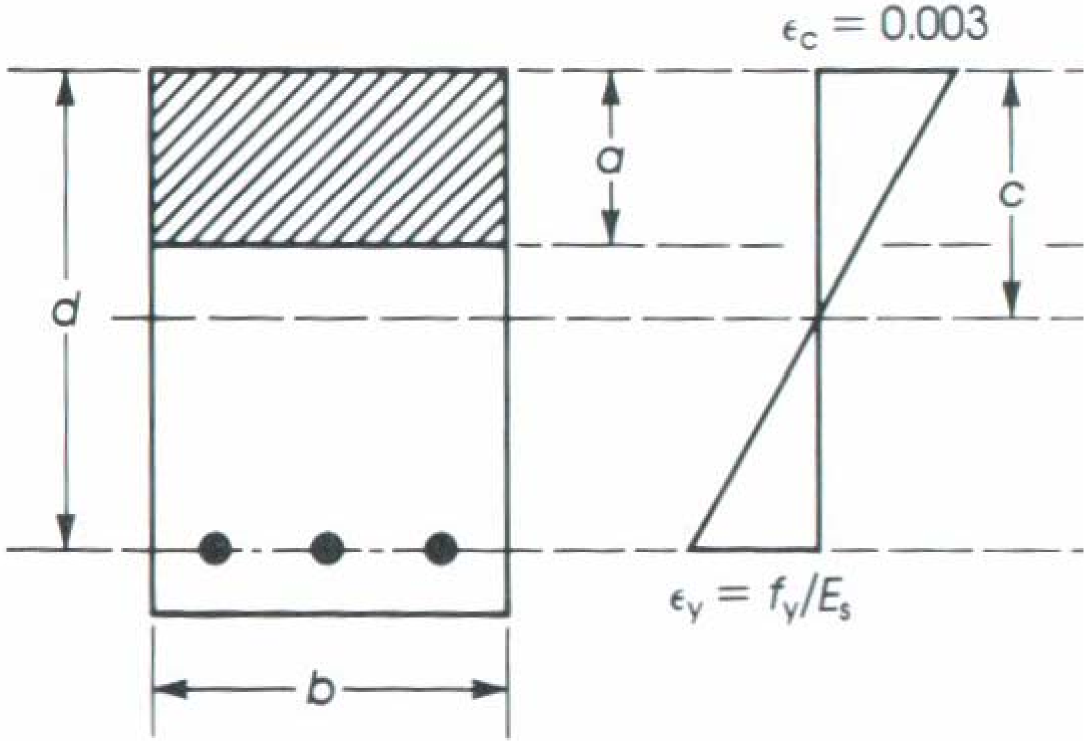
Balanced Reinforcement Ratio, ρ_{bal}

ρ_{bal} = unique ρ value to get simultaneous $\varepsilon_c = 0.003$
& $\varepsilon_s = \varepsilon_y$

Use similar triangles:

$$\frac{0.003}{c_b} = \frac{\varepsilon_y}{d - c_b}$$



Balanced Reinforcement Ratio, ρ_{bal}

The equation can be rewritten to find c_b

$$0.003d - 0.003c_b = \varepsilon_y c_b$$

$$c_b\left(0.003 + \varepsilon_y\right) = 0.003d$$

$$c_b = \frac{0.003d}{\left(0.003 + \varepsilon_y\right)} \quad \Rightarrow \quad \frac{c_b}{d} = \frac{0.003}{\left(0.003 + \varepsilon_y\right)}$$

$$\frac{c_b}{d} = \left(\frac{0.003}{\left(0.003 + \varepsilon_y\right)}\right)\left(\frac{E_s}{E_s}\right) = \frac{87000}{\left(87000 + f_y\right)}$$

