Limitations on Reinforcement Ratio for Doubly Reinforced beams

Lower limit on ρ (ACI 10.5)

same as for single reinforce beams.

$$\rho_{\mathrm{min}} = \frac{3\sqrt{f_{\mathrm{c}}'}}{f_{\mathrm{y}}} \geq \frac{200}{f_{\mathrm{y}}}$$

Example: Doubly Reinforced Section

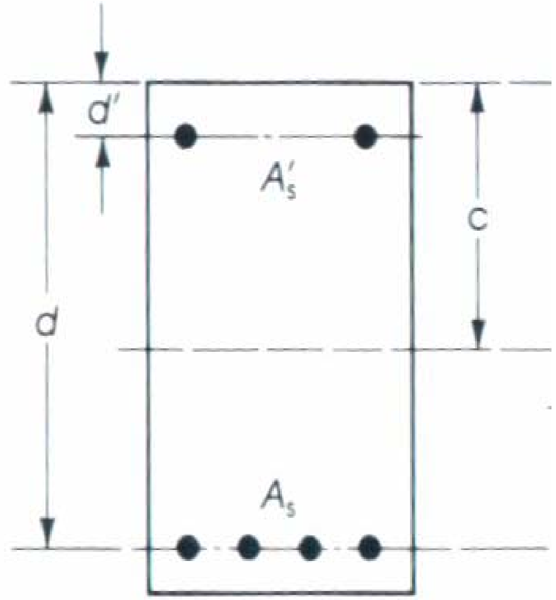
Given:

f'_c= 4000 psi f_y = 60 ksi

A'_s = 2 #5 A_s = 4 #7

d'= 2.5 in. d = 15.5 in

h=18 in. b =12 in.



Calculate M_n for the section for the given compression steel.

Example: Doubly Reinforced Section

Compute the reinforcement coefficients, the area of the bars #7 (0.6 in^2) and #5 (0.31 in^2)

$$A_s = 4\left(0.6\text{ in}^2\right) = 2.4\text{ in}^2$$

$$A'_s = 2\left(0.31\text{ in}^2\right) = 0.62\text{ in}^2$$

$$\rho = \frac{A_s}{bd} = \frac{2.4\text{ in}^2}{\left(12\text{ in.}\right)\left(15.5\text{ in.}\right)} = 0.0129$$

$$\rho' = \frac{A'_s}{bd} = \frac{0.62\text{ in}^2}{\left(12\text{ in.}\right)\left(15.5\text{ in.}\right)} = 0.0033$$

Example: Doubly Reinforced Section

Compute the effective reinforcement ratio and minimum ρ

$$\rho_{eff} = \rho - \rho' = 0.0129 - 0.0033 = 0.00957$$

$$\rho = \frac{200}{f_{\mathrm{y}}} = \frac{200}{60000} = 0.00333 \text{ or } \frac{3\sqrt{f_{\mathrm{c}}}}{f_{\mathrm{y}}} = \frac{3\sqrt{4000}}{60000} = 0.00316$$

$$\rho \geq \rho_{\mathrm{min}} \Rightarrow 0.0129 \geq 0.00333 \text{ OK!}$$

Example: Doubly Reinforced Section

Compute the effective reinforcement ratio and minimum ρ

$$(\rho - \rho') \geq \left(\frac{\beta_1 (0.85 f_c') d'}{d\ f_y} \right) \left(\frac{87000}{87000 - f_y} \right)$$

$$\geq \left(\frac{0.85(0.85(4\text{ ksi}))(2.5\text{ in.})}{60\text{ ksi}(15.5\text{ in.})} \right) \left(\frac{87}{87 - 60} \right) = 0.0398$$

$0.00957 \not\geq 0.0398$ Compression steel has not yielded.

Example: Doubly Reinforced Section

Compute the effective reinforcement ratio and minimum ρ

$$f_s' = 29 \text{ x } 10^6 \left(1 - \frac{\beta_1 \left(0.85 f_c'\right) d'}{\left(\rho - \rho'\right) d \ f_y} \right) 0.003$$

$$= 87 \left(1 - \frac{0.85\left(0.85\left(4 \text{ ksi}\right)\right)\left(2.5 \text{ in.}\right)}{\left(0.00957\right)\left(15.5 \text{ in.}\right)\left(60 \text{ ksi}\right)} \right) = 16.37 \text{ ksi}$$

