Overall Heat-Transfer Coefficient**
- The overall heat transfer Coefficient is combined coefficient that total coefficients of the wall and it is denoted as U
- $\dot{Q} = AU(T_{\infty o} - T_{\infty i}) = \frac{(T_{\infty o} - T_{\infty i})}{R_{conv,o} + R_{w} + R_{conv,i}} = \frac{(T_{\infty o} - T_{\infty i})}{\frac{1}{h_o A} + \frac{\Delta x}{kA} + \frac{1}{h_i A}} = \frac{A(T_{\infty o} - T_{\infty i})}{\frac{1}{h_o} + \frac{\Delta x}{k} + \frac{1}{h_i}}$
- Then $U = \frac{1}{\frac{1}{h_o} + \frac{\Delta x}{k} + \frac{1}{h_i}} = \left[\frac{1}{h_o} + \frac{\Delta x}{k} + \frac{1}{h_i}\right]^{-1}$ W/m^2.^{o}C

Steady State Heat Conduction in One Dimension

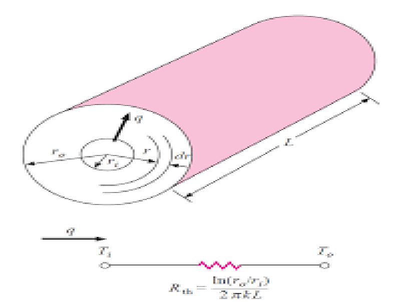
Heat conduction through cylindrical wall

To determine heat flow through a cylindrical wall like tube and pipe and cylinders that assume the heat flow radially we will begin with the Fourier's law of conduction.

$\dot{Q} = -kA\frac{dT}{dx}$ the flow is radially, the we dr instead of dx and A=2πrL

$\dot{Q} = -k(2\pi rL)\frac{dT}{dr}$ to integrate this we separate the variables

$\dot{Q}\int_{r_i}^{r_o}\frac{dr}{r} = -k(2\pi L)\int_{T_i}^{T_o} dT$



Steady State Heat Conduction in One Dimension

The integration gives

$$\dot{Q}\,ln\frac{r_o}{r_i} = -2\pi Lk(T_o - T_i)$$

$$\dot{Q} = \frac{(T_i - T_o)}{\frac{1}{2\pi Lk}ln\frac{r_o}{r_i}} = \frac{(T_i - T_o)}{R_{th}}$$ Where: $R_{th} = \frac{1}{2\pi Lk}ln\frac{r_o}{r_i}$

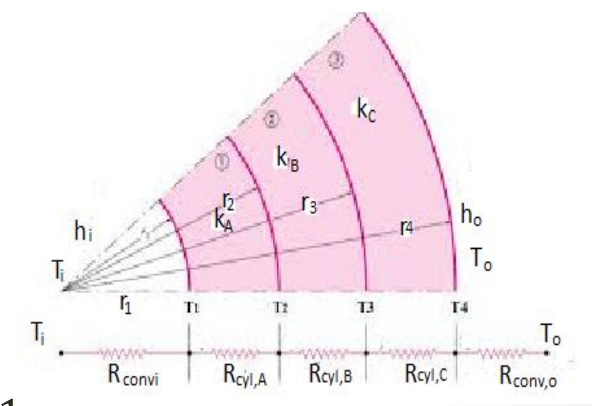
If the pipe is made of many layers as in the figure

$$\dot{Q} = \frac{T_{\infty i} - T_{\infty o}}{R_{conv,i} + R_{cyl.A} + R_{cyl.B} + R_{cyl.C} + R_{conv,o}}$$

$$R_{conv,i} = \frac{1}{h_i A_i} = \frac{1}{2\pi L r_i h_i},$$

$$R_{cyl.A} = \frac{1}{2\pi Lk_A}ln\frac{r_2}{r_1}, \quad R_{cyl.A} = \frac{1}{2\pi Lk_B}ln\frac{r_3}{r_2}$$

$$R_{cyl.A} = \frac{1}{2\pi Lk_C}ln\frac{r_4}{r_3}, \quad R_{conv,o} = \frac{1}{h_o A_o} = \frac{1}{2\pi L r_o h_o}$$



Steady State Heat Conduction in One Dimension

- $A_i = 2\pi r_i L = \pi D_i L,\ A_o = 2\pi r_o L = \pi D_o L$
- $\dot{Q} = AU(T_{\infty i} - T_{\infty o}) = \frac{T_{\infty i} - T_{\infty o}}{R_{conv,i} + R_{cyl.A} + R_{cyl.B} + R_{cyl.C} + R_{conv,o}}$
- As the outer surface area in not equal ton inner surface area
- $AU = A_i U_i = A_o U_o = \frac{1}{\frac{1}{2\pi L r_i h_i} + \frac{1}{2\pi L k_A} ln\frac{r_2}{r_1} + \frac{1}{2\pi L k_B} ln\frac{r_3}{r_2} + \frac{1}{2\pi L k_C} ln\frac{r_4}{r_3} + \frac{1}{2\pi L r_o h_o}}$
- $U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i}{k_A} ln\frac{r_2}{r_1} + \frac{r_i}{k_B} ln\frac{r_3}{r_2} + \frac{r_i}{k_C} ln\frac{r_4}{r_3} + \frac{r_i}{r_o h_o}}$ Overall heat transfer coefficient based on inner area
- $U_o = \frac{1}{\frac{r_o}{r_i h_i} + \frac{r_o}{k_A} ln\frac{r_2}{r_1} + \frac{r_o}{k_B} ln\frac{r_3}{r_2} + \frac{r_o}{k_C} ln\frac{r_4}{r_3} + \frac{1}{h_o}}$ Overall heat transfer coefficient based on outer area

