Lecture 7 - Flexure

Lecture Goals

- Basic Concepts
- Rectangular Beams
- Non-uniform beams
- Safety factors
- Loading and Resistance
- Balanced Beams

Flexural Stress Example

Example of rectangular reinforced concrete beam.

Given a rectangular beam

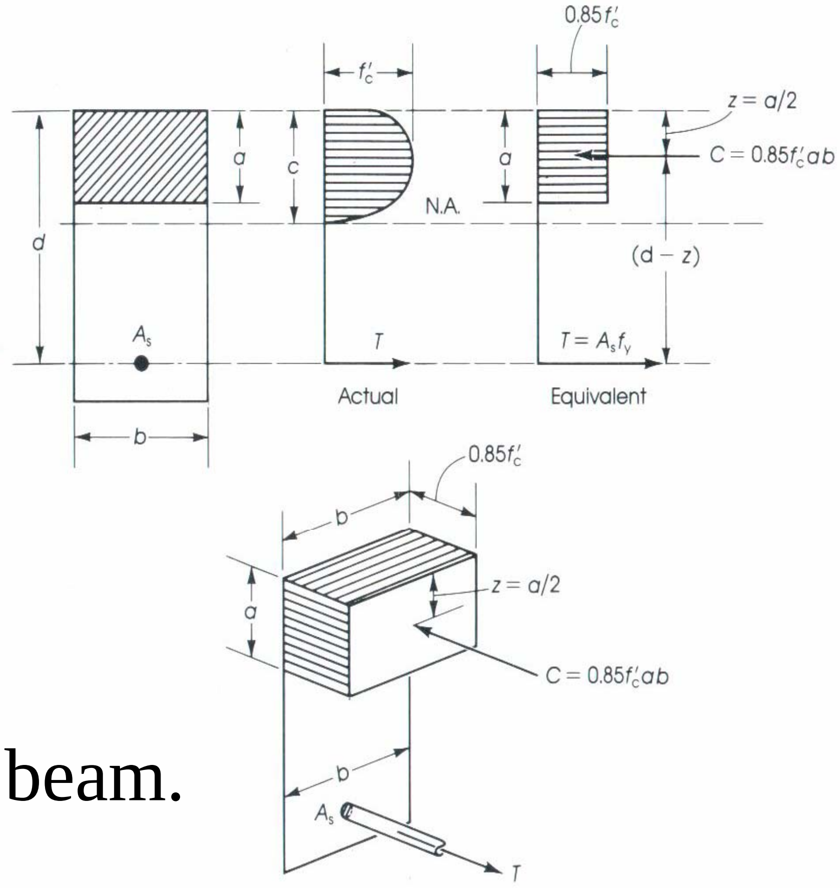
f_c = 4000 psi

f_y = 60 ksi   (4 #7 bars)

b = 12 in. d = 15.5 in. h= 18 in.

Find the neutral axis.

Find the moment capacity of the beam.



Example

Determine the area of steel, #7 bar has 0.6 in^2.

$$A_s = 4\left(0.6\ \text{in}^2\right) = 2.4\ \text{in}^2$$

The β value is β_1 = 0.85 because the concrete has a f_c =4000 psi.

Example

From equilibrium

$$C = T$$

$$0.85 f_c b a = f_y A_s$$

$$a = \frac{f_y A_s}{0.85 f_c b} = \frac{(60\text{ ksi})(2.4\text{ in}^2)}{0.85(4\text{ ksi})(12\text{ in})} = 3.53\text{ in.}$$

The neutral axis is

$$c = \frac{a}{\beta_1} = \frac{3.53\text{ in.}}{0.85} = 4.152\text{ in.}$$

Example

Check to see whether or not the steel has yielded.

$$\varepsilon_{\text{y}} = \frac{f_{\text{y}}}{\text{E}_{\text{s}}} = \frac{60 \text{ ksi}}{29000 \text{ ksi}} = 0.00207$$

Check the strain in the steel

$$\varepsilon_{\text{s}} = \left(\frac{d - c}{c}\right)(0.003)$$

Steel yielded!

$$= \left(\frac{15.5 \text{ in.} - 4.152 \text{ in.}}{4.152 \text{ in.}}\right)(0.003) = 0.0082 > 0.000207$$

Example

Compute moment capacity of the beam.

