Module Title: Fundamental of Electrical Engineering (DC)

Module Code:	**UOMU024011**

Chapter -1-2
Lectures (week 4, 5, 6)
Parallel Elements and Current Division

Parallel Circuits - Current Divider Rule

Power dissipation in resistive network

Delta to star and star to delta Network conversion

Definition:

Parallel Circuits are two elements, branches, or networks are connected in parallel if they have two points in common. See figure 2.1 below.

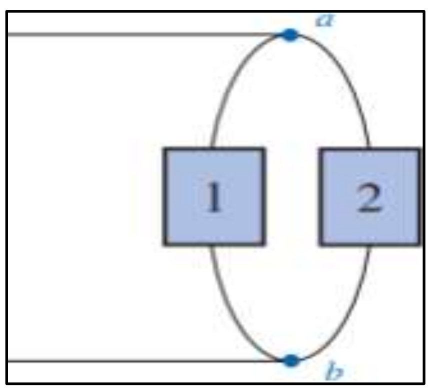

Figure 2.1 Parallel Circuits are two elements

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There are different ways of connections: see Figure 2.2 Below

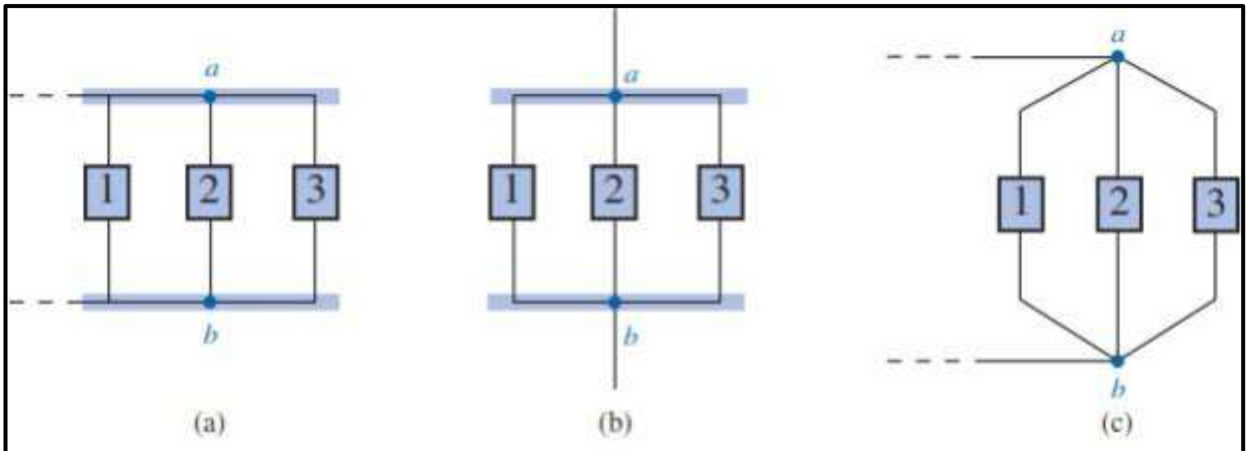


Figure 2.2 different ways of connections

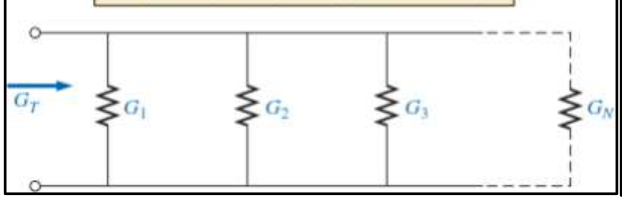
2.1 Total Conductance and Resistance

Recall that for series resistors, the total resistance is the sum of the resistor values.

For parallel elements, the total conductance is the sum of the individual conductance's.