Nominal Moment Equation

The equation can be rewritten in the form:

$$C = T \quad \Rightarrow \quad 0.85 f_c' \mathrm{ba} = \mathrm{A_s} f_y$$

$$\mathrm{a} = \frac{f_y \mathrm{A_s}}{0.85 f_c' \mathrm{b}}$$

$$M_n = \mathrm{A_s} f_y \left(\mathrm{d} - \frac{\mathrm{a}}{2} \right)$$

Nominal Moment Equation

The equation can be rewritten in the form:

$$M_n = f_y \left(\frac{A_s}{bd}\right)\left(\frac{b}{d}\right)d^2\left(d - \frac{f_y A_s d}{1.7 f_c' bd}\right)$$

Use the ratio r = b/d and ρ

$$M_n = \left(\rho f_y\right)\left(r\right)d^2\left(d - \frac{f_y \rho d}{1.7 f_c'}\right)$$

Nominal Moment Equation

Use $\omega = \rho\, f_y/f_c$ and

$$M_n = \omega(r)\, f_c' d^3 \left(1 - \frac{\omega}{1.7}\right) = \omega(r)\, f_c' (1 - 0.59\omega)\, d^3$$

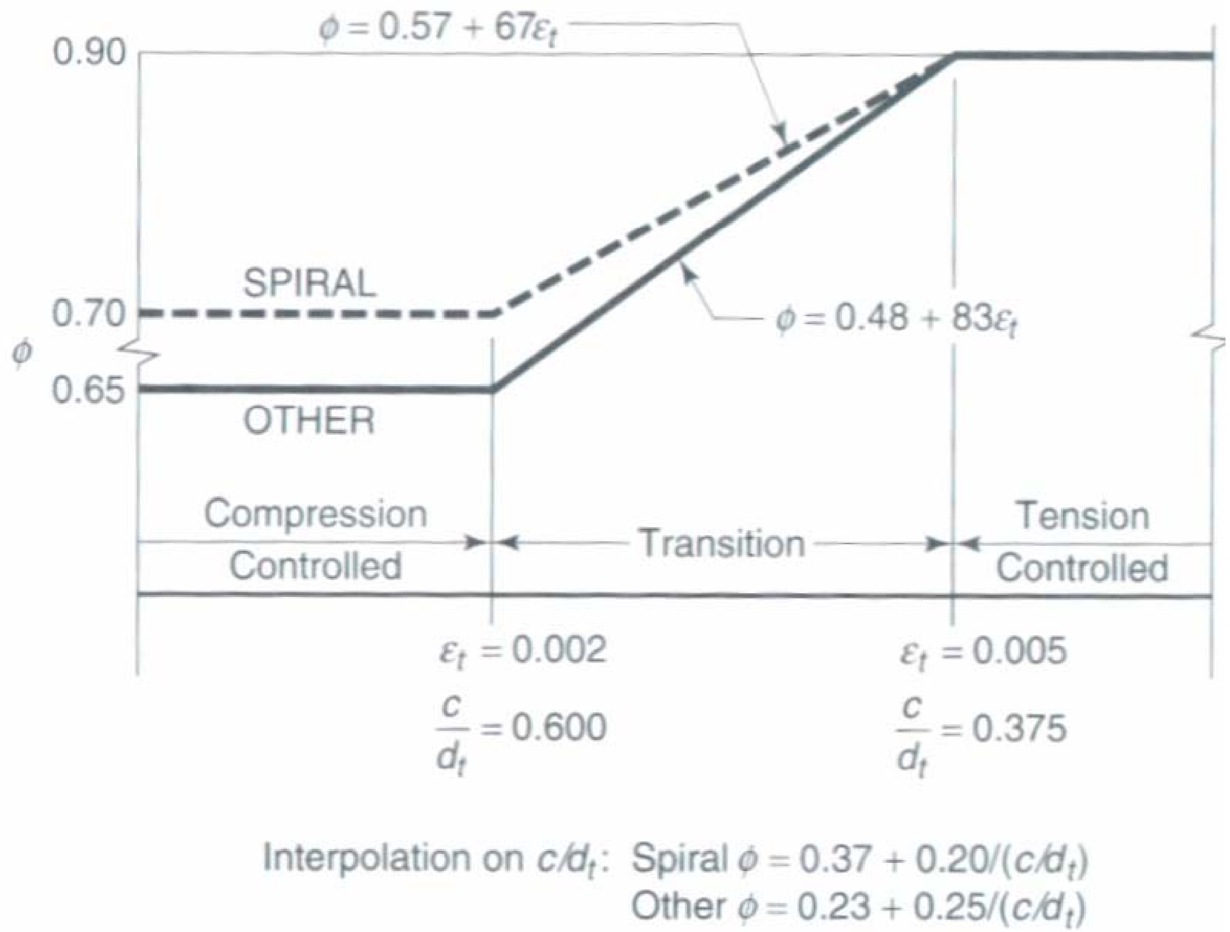
Use the ratio r = b/d and R

$$M_n = Rbd^2$$

$$R = \omega f_c' (1 - 0.59\omega)$$

Strain Limits Method for Analysis

The strength reduction factor, ϕ, will come into the calculation of the strength of the beam.



Limitations on Reinforcement Ratio, ρ

The selection of the steel will be determined by the

<u>Lower Limit on ρ</u> ACI 10.5.1