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

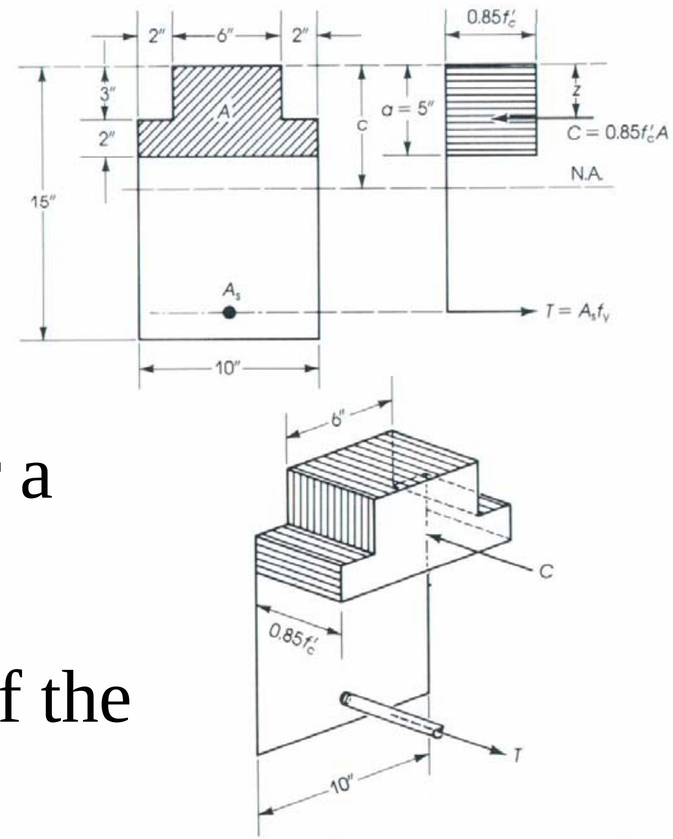
$$= \left(2.4 \text{ in}^2\right)\left(60 \text{ ksi}\right)\left(15.5 \text{ in.} - \frac{3.53 \text{ in.}}{2}\right)$$

$$= 1979 \text{ k-in.} \Rightarrow 164.8 \text{ k-ft.}$$

Flexural Stress - Example

For a non-rectangular beam

For the given beam with concrete rated at f_c = 6000 psi and the steel is rated at f_s = 60,000 psi. d = 12.5 in.



(a) Determine the area of the steel for a balanced system.

(b) Determine the moment capacity of the beam. M_n

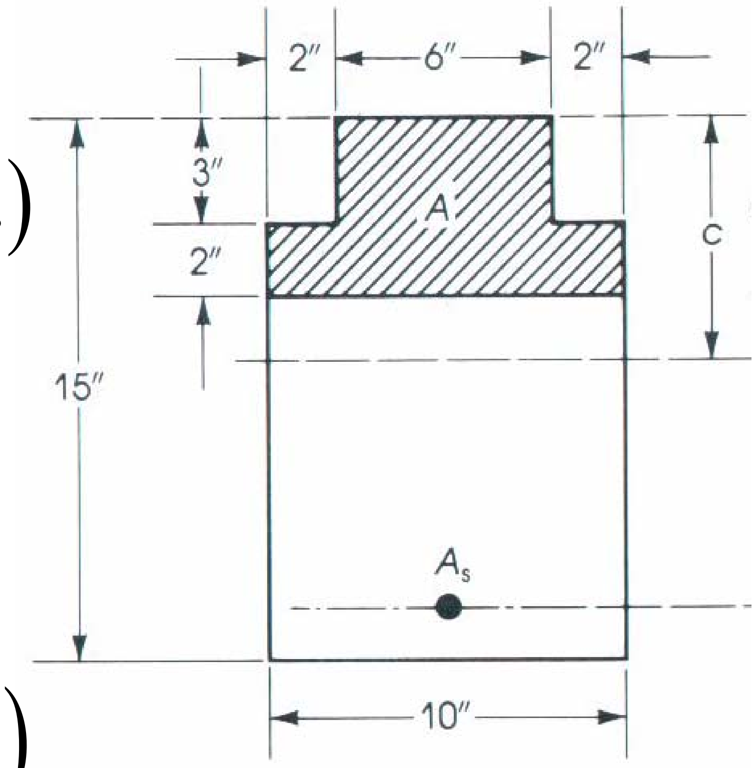
(c) Determine the NA.

Flexural Stress - Example

For a non-rectangular beam

The area of the concrete section is

$$A_c = (6 \text{ in.})(3 \text{ in.}) + (10 \text{ in.})(2 \text{ in.})$$
$$= 38 \text{ in}^2$$



The force due to concrete forces.

$$C = 0.85 f_c A_c$$
$$= 0.85(6000 \text{ psi})(38 \text{ in}^2)$$
$$= 193,000 \text{ lb.}$$