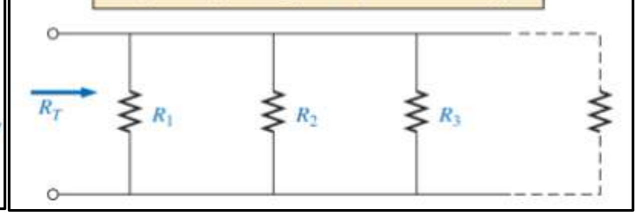
Parallel Conductance

$$G_T = G_1 + G_2 + G_3 + \cdots + G_N$$

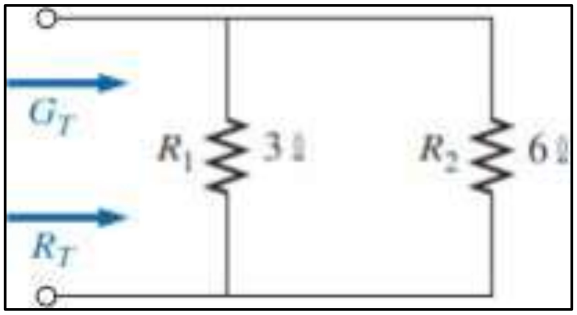


Parallel Resistance

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$



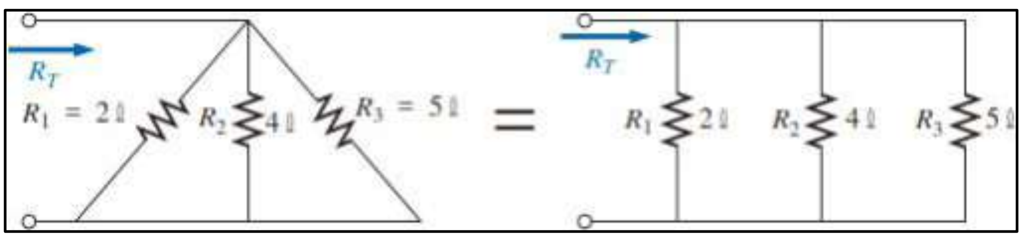
Example 2.1 Determine the total conductance and resistance for the parallel network of Fig. shown.



$$G_T = G_1 + G_2 = \frac{1}{3\,\Omega} + \frac{1}{6\,\Omega} = 0.333\text{ S} + 0.167\text{ S} = \mathbf{0.5\ S}$$

and $$R_T = \frac{1}{G_T} = \frac{1}{0.5\text{ S}} = \mathbf{2\ \Omega}$$

Example 2.2 Determine the total resistance for the network of Fig. shown.



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{2\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{5\,\Omega} = 0.5\text{ S} + 0.25\text{ S} + 0.2\text{ S}$$
$$= 0.95\text{ S}$$

and $$R_T = \frac{1}{0.95\text{ S}} = \mathbf{1.053\ \Omega}$$

<u>Notes' : The total resistance of parallel resistors is always less than the value smallest resistor.</u>

For equal resistors in parallel, the equation becomes significantly easier to apply. For N equal resistors in parallel,

$$\frac{1}{R_T} = \underbrace{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots + \frac{1}{R}}_{N}$$

$$= N\left(\frac{1}{R}\right)$$

and $$R_T = \frac{R}{N}$$

and $$G_T = NG$$

For two parallel resistors, we write:

and $$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_T} = \left(\frac{R_2}{R_2}\right)\frac{1}{R_1} + \left(\frac{R_1}{R_1}\right)\frac{1}{R_2} = \frac{R_2}{R_1 R_2} + \frac{R_1}{R_1 R_2}$$

$$= \frac{R_2 + R_1}{R_1 R_2}$$

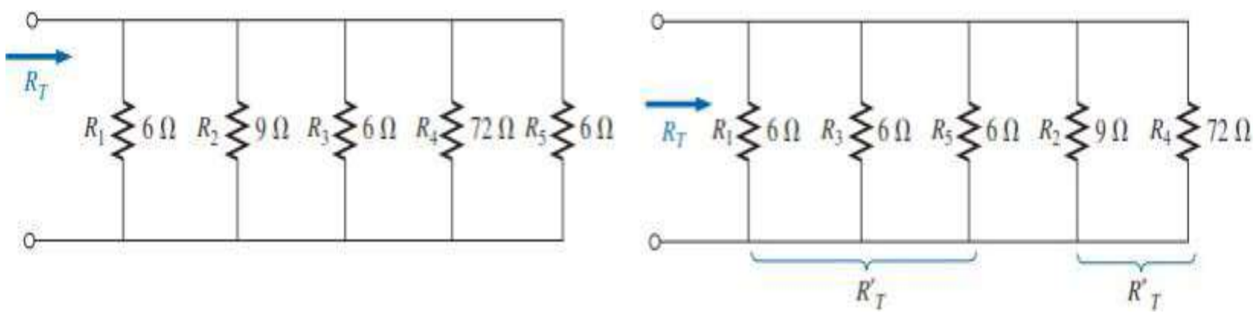
In words, the total resistance of two parallel resistors is the product of the two divided by their sum.