Steady State Heat Conduction in One Dimension

- **Ex.4. A tube of inner diameter 10cm and outer diameter 20cm and 2m has thermal conductivity of 20W/m.°C. The temperature at outer surface is 30°C and the temperature at inner surface is 120°C. Find the heat transfer from this tube.**
- Solution: tube with D_i=10cm, r_i=5cm, D_o=20cm, r_o=10cm
  k=20W/m.°C, T_i=120°C, T_o=30°C
- Requirement: heat transfer from the tube wall
- $\dot{Q} = \frac{(T_i - T_o)}{\frac{1}{2\pi Lk} ln\frac{r_o}{r_i}} = \frac{(120-30)}{\frac{1}{2\pi\times2\times20} ln\frac{10}{5}} = 32633W$

Steady State Heat Conduction in One Dimension

Ex.5. Hot air at a temperature of 60°C is flowing through a steel pipe of inner diameter 110 mm and outer diameter 120mm, with k=40W/m.°C. The pipe is covered with two layers of different insulating materials of thickness 60mm and 40mm, and their corresponding thermal conductivity 0.24W/m.°C and 0.4W/m.°C. The inside and outside heat transfer coefficients are 60W/m².°C and 12W/m².°C respectively. The atmospheric temperature is 20°C. Find (i) overall heat transfer coefficient based of the inner surface area, (ii) the total thermal resistance, (iii) rate of heat losses from 60m length tube.

Solution: D_i=110mm=0.110m, r_i=0.05m, D_1=120mm=0.12m, r_1=.06m, k_A=40W/m.°C, Δr_1=60mm=0.06m, $r_2=r_1+\Delta r_1$=0.06+0.06=0.12m, k_B=0.24W/m.°C. Δr_2=40mm=0.04m, $r_3=r_2+\Delta r_2$=0.12+0.04=0.16m, k_c=0.4W/m.°C, h_i=60W/m².°C, h_o=12W/m².°C, $T_{\infty i}$=65°C, $T_{\infty o}$=20°C. L=60m, $T_{\infty i}$=60°C

Requirements: (i) overall heat transfer coefficient based on inner area, (ii) total thermal resistance, (iii) heat losses from the tube

Steady State Heat Conduction in One Dimension

Analysis: to find overall heat transfer coefficient based of inner area

$$U_i = \frac{1}{\frac{1}{h_i}+\frac{r_i}{k_A}ln\frac{r_2}{r_1}+\frac{r_i}{k_B}ln\frac{r_3}{r_2}+\frac{r_i}{k_C}ln\frac{r_4}{r_3}+\frac{r_i}{r_o h_o}}$$

$$U_i = \frac{1}{\frac{1}{60}+\frac{0.05}{80}ln\frac{0.06}{0.05}+\frac{0.05}{0.24}ln\frac{0.12}{0.06}+\frac{0.05}{0.4}ln\frac{0.16}{0.12}+\frac{0.05}{0.16\times 12}} =$$

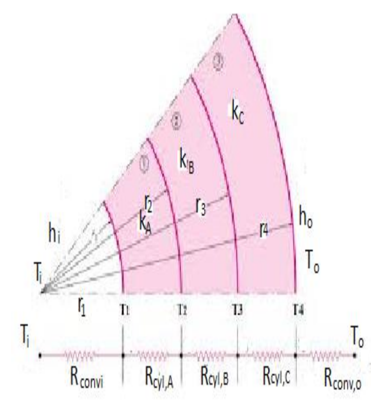
$4.48 W/m^2.°C$

(ii) $\frac{1}{R_{total}} = A_i U_i = (2\pi L r_i)U_i = (2\pi \times 60 \times 0.5) \times 4.48 =$

$84.456 W/°C$

$R_{tatal} = \frac{1}{A_i U_i} = \frac{1}{84.456} = 0.01184°C/W$

$\dot{Q} = A_i U_i (T_{\infty i} - T_{\infty o}) = 84.456(60-20) = 3378.24W$



Steady State Heat Conduction in One Dimension

Heat conduction through spherical wall

Also we shall begin from Fourier's law to drive the relation for heat conduction in a spherical shell