Use an iterative technique to find f'_s

Example: Doubly Reinforced Section

Compute the iterative values (1)

$$c = \frac{A_s f_y - A_s' f_s'}{0.85 f_c \beta_1 b} = \frac{\left(2.4 \text{ in}^2\right)\left(60 \text{ ksi}\right) - \left(0.62 \text{ in}^2\right)\left(16.37 \text{ ksi}\right)}{0.85\left(4 \text{ ksi}\right)\left(0.85\right)\left(12 \text{ in.}\right)}$$

$$c = 3.86 \text{ in.}$$

$$f_s' = E_s \varepsilon_s' = 29 \text{ x } 10^6 \left(1 - \frac{d'}{c}\right) 0.003 = 87\left(1 - \frac{\left(2.5 \text{ in.}\right)}{\left(3.86 \text{ in.}\right)}\right)$$

$$= 30.65 \text{ ksi}$$

Use an iterative technique to find f'_s

Example: Doubly Reinforced Section

Compute the iterative values (2)

$$c = \frac{A_s f_y - A_s' f_s'}{0.85 f_c \beta_1 b} = \frac{\left(2.4 \text{ in}^2\right)\left(60 \text{ ksi}\right) - \left(0.62 \text{ in}^2\right)\left(30.65 \text{ ksi}\right)}{0.85\left(4 \text{ ksi}\right)\left(0.85\right)\left(12 \text{ in.}\right)}$$

$$c = 3.604 \text{ in.}$$

$$f_s' = E_s \varepsilon_s' = 29 \text{ x } 10^6 \left(1 - \frac{d'}{c}\right) 0.003 = 87\left(1 - \frac{\left(2.5 \text{ in.}\right)}{\left(3.604 \text{ in.}\right)}\right)$$

$$= 26.66 \text{ ksi}$$

Use an iterative technique to find f’$_s$

Example: Doubly Reinforced Section

Compute the iterative values (2)

$$c = \frac{A_s f_y - A_s' f_s'}{0.85 f_c \beta_1 b} = \frac{\left(2.4 \text{ in}^2\right)\left(60 \text{ ksi}\right) - \left(0.62 \text{ in}^2\right)\left(30.66 \text{ ksi}\right)}{0.85\left(4 \text{ ksi}\right)\left(0.85\right)\left(12 \text{ in.}\right)}$$

$$c = 3.676 \text{ in.}$$

$$f_s' = E_s \varepsilon_s' = 29 \text{ x } 10^6 \left(1 - \frac{d'}{c}\right) 0.003 = 87\left(1 - \frac{\left(2.5 \text{ in.}\right)}{\left(3.676 \text{ in.}\right)}\right)$$

$$= 27.82 \text{ ksi}$$

Use an iterative technique to find f'_s

Example: Doubly Reinforced Section

Compute the iterative values (etc)

$$c = \frac{A_s f_y - A_s' f_s'}{0.85 f_c \beta_1 b} = \frac{\left(2.4 \text{ in}^2\right)\left(60 \text{ ksi}\right) - \left(0.62 \text{ in}^2\right)\left(27.49 \text{ ksi}\right)}{0.85\left(4 \text{ ksi}\right)\left(0.85\right)\left(12 \text{ in.}\right)}$$

$$c = 3.66 \text{ in.}$$

$$f_s' = E_s \varepsilon_s' = 29 \text{ x } 10^6 \left(1 - \frac{d'}{c}\right) 0.003 = 87\left(1 - \frac{\left(2.5 \text{ in.}\right)}{\left(3.66 \text{ in.}\right)}\right)$$

$$= 27.57 \text{ ksi}$$

Use f'_s=27.56 ksi c=3.66 in

Example: Doubly Reinforced Section

Compute the moment capacity of the beam

$$M_n = \left(A_s f_y - A_s' f_s'\right)\left(d - \frac{a}{2}\right) + A_s' f_s' \left(d - d'\right)$$

$$= \left(\left(2.4\ \text{in}^2\right)\left(60\ \text{ksi}\right) - \left(0.62\ \text{in}^2\right)\left(27.56\ \text{ksi}\right)\right)\left(15.5\ \text{in.} - \frac{0.85\left(3.66\ \text{in.}\right)}{2}\right)$$

$$+\left(0.62\ \text{in}^2\right)\left(27.56\ \text{ksi}\right)\left(15.5\ \text{in.} - 2.5\ \text{in.}\right)$$

$$= 1991.9\ \text{k-in.} \Rightarrow 166\ \text{k-ft}$$