For three parallel resistors,

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Example 2.3 Calculate the total resistance of the parallel network of Fig. shown.



$$R'_T = \frac{R}{N} = \frac{6\ \Omega}{3} = 2\ \Omega$$

$$R''_T = \frac{R_2 R_4}{R_2 + R_4} = \frac{(9\ \Omega)(72\ \Omega)}{9\ \Omega + 72\ \Omega} = \frac{648\ \Omega}{81} = 8\ \Omega$$

$$R_T = R'_T \parallel R''_T$$

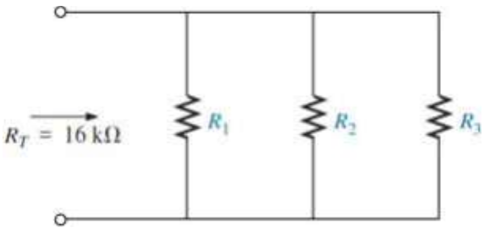
↑ In parallel with

$$= \frac{R'_T R''_T}{R'_T + R''_T} = \frac{(2\ \Omega)(8\ \Omega)}{2\ \Omega + 8\ \Omega} = \frac{16\ \Omega}{10} = \mathbf{1.6\ \Omega}$$

Home work

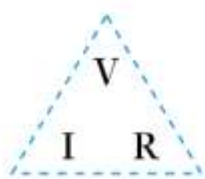
Example 2.4 Determine the values of R1, R2, and R3 in Figure shown below.

if R2= 2R1and R3 = 2R2 and the total resistance is 16 kΩ.



2.1.1 Power Dissipation in Resistors

- *The current flowing in a conductor is directly proportional to the applied voltage V and inversely proportional to its resistance R*



$I = V/R$

$V = IR$

$R = V/I$

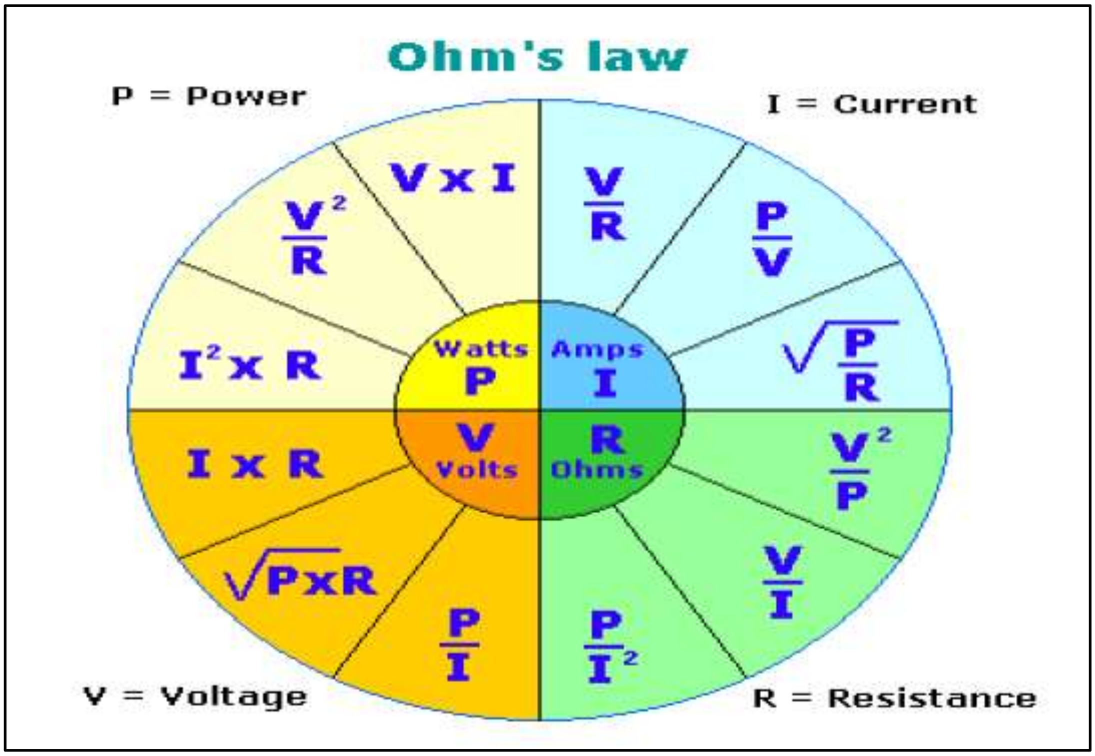
- The instantaneous power dissipation *P* of a resistor is given by the product of the voltage across it and the current passing through it. Combining this result with Ohm's law gives:

$P = VI$

$P = I^2R$

$P = V^2/R$

2.2.2 Ohm's Law (V I R) and Power Triangle



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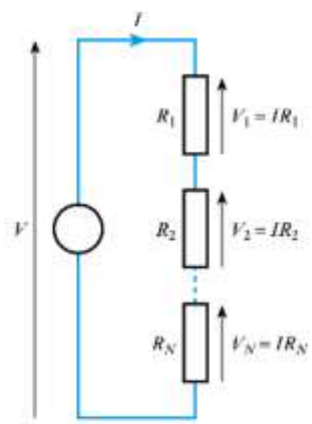
2.2.3 Resistors in Series

- Series

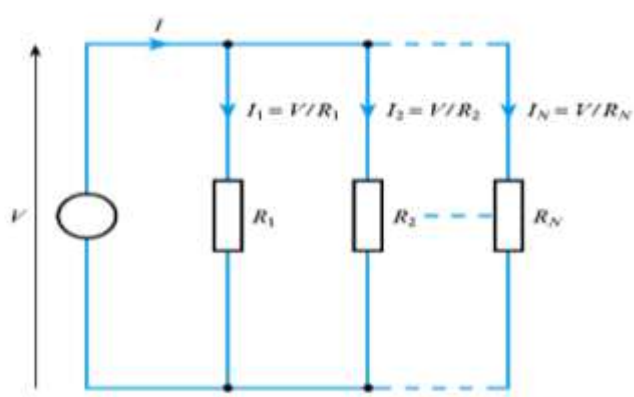


$$R = R_1 + R_2 + R_3$$

$$VT = V1 + V2 + V3$$

$$IT = I1 = I2 = I3$$

2.2.4 Resistors in Parallel



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$IT = I1 + I2 + I3$$

$$VT = V1 = V2 = V3$$

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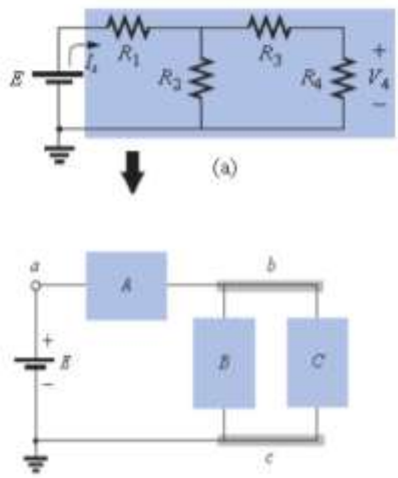
2.2.5 Resistors in Series and Parallel



$$C = (R3 + R4)$$
$$RT = (B * C / B + A)$$

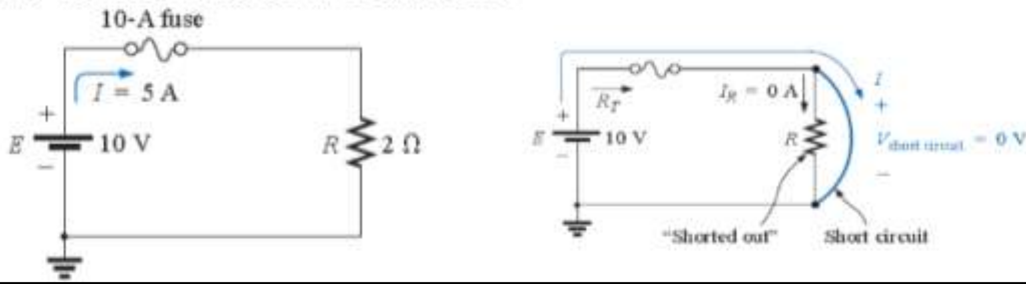
$$VB = VC$$
$$VT = VA + VB$$

$$IT = IB + IC$$
$$IT = IA$$

2.2.6 Open and Short Circuits

An open circuit is simply two isolated terminals not connected by
an element of any kind

A short circuit is a very low resistance, direct connection between
two terminals of a network



2.2.7 Resistive Potential Dividers

The voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements (see figure 2.1)