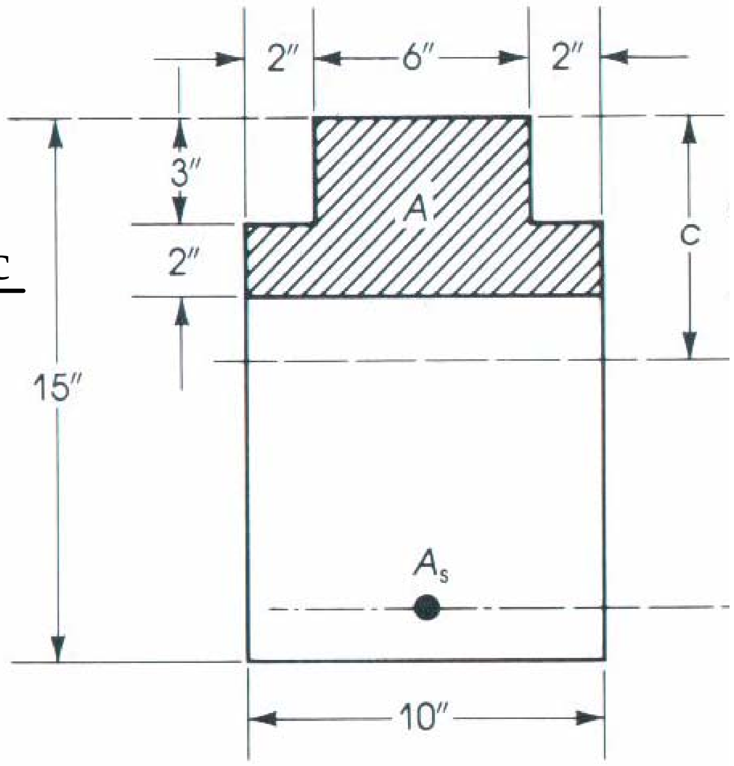
Flexural Stress - Example

Using equilibrium, the area of the steel can be found

$$T = C$$

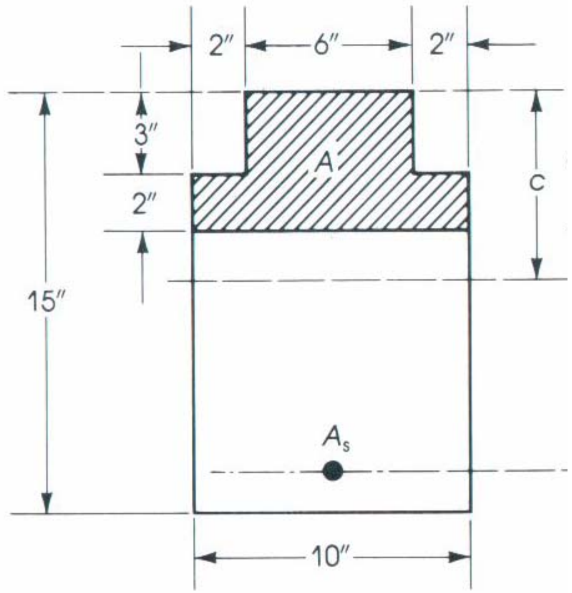
$$f_s A_s = 0.85 f_c A_c \Rightarrow A_s = \frac{0.85 f_c A_c}{f_s}$$

$$A_s = \frac{193800 \text{ lb}}{60000 \text{ psi}} = 3.23 \text{ in}^2$$



Flexural Stress - Example

Find the center of the area of concrete area



$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i}$$

$$= \frac{(6\text{ in.})(3\text{ in.})(1.5\text{ in.}) + (10\text{ in.})(2\text{ in.})(4\text{ in.})}{(6\text{ in.})(3\text{ in.}) + (10\text{ in.})(2\text{ in.})}$$

$$= 2.8158\text{ in.}$$

Flexural Stress - Example

The moment capacity of the beam is

$$M_{\mathrm{n}} = T\left(d - \bar{y}\right)$$

$$= 193800 \text{ lb.}\left(12.5 \text{ in. } - 2.8158 \text{ in.}\right)\left(\frac{1 \text{ kip}}{1000 \text{ lb}}\right)$$

$$= 1869 \text{ k-in.} \Rightarrow 155.75 \text{ k-ft.}$$

Flexural Stress - Example

Compute the β_1 value

$$\beta_1 = 0.85 - 0.05 * \left(\frac{f_c - 4000 \text{ psi}}{1000 \text{ psi}} \right)$$

$$= 0.85 - 0.05 * \left(\frac{6000 \text{ psi} - 4000 \text{ psi}}{1000 \text{ psi}} \right)$$