$$A_{s(\min)} = \frac{3\sqrt{f_c'}}{f_y} * b_w d \geq \frac{200}{f_y} * b_w d \qquad \text{ACI Eqn. (10-3)}$$

f_c & f_y are in psi

Limitations on Reinforcement Ratio, ρ

Lower Limit on ρ ACI 10.5.1

$$\rho_{\min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y}$$

Lower limit used to avoid “Piano Wire” beams.

Very small A_s ($M_n < M_{cr}$)

ε_s is huge (large deflections)

when beam cracks ($M_n > M_{cr}$) beam fails right away because $M_n < M_{cr}$

Additional Requirements for Lower Limit on ρ

If A_s (provided) $\geq$ 4/3 A_s (required) based on analysis, then A_s (min) is not required.

$$\phi M_n \geq \frac{4}{3} M_u$$

for A_s (provided)

See ACI 10.5.3

Additional Requirements for Lower Limit on ρ

Temperature and Shrinkage reinforcement in structural slabs and footings (ACI 7.12) place perpendicular to direction of flexural reinforcement.

GR 40 or GR 50 Bars: A_s (T&S) = 0.0020 A_g

GR 60 or Welded Wire Fabric (WWF):

$$A_s \text{ (T\&S)} = 0.0018\, A_g$$

A_g - Gross area of the concrete

Example

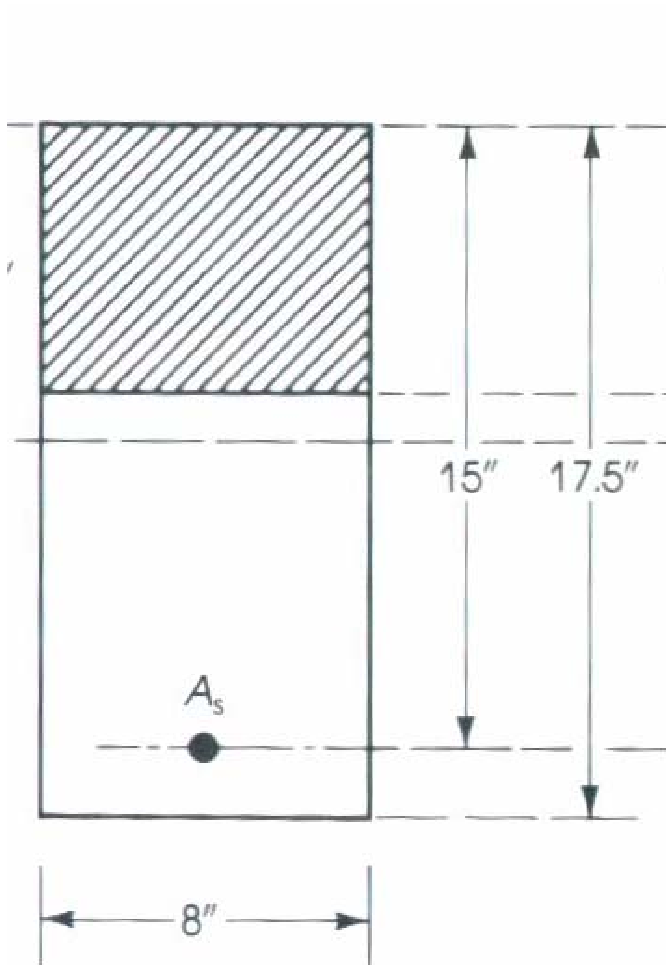
Given:

f_c = 3 ksi & f_y = 40 ksi
and $A_s = 4$ in^2

Determine:

(1) Determine if the beam will satisfy ACI code.