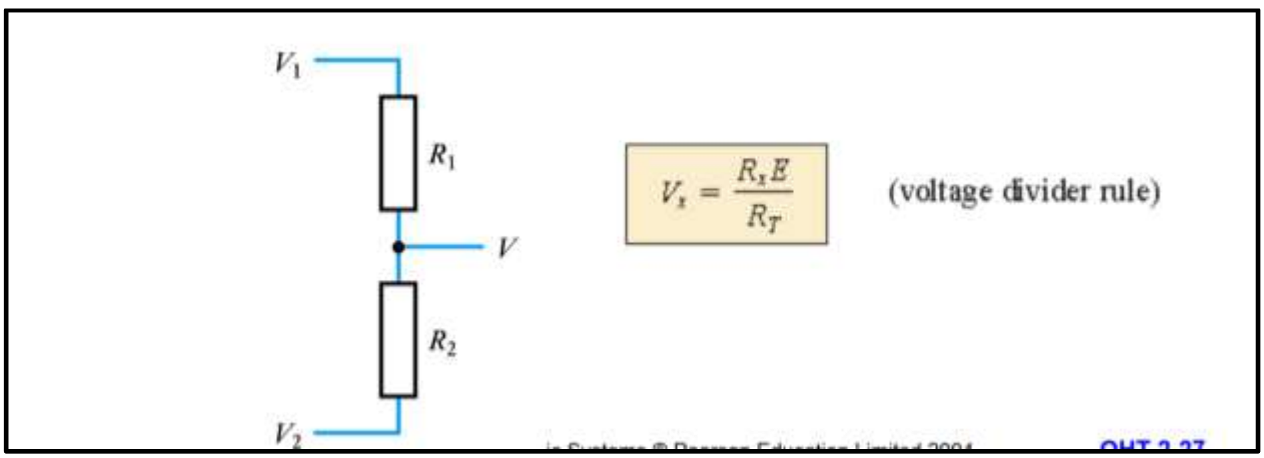


Figure 2.1

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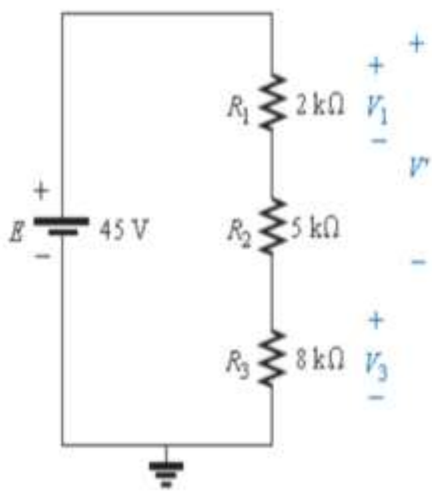
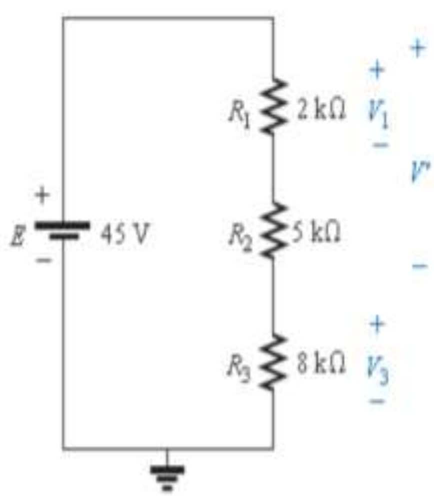
Example 2.2

Figure below shows the circuit potential divider. Find the voltage across the resistance R1 and R3 .



$$V_1 = \frac{R_1 E}{R_T} = \frac{(2\,\text{k}\Omega)(45\,\text{V})}{2\,\text{k}\Omega + 5\,\text{k}\Omega + 8\,\text{k}\Omega} = \frac{(2\,\text{k}\Omega)(45\,\text{V})}{15\,\text{k}\Omega}$$

$$= \frac{(2 \times 10^3\,\Omega)(45\,\text{V})}{15 \times 10^3\,\Omega} = \frac{90\,\text{V}}{15} = \mathbf{6\,V}$$



$$= \frac{360\,\text{V}}{15} = \mathbf{24\,V}$$

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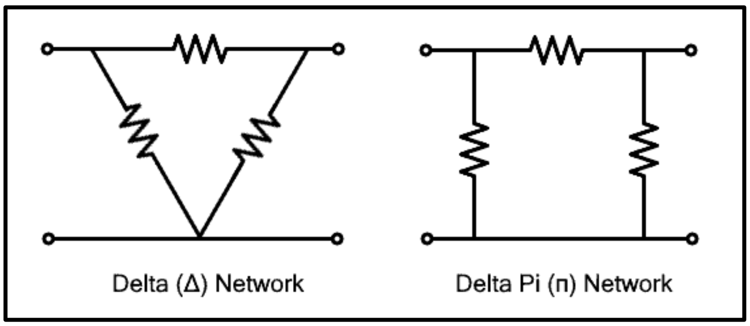
2.2.8 Delta and Star connection network

