Department of biology

((GENERAL MATHEMATICS))

1 stage

Week 4- lecture 4

Limits

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By

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1. Limits

Limit is the value that a function or sequence "approaches" as the input or index approaches some value.

We say that the limit of (x) is L as x approaches a and write this as

$$\lim_{x \to a} f(x) = L$$

1. Properties of the limits

1. If a and c are constants then $\lim_{x \to a} c = c$
2. $\lim_{x \to a} x = a$

Let $f_1(x) = L_1$ and $f_2(x) = L_2$ then

3. $\lim_{x \to a}\left(f_1(x) + f_2(x)\right) = L_1 + L_2$
4. $\lim_{x \to a}\left(f_1(x) - f_2(x)\right) = L_1 - L_2$
5. $\lim_{x \to a}\left(f_1(x).f_2(x)\right) = L_1.L_2$
6. $\lim_{x \to a}\left(\frac{f_1(x)}{f_2(x)}\right) = \frac{L_1}{L_2}$; $L_2 \neq 0$
7. If $\lim_{x \to a} g(x) = L$ then $\lim_{x \to a} f\left(g(x)\right) = f(L)$

2. Evaluating Limits

There are several methods to evaluate limits:

1. Direct Substitution:

If f(x) is continuous at x=a, substitute a directly into f(x).

- Example:
  $\lim_{x \to 2} (3x+1) = 3(2) + 1 = 7$

2. Factoring:

If direct substitution results in an indeterminate form ($\frac{0}{0}$), factor and simplify the expression.

- Example:
  $\lim_{x\to1}\frac{x^2-1}{x-1}$:
  Factor $x^2-1=(x-1)(x+1)$:

$$\lim_{x\to1}\frac{(x-1)(x+1)}{x-1}=\lim_{x\to1}(x+1)=2$$

3. Rationalization:

Use this technique if the function involves square roots.

- Example:
  $\lim_{x\to4}\frac{\sqrt{x}-2}{x-4}$:
  Multiply numerator and denominator by the conjugate:

$$\frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}=\frac{x-4}{(x-4)(\sqrt{x}+2)}=\frac{1}{\sqrt{x}+2}$$

Now, substitute $x=4$:

$$\lim_{x\to4}\frac{1}{\sqrt{x}+2}=\frac{1}{4}$$

4. Limits at Infinity:

For limits as x→∞, analyze the growth of numerator and denominator.

- Example:
  $\lim_{x\to\infty}\frac{3x^2+2x}{5x^2-4}$:
  Divide numerator and denominator by x^2:

$$\lim_{x\to\infty}\frac{3+\frac{2}{x}}{5-\frac{4}{x}}=\frac{3}{5}$$

➢ Examples: Evaluate

1. $$\lim_{x\to-2}\frac{x^2+x-2}{x^2+5x+6}$$

$$\lim_{x\to-2}\frac{x^2+x-2}{x^2+5x+6}$$ fraction undefined at $x=-2$

Factor numerator and denominator. (See Section P.6.)

$$=\lim_{x\to-2}\frac{(x+2)(x-1)}{(x+2)(x+3)}$$ Cancel common factors.

$$=\lim_{x\to-2}\frac{x-1}{x+3}$$ Evaluate this limit by substituting $x=-2$.

$$=\frac{-2-1}{-2+3}=-3.$$

2. $$\lim_{x\to4}\frac{\sqrt{x}-2}{x^2-16}$$

$$\lim_{x\to4}\frac{\sqrt{x}-2}{x^2-16}$$ fraction undefined at $x=4$

Multiply numerator and denominator by the conjugate of the expression in the numerator.

$$=\lim_{x\to4}\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x^2-16)(\sqrt{x}+2)}$$

$$=\lim_{x\to4}\frac{x-4}{(x-4)(x+4)(\sqrt{x}+2)}$$

$$=\lim_{x\to4}\frac{1}{(x+4)(\sqrt{x}+2)}=\frac{1}{(4+4)(2+2)}=\frac{1}{32}$$

3. $$\lim_{x\to2}\frac{\sqrt{x}-\sqrt{2}}{x-2}=\lim_{x\to2}\frac{\sqrt{x}-\sqrt{2}}{(\sqrt{x}-\sqrt{2})(\sqrt{x}+\sqrt{2})}=\lim_{x\to2}\frac{1}{(\sqrt{x}+\sqrt{2})}=\frac{1}{2\sqrt{2}}$$

