Department of biology

Department of biology

((GENERAL MATHEMATICS))

1 stage

Week 5- lecture 5

Derivative

المشتقة

By

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Derivative

The derivative of a function measures how the function changes as its input changes. If y = f (x), the derivative f'(x) gives the slope of the tangent line to the curve at a given point x. This can be expressed as:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1. Differentiation Rules:

1. **Constant Rule:**

   If $f(x) = c$ (where c is a constant), then $f'(x) = 0$.

2. **Power Rule:**

   If $f(x) = x^n$ (where n is a real number), then:

$$f'(x) = n \cdot x^{n-1}$$

3. **Sum and Difference Rule:**

   If $f(x) = g(x) \pm h(x)$, then:

$$f'(x) = g'(x) \pm h'(x)$$

4. **Product Rule:**

   If $f(x) = g(x) \cdot h(x)$, then:

$$f'(x) = g'(x)h(x) + g(x)h'(x)$$

5. **Quotient Rule:**

   If $f(x) = \frac{g(x)}{h(x)}$, then:

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}$$

6. **Chain Rule:**

   If $f(x) = g(h(x))$, then:

$$f'(x) = g'(h(x)) \cdot h'(x)$$

7. **Trig Derivatives:**

- $f(x) = \sin(x)$ then $f'(x) = \cos(x)$
- $f(x) = \cos(x)$ then $f'(x) = -\sin(x)$
- $f(x) = \tan(x)$ then $f'(x) = \sec^2(x)$
- $f(x) = \sec(x)$ then $f'(x) = \sec(x)\tan(x)$
- $f(x) = \cot(x)$ then $f'(x) = -\csc^2(x)$
- $f(x) = \csc(x)$ then $f'(x) = -\csc(x)\cot(x)$

8. **Exponential Derivatives**

- $f(x) = a^x$ then $f'(x) = \ln(a)a^x$
- $f(x) = e^x$ then $f'(x) = e^x$
- $f(x) = a^{g(x)}$ then $f'(x) = \ln(a)a^{g(x)}g'(x)$
- $f(x) = e^{g(x)}$ then $f'(x) = e^{g(x)}g'(x)$

9. **Logarithm Derivatives**

- $f(x) = \log_a(x)$ then $f'(x) = \dfrac{1}{\ln(a)x}$
- $f(x) = \ln(x)$ then $f'(x) = \dfrac{1}{x}$
- $f(x) = \log_a(g(x))$ then $f'(x) = \dfrac{g'(x)}{\ln(a)g(x)}$
- $f(x) = \ln(g(x))$ then $f'(x) = \dfrac{g'(x)}{g(x)}$

Now let's see and talk about the first five rules only, we will mention the next ones in the next lectures.

Examples:

1. Constant Rule

1. $f(x) = 7 \quad \Rightarrow \quad f'(x) = 0$
2. $f(x) = -4 \quad \Rightarrow \quad f'(x) = 0$
3. $f(x) = 12 \quad \Rightarrow \quad f'(x) = 0$
4. $f(x) = 0 \quad \Rightarrow \quad f'(x) = 0$
5. $f(x) = 100 \quad \Rightarrow \quad f'(x) = 0$

2. Power Rule

1. $f(x) = x^3 \quad \Rightarrow \quad f'(x) = 3x^2$
2. $f(x) = x^5 \quad \Rightarrow \quad f'(x) = 5x^4$
3. $f(x) = x^{1/2} \quad \Rightarrow \quad f'(x) = \frac{1}{2}x^{-1/2}$
4. $f(x) = x^{-2} \quad \Rightarrow \quad f'(x) = -2x^{-3}$
5. $f(x) = x^{4/3} \quad \Rightarrow \quad f'(x) = \frac{4}{3}x^{1/3}$
6. $f(x) = x^8 \quad \Rightarrow \quad f'(x) = 8x^7$
7. $f(x) = x^{-5} \quad \Rightarrow \quad f'(x) = -5x^{-6}$
8. $f(x) = x^{2.5} \quad \Rightarrow \quad f'(x) = 2.5x^{1.5}$