$\dot{Q} = -kA\frac{dT}{dr}$ A=$4\pi r^2$ for spherical shell

$\dot{Q} = -k(4\pi r^2)\frac{dT}{dr}$

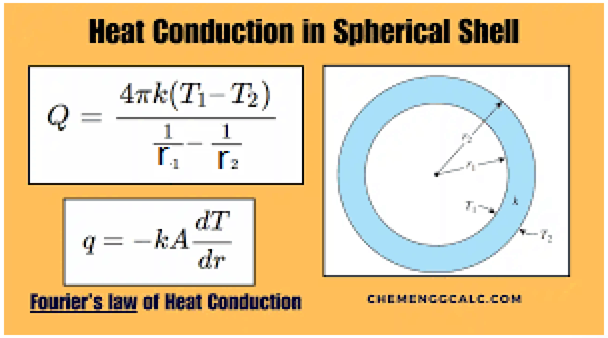
And by separation of variables and integrating:

$\dot{Q}\int_{r_1}^{r_2}\frac{dr}{r^2} = -4\pi k\int_{T_1}^{T_2} dT$

$\dot{Q}\left(\frac{-1}{r}\right)\Big|_{r_1}^{r_2} = -4\pi k(T_2 - T_1)$

$\dot{Q}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 4\pi k(T_1 - T_2)$

$\dot{Q} = \frac{(T_1 - T_2)}{\frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$



Steady State Heat Conduction in One Dimension

The spherical thermal resistance is: $R_{sph} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{(r_2 - r_1)}{2\pi k r_1 r_2}$

When there is convection inside and outside the sphere the relation becomes

$$\dot{Q} = \frac{T_{\infty i} - T_{\infty o}}{R_{conv,i} + R_{sph} + R_{conv,o}} = \frac{T_{\infty i} - T_{\infty o}}{\frac{1}{A_i h_i} + \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{A_o h_o}}$$

$$A_i = 4\pi r_i^2, \; A_o = 4\pi r_o^2$$

$$A_i U_i = A_o U_o = \frac{1}{\frac{1}{A_i h_i} + \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{A_o h_o}}$$

$$U_i = \frac{1}{\frac{A_i}{A_i h_i} + \frac{A_i}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{A_i}{A_o h_o}} = \frac{1}{\frac{1}{h_i} + \frac{r_i^2}{k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{r_i^2}{r_o^2 h_o}}$$

Overall heat transfer coefficient based on inside area

$$U_o = \frac{1}{\frac{A_o}{A_i h_i} + \frac{A_o}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{A_o}{A_o h_o}} = \frac{1}{\frac{r_o^2}{r_i^2 h_i} + \frac{r_o^2}{k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{h_o}}$$

Overall heat transfer coefficient based on outside area

Steady State Heat Conduction in One Dimension

- **Ex.6. spherical tank containing water at 90°C and the heat transfer coefficient is 27W/m^2.°C. The inner diameter of the tank is 1m, and the thickness of the wall 0.01m. Thermal conductivity of tank material is 0.8W/m.K. The atmospheric air temperature is 20°C and the heat transfer coefficient of the outside air is 8W/m^2.°C. Find the overall heat transfer area based on outside area and the rate of heat losses from the tank.**
- **Solution**: spherical tank $T_{\infty i}$=90°C, h_i=27W/m^2.°C, D_i=1.0m, r_i=0.5m, Δr=0.01m, r_o=0.51m, k=0.8W/m.°C, $T_{\infty o}$=20°C, h_o=8W/m^2.°C
- **Requirements**: U_o, $\dot{Q}$,

Steady State Heat Conduction in One Dimension

$$U_o = \frac{1}{\frac{r_o^2}{r_i^2 h_i} + \frac{r_o^2}{k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{h_o}} = \frac{1}{\frac{(0.51)^2}{(0.5)^2 \times 27} + \frac{(0.51)^2}{0.8}\left(\frac{1}{0.5} - \frac{1}{0.51}\right) + \frac{1}{8}} = 5.673 W/m^2.{}^{o}\mathrm{C}$$

$$A_o = 4\pi {r_o}^2 = 4\pi \times (0.51)^2 = 3.268 m^2$$

$$\dot{Q} = A_o U_o (T_{\infty i} - T_{\infty o}) = 3.268 \times 5.673(90 - 20) = 1297.7555 W$$

$$U_o = \frac{1}{\frac{r_o^2}{r_i^2 h_i} + \frac{r_o^2}{k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{h_o}} = \frac{1}{\frac{(0.51)^2}{(0.5)^2 \times 27} + \frac{(0.51)^2}{0.8}\left(\frac{1}{0.5} - \frac{1}{0.51}\right) + \frac{1}{8}} = 5.673 W/m^2.{}^{o}\mathrm{C}$$

$$A_o = 4\pi {r_o}^2 = 4\pi \times (0.51)^2 = 3.268 m^2$$

$$\dot{Q} = A_o U_o (T_{\infty i} - T_{\infty o}) = 3.268 \times 5.673(90 - 20) = 1297.7555 W$$