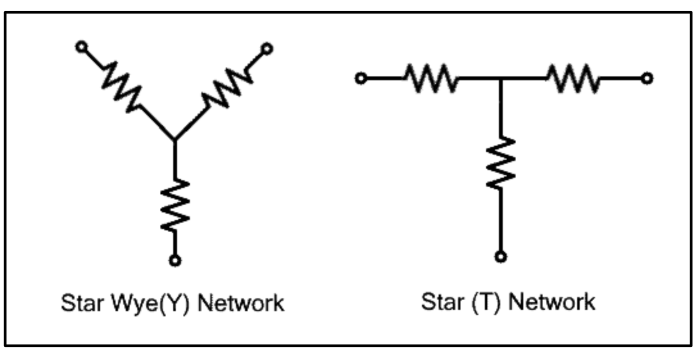
Delta Connected Network

The delta connected network is formed when three network branches or impedances are connected to form a loop in such a way that their heads are connected to the tails of the adjacent branch. The resultant network forms a triangle shape that resembles a Greek letter Delta "Δ" which is why it's named after it. It is also known as π (pi) network because it resembles the letter after rearranging the branches. Know more about **Delta Connection** in the previous post.



Star Connected Network

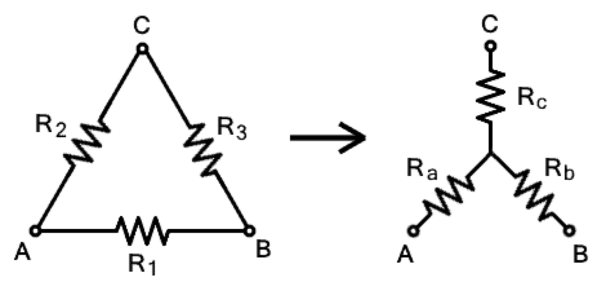
The Star connected network is formed when three branches or impedances are connected together at a common point. The other ends of the branch networks are free. The resultant shape resembles the letter "Y" which is why it is also called "Y" or "Wye" connected network. It is also known as "T" connected network due to its shape after rearranging the network branches. Know more about **Star Connection .**



The circuits given above can be converted using the following transformation. During the transformation, the terminals A, B, C must remain in the same position, only the impedance & their arrangement changes. The following figure illustrates the statement given above.

Delta to Star Conversion

The delta connected network can be transformed into star configuration using a set of electrical formula. Let's derive the equation for each impedance.



Delta to Star Conversion

The given figure shows a delta network having A, B, C terminals with the impedances R_1, R_2, R_3. The equivalent star connected network with R_A, R_B & R_C where they are connected to their corresponding terminals as shown in the figure.
As mentioned earlier, the terminals A, B, C remains the same, as well as the impedance between them, must remain the same.

The total impedance between A-B in the delta network;

$$R_{AB} = R_1 \parallel (R_2 + R_3)$$

$$R_{AB} = \frac{R_1\,(R_2 + R_3)}{R_1 + R_2 + R_3} \quad \ldots\ldots\ldots\ldots\ldots (i)$$

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Similarly, the impedance between terminals B-C

$$R_{BC} = R_3 \,||\, (R_1 + R_2)$$

$$R_{BC} = \frac{R_3\,(R_1 + R_2)}{R_1 + R_2 + R_3} \quad \ldots\ldots\ldots\ldots\ldots\ (ii)$$

Similarly, the impedance between A-C

$$R_{AC} = R_2 \,||\, (R_1 + R_3)$$

$$R_{AC} = \frac{R_2\,(R_1 + R_3)}{R_1 + R_2 + R_3} \quad \ldots\ldots\ldots\ldots\ldots\ (iii)$$

According to star network;

$$\mathbf{R_{AB} = R_A + R_B}$$
$$\mathbf{R_{BC} = R_B + R_C}$$
$$\mathbf{R_{AC} = R_A + R_C}$$

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Now adding equation (i), (ii) & (iii) together.