3. Sum and Difference Rule

1. $f(x) = x^3 + x^2 \quad \Rightarrow \quad f'(x) = 3x^2 + 2x$
2. $f(x) = 4x^5 - 3x^2 \quad \Rightarrow \quad f'(x) = 20x^4 - 6x$
3. $f(x) = x^4 + x^3 - x^2 \quad \Rightarrow \quad f'(x) = 4x^3 + 3x^2 - 2x$
4. $f(x) = 5x^7 - 2x^3 + x \quad \Rightarrow \quad f'(x) = 35x^6 - 6x^2 + 1$
5. $f(x) = x^6 - x^3 + x^2 \quad \Rightarrow \quad f'(x) = 6x^5 - 3x^2 + 2x$
6. $f(x) = 2x^2 + 3x - 4 \quad \Rightarrow \quad f'(x) = 4x + 3$
7. $f(x) = x^4 - 5x^2 + 7x \quad \Rightarrow \quad f'(x) = 4x^3 - 10x + 7$

4. Product Rule

1. $f(x) = (x^2)(x^3) \quad \Rightarrow \quad f'(x) = (2x)(x^3) + (x^2)(3x^2) = 2x^4 + 3x^4 = 5x^4$
2. $f(x) = (x^2 + 1)(x^3 - 2)$

$$f'(x) = (2x)(x^3 - 2) + (x^2 + 1)(3x^2)$$

$$f'(x) = 2x^4 - 4x + 3x^4 + 3x^2 = 5x^4 + 3x^2 - 4x$$

3. $f(x) = (3x + 1)(x^2 - 5)$

$$f'(x) = (3)(x^2 - 5) + (3x + 1)(2x)$$

$$f'(x) = 3x^2 - 15 + 6x^2 + 2x = 9x^2 + 2x - 15$$

4. $f(x) = (x^2 - 1)(x^3 + x)$

$$f'(x) = (2x)(x^3 + x) + (x^2 - 1)(3x^2 + 1)$$

$$f'(x) = 2x^4 + 2x + 3x^3 + x^2 - 3x^2 - 1 = 2x^4 + 3x^3 - 2x^2 + 2x - 1$$

5. $f(x) = (x^4 + x^2)(x^2 - 3)$

$$f'(x) = (4x^3 + 2x)(x^2 - 3) + (x^4 + x^2)(2x)$$

$$f'(x) = 4x^5 - 12x^3 + 2x^3 - 6x + 2x^5 + 2x^3$$

$$f'(x) = 6x^5 - 8x^3 - 6x$$

5. Quotient Rule

1. $f(x) = \frac{x^2}{x+1}$

$$f'(x) = \frac{(2x)(x+1) - (x^2)(1)}{(x+1)^2}$$

$$f'(x) = \frac{2x^2 + 2x - x^2}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2}$$

2. $f(x) = \frac{3x^2+1}{x^3}$

$$f'(x) = \frac{(6x)(x^3) - (3x^2+1)(3x^2)}{x^6}$$

$$f'(x) = \frac{6x^4 - (9x^4 + 3x^2)}{x^6}$$

$$f'(x) = \frac{-3x^4 - 3x^2}{x^6} = \frac{-3x^2(x^2+1)}{x^6} = \frac{-3(x^2+1)}{x^4}$$

3. $f(x) = \frac{x^2+2x+1}{x-1}$

$$f'(x) = \frac{(2x+2)(x-1) - (x^2+2x+1)(1)}{(x-1)^2}$$

$$f'(x) = \frac{(2x^2 - 2x + 2x - 2) - (x^2 + 2x + 1)}{(x-1)^2}$$

$$f'(x) = \frac{2x^2 - 2 - x^2 - 2x - 1}{(x-1)^2}$$

$$f'(x) = \frac{x^2 - 2x - 3}{(x-1)^2}$$

4. $f(x) = \frac{x^3-4}{x^2+1}$

$$f'(x) = \frac{(3x^2)(x^2+1) - (x^3-4)(2x)}{(x^2+1)^2}$$

$$f'(x) = \frac{3x^4 + 3x^2 - 2x^4 + 8x}{(x^2+1)^2}$$

$$f'(x) = \frac{x^4 + 3x^2 + 8x}{(x^2+1)^2}$$