4. $$\lim_{x\to\infty}\frac{3x^2+5}{7x^2+2x-3}$$

Divide top and bottom by x^2, then we get

$$\lim_{x\to\infty}\frac{3x^2+5}{7x^2+2x-3}=\lim_{x\to\infty}\frac{3+(5/x^2)}{7+(2/x)-(3/x^2)}=\frac{3}{7}$$

5. $\lim_{x\to\infty} \frac{(2x+1)^4}{(x^2+3x-1)^2} = \left(\lim_{x\to\infty} \frac{(2x+1)^2}{x^2+3x-1}\right)^2 = \left(\lim_{x\to\infty} \frac{4x^2+4x+1}{x^2+3x-1}\right)^2$

$$= \left(\lim_{x\to\infty} \frac{4+(4/x)+(1/x^2)}{1+(3/x)-(1/x^2)}\right)^2 = 4^2 = 16$$

6.. $\lim_{x\to\infty} \frac{3x}{\sqrt{16x^2+1}} = \lim_{x\to\infty} \frac{\frac{3x}{x}}{\frac{\sqrt{16x^2+1}}{x}} = \lim_{x\to\infty} \frac{3}{\sqrt{\frac{16x^2+1}{x^2}}} = \lim_{x\to\infty} \frac{3}{\sqrt{16+\frac{1}{x^2}}} = \frac{3}{4}$

7. $\lim_{x\to\infty} \sqrt{x^2+x+1} - x = \lim_{x\to\infty} \left(\sqrt{x^2+x+1}-x\right) \times \frac{\left(\sqrt{x^2+x+1}+x\right)}{\left(\sqrt{x^2+x+1}+x\right)}$

$$= \lim_{x\to\infty} \frac{x^2+x+1-x^2}{\sqrt{x^2+x+1}+x} = \lim_{x\to\infty} \frac{x+1}{\sqrt{x^2+x+1}+x}$$

$$= \lim_{x\to\infty} \frac{\frac{x+1}{x}}{\frac{\sqrt{x^2+x+1}+x}{x}} = \lim_{x\to\infty} \frac{1+\frac{1}{x}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1} = \frac{1}{2}$$

8. $\lim_{x\to\infty} \left(\sqrt{x^2+2x}-x\right) = \lim_{x\to\infty} \left(\sqrt{x^2+2x}-x\right) \times \frac{\left(\sqrt{x^2+2x}+x\right)}{\left(\sqrt{x^2+2x}+x\right)}$

$$= \lim_{x\to\infty} \frac{x^2+2x-x^2}{\sqrt{x^2+2x}+x} = \lim_{x\to\infty} \frac{2x}{\sqrt{x^2+2x}+x}$$

$$= \lim_{x\to\infty} \frac{\frac{2x}{x}}{\frac{\sqrt{x^2+2x}+x}{x}} = \lim_{x\to\infty} \frac{2}{\sqrt{1+\frac{2}{x}}+1} = 1$$

9. $\lim_{x\to\infty} \frac{3x^3-2x+1}{x^3+3x^2+x}$

$$\lim_{x\to\infty} \frac{3x^3-2x+1}{x^3+3x^2+x} = \frac{3-\frac{2}{x^2}+\frac{1}{x^3}}{1+\frac{3}{x}+\frac{1}{x^2}} = \frac{3}{1} = 3$$

10. $\lim_{x\to\infty} \frac{(x+4)^6}{(4x^2+4x+1)^3}$

$$\lim_{x\to\infty} \frac{(x+4)^6}{(4x^2+4x+1)^3}$$

$$\frac{x^6}{64x^6} = \frac{1}{64}$$

11. $\lim_{x\to\infty} \sqrt{x^2+x} - \sqrt{x^2-x}$

$$\lim_{x\to\infty} \left(\sqrt{x^2+x} - \sqrt{x^2-x}\right)$$

$$= \frac{2x}{\sqrt{x^2+x} + \sqrt{x^2-x}}$$

$$= \frac{2}{\sqrt{1+\frac{1}{x}} + \sqrt{1-\frac{1}{x}}}$$

$$= \frac{2}{2} = 1$$