$$R_{AB} + R_{BC} + R_{AC} = \frac{R_1\,(R_2 + R_3)}{R_1 + R_2 + R_3} + \frac{R_3\,(R_1 + R_2)}{R_1 + R_2 + R_3} + \frac{R_2\,(R_1 + R_3)}{R_1 + R_2 + R_3}$$

$$(R_A + R_B) + (R_B + R_C) + (R_A + R_C) = \frac{R_1\,(R_2 + R_3) + R_3\,(R_1 + R_2) + R_2\,(R_1 + R_3)}{R_1 + R_2 + R_3}$$

$$2R_A + 2R_B + 2R_C = \frac{R_1R_2 + R_1R_3 + R_1R_3 + R_2R_3 + R_1R_2 + R_2R_3}{R_1 + R_2 + R_3}$$

$$2R_A + 2R_B + 2R_C = \frac{2(R_1R_2) + 2(R_2R_3) + 2(R_1R_3)}{R_1 + R_2 + R_3}$$

$$2(R_A + R_B + R_C) = \frac{2(R_1R_2 + R_2R_3 + R_1R_3)}{R_1 + R_2 + R_3}$$

$$R_A + R_B + R_C = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1 + R_2 + R_3} \cdots\cdots\cdots (iv)$$

Now subtract equation (i), (ii), & (iii) one by one from equation (iv) First, Subtract (ii) from (iv).

$$R_A + R_B + R_C - (R_B + R_C) = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1 + R_2 + R_3} - \frac{R_3\,(R_1 + R_2)}{R_1 + R_2 + R_3}$$

$$R_A = \frac{R_1R_2 + R_2R_3 + R_1R_3 - R_1R_3 - R_2R_3}{R_1 + R_2 + R_3}$$

$$R_A = \frac{R_1R_2}{R_1 + R_2 + R_3} \cdots\cdots\cdots (v)$$

Similarly subtracting (i) & (iii) from (iv) results in:

$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3} \cdots\cdots\cdots (vi)$$

$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3} \cdots\cdots\cdots (vii)$$

From the derived equations for star-equivalent impedances R_A, R_B, & R_C we can conclude the relation between delta-to-star conversions as; the equivalent star impedance is equal to the product of the adjacent delta impedances with a terminal divide by the sum of all three delta impedances. In case **all three Impedances are same** in a delta network, the equivalent star impedance would become.

$$R_{star} = \frac{RR}{R + R + R}$$

$$R_{star} = \frac{R^2}{3R}$$

$$R_{star} = \frac{R_{delta}}{3}$$

Since all the ***impedances*** throughout the delta network are equal, each three equivalent star resistance would be 1/3 times the delta impedance.

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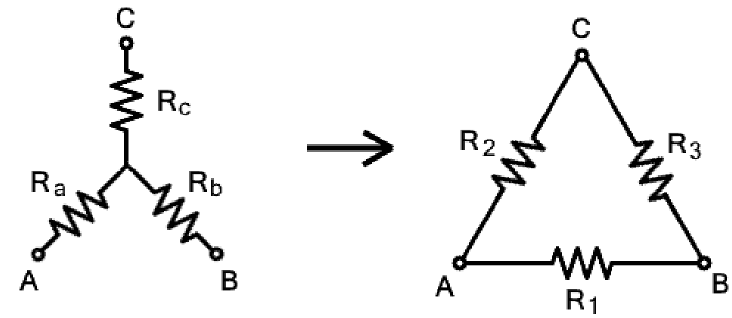
Star to Delta Conversion

Now we are going to convert the star connected impedance into delta connected impedance. Let's derive the equations used for a star to delta conversion.



Star to Delta Conversion

The given figure shows star connected impedance R_A, R_B & R_C. While the required delta equivalent impedance is R_1, R_2 & R_3 as shown in the figure.

In order to find the equivalent delta resistance, multiply the previous equation (v) & (vi), as well as (vi) & (vii) & (v) & (vii) together.

Multiplying (v) & (vi).

$$R_A R_B = \frac{R_1 R_2}{R_1 + R_2 + R_3} \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_A R_B = \frac{R_1^2 R_2 R_3}{(R_1 + R_2 + R_3)^2} \cdots\cdots\cdots (viii)$$

Similarly multiplying (vi) with (vii) & (v) with (vii).

$$R_B R_C = \frac{R_1 R_2 R_3^2}{(R_1 + R_2 + R_3)^2} \cdots\cdots\cdots (ix)$$

$$R_A R_C = \frac{R_1 R_2^2 R_3}{(R_1 + R_2 + R_3)^2} \cdots\cdots\cdots (x)$$

Now add equation (viii), (ix) & (x) together.