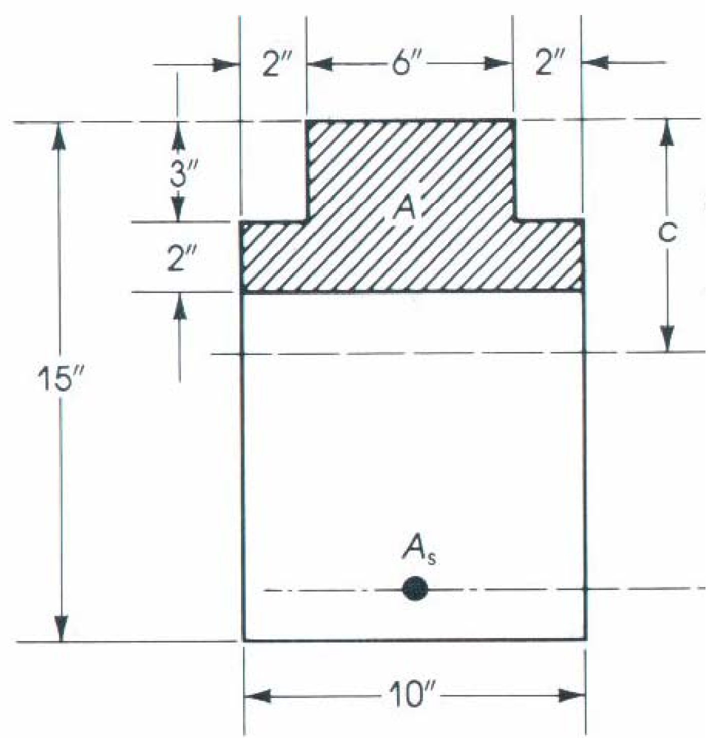
$$= 0.75$$

Flexural Stress - Example

Find the neutral axis

$$c = \frac{a}{\beta_1}$$

$$= \frac{5.0 \text{ in.}}{0.75} = 6.67 \text{ in.}$$



Safety Provisions

Structures and structural members must always be designed to carry some reserve load above what is expected under normal use.

Safety Provisions

There are three main reasons why some sort of safety factor are necessary in structural design.

[1] Consequences of failure.

[2] Variability in loading.

[3] Variability in resistance.

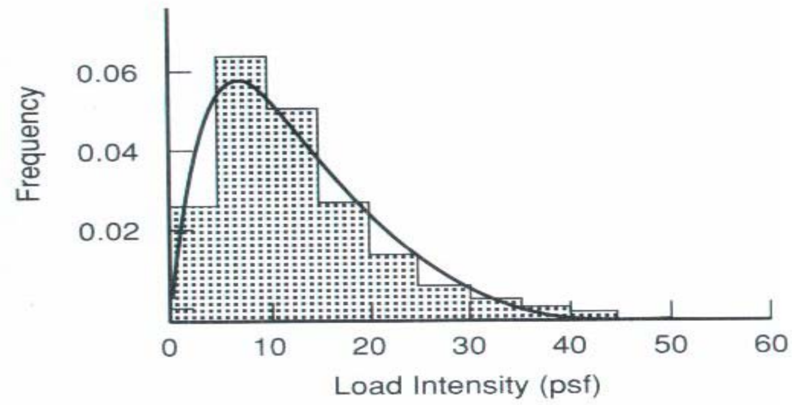
Consequences of Failure

A number of subjective factors must be considered in determining an acceptable level of safety.

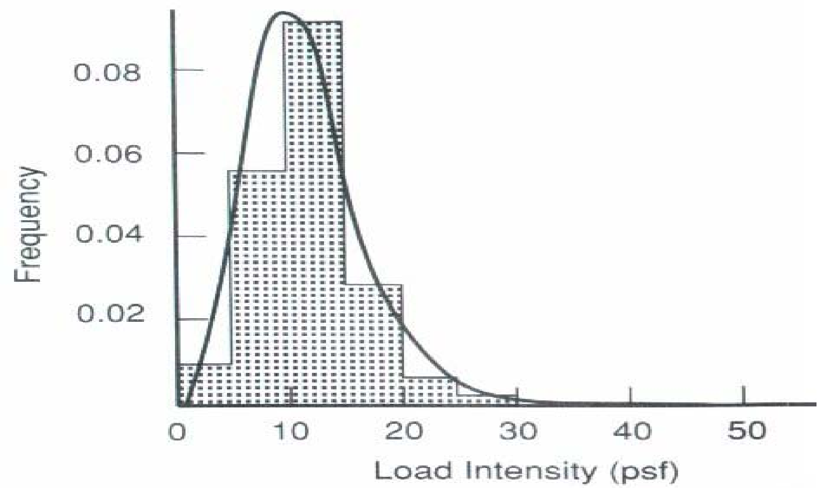
- Potential loss of life.
- Cost of clearing the debris and replacement of the structure and its contents.
- Cost to society.
- Type of failure warning of failure, existence of alternative load paths.

Variability in Loading

Frequency distribution of sustained component of live loads in offices.



(a) Area = 151 ft^2



(b) Area = 2069 ft^2

Variability in Resistance

- Variability of the strengths of concrete and reinforcement.
- Differences between the as-built dimensions and those found in structural drawings.
- Effects of simplification