(2) If f_c = 6 ksi?



Example

Given:

f_c = 3 ksi & f_y = 40 ksi and A_s = 4 in^2

$$\rho = \frac{A_s}{bd} = \frac{4 \text{ in}^2}{(8 \text{ in.})(15 \text{ in.})} = 0.0333$$

The minimum steel ratio is

$$\rho_{min} = \frac{3\sqrt{3000}}{40000} = 0.00411 \quad \geq \quad \frac{200}{40000} = 0.005$$

$$\therefore \quad \rho_{min} = 0.005 \quad \Rightarrow 0.0333 > 0.005 \text{ OK!}$$

Example

Given:

f_c = 3 ksi & f_y = 40 ksi and A_s =4 in^2

$$a = \frac{f_y A_s}{0.85 f_c' b} = \frac{(40 \text{ ksi})(4 \text{ in}^2)}{0.85(3 \text{ ksi})(8 \text{ in})} = 7.843 \text{ in.}$$

The neutral axis is

$$c_b = \frac{a}{\beta_1} = \frac{7.843 \text{ in.}}{0.85} = 9.23 \text{ in.} \Rightarrow \frac{c_b}{d} = \frac{9.23 \text{ in.}}{15 \text{ in.}} = 0.615$$

Example

The strain in the steel is

$$\varepsilon_{\mathrm{t}} = \left(\frac{d - c_{\mathrm{b}}}{c_{\mathrm{b}}} \right) (0.003) = \left(\frac{15 \text{ in.} - 7.843 \text{ in.}}{7.843 \text{ in.}} \right) (0.003)$$

$$= 0.0027$$

There for the beam is in the compression zone and ϕ would be 0.65, however c/d ratio is greater than 0.375 so the beam will need to be redesigned.

Example

Given:

f_c = 6 ksi & f_y = 40 ksi and A_s =4 in^2

$$\rho = \frac{A_s}{bd} = \frac{4 \text{ in}^2}{(8 \text{ in.})(15 \text{ in.})} = 0.0333$$

The minimum steel ratio is

$$\rho_{\min} = \frac{3\sqrt{6000}}{40000} = 0.00581 \quad \geq \quad \frac{200}{40000} = 0.005$$

$$\therefore \quad \rho_{\min} = 0.00581 \quad \Rightarrow 0.0333 > 0.00581 \ \text{ OK!}$$

Example

Given:

f_c = 6 ksi & f_y = 40 ksi and A_s =4 in^2

$$a = \frac{f_y A_s}{0.85 f_c' b} = \frac{(40\text{ ksi})(4\text{ in}^2)}{0.85(6\text{ ksi})(8\text{ in})} = 3.922\text{ in.}$$

The neutral axis is at

$$c_b = \frac{a}{\beta_1} = \frac{3.922\text{ in.}}{0.75} = 5.22\text{ in.} \Rightarrow \frac{c_b}{d} = \frac{5.22\text{ in.}}{15\text{ in.}} = 0.349$$

Example

The strain in the steel will be

$$\varepsilon_t = \left(\frac{d - c_b}{c_b} \right)(0.003) = \left(\frac{15 \text{ in.} - 5.22 \text{ in.}}{5.22 \text{ in.}} \right)(0.003)$$

$$= 0.0056$$

There for the beam is in the tension zone and ϕ will be 0.9.

Homework (Due: 9/27/02)

Do the following problems:

1. Problem 5.2 from the text.
2. Problem 5.3 from the text.