$$R_AR_B + R_BR_C + R_AR_C = \frac{R_1^2R_2R_3}{(R_1+R_2+R_3)^2} + \frac{R_1R_2R_3^2}{(R_1+R_2+R_3)^2} + \frac{R_1R_2^2R_3}{(R_1+R_2+R_3)^2}$$

$$R_AR_B + R_BR_C + R_AR_C = \frac{R_1^2R_2R_3 + R_1R_2R_3^2 + R_1R_2^2R_3}{(R_1+R_2+R_3)^2}$$

$$R_AR_B + R_BR_C + R_AR_C = \frac{R_1R_2R_3(R_1+R_2+R_3)}{(R_1+R_2+R_3)^2}$$

$$R_AR_B + R_BR_C + R_AR_C = \frac{R_1R_2R_3}{(R_1+R_2+R_3)} \cdots\cdots\cdots (xi)$$

In order to get the individual equivalent delta impedance, we divide equation (xi) with (v), (vi) & (vii) separately such as.

$$\frac{R_AR_B + R_BR_C + R_AR_C}{R_AR_B} = \frac{R_1R_2R_3}{(R_1+R_2+R_3)} \div \frac{R_1R_2}{(R_1+R_2+R_3)}$$

$$\frac{R_AR_B + R_BR_C + R_AR_C}{R_A} = \frac{R_1R_2R_3}{(R_1+R_2+R_3)} \times \frac{(R_1+R_2+R_3)}{R_1R_2}$$

$$\frac{R_AR_B + R_BR_C + R_AR_C}{R_A} = R_3$$

$$R_3 = \frac{R_AR_B + R_BR_C + R_AR_C}{R_A}$$

Dividing (xi) with (v).

Similarly dividing equation (xi) with (vi) & (vii) separately results in.

$$R_2 = \frac{R_A R_B + R_B R_C + R_A R_C}{R_B}$$

$$R_1 = \frac{R_A R_B + R_B R_C + R_A R_C}{R_C}$$

The relation between star to delta equivalent impedance is clear from the given equation. The sum of the two-product of all star-impedances divide by the star impedance of the corresponding terminal is equal to the delta impedance connected with the opposite terminal.

$$R_1 = \frac{R_A R_B}{R_C} + R_B + R_A$$

$$R_2 = \frac{R_A R_C}{R_B} + R_A + R_C$$

$$R_3 = \frac{R_B R_C}{R_A} + R_B + R_C$$

Simplifying the equations will lead toIn case all the star impedances are equal; the equivalent delta impedance would be;

Using the previous equation,

$$R_{star} = \frac{R_{delta}}{3}$$

$$R_{delta} = 3R_{star}$$

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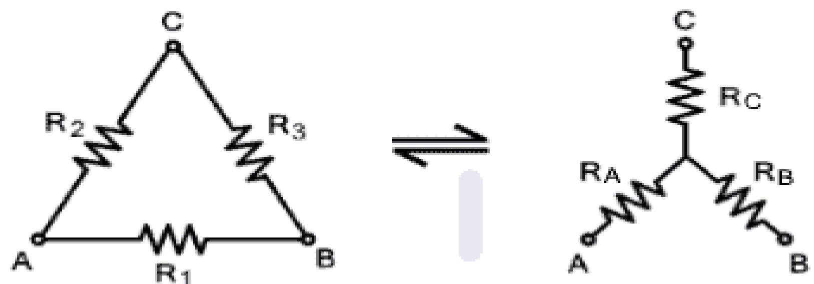
This equation suggests that each equivalent delta impedance is equal to 3 times the star impedance. See the table below



Delta	Star
$R_1 = \dfrac{R_A R_B + R_B R_C + R_A R_C}{R_C}$	$R_A = \dfrac{R_1 R_2}{R_1 + R_2 + R_3}$
$R_2 = \dfrac{R_A R_B + R_B R_C + R_A R_C}{R_B}$	$R_B = \dfrac{R_1 R_3}{R_1 + R_2 + R_3}$
$R_3 = \dfrac{R_A R_B + R_B R_C + R_A R_C}{R_A}$	$R_C = \dfrac{R_2 R_3}{R_1 + R_2 + R_3}$
If, $R_A = R_B = R_C = R_{star}$ $R_{delta} = 3R_{star}$	If, $R_1 = R_2 = R_3 = R_{delta}$ $R_{star} = \dfrac{R_{delta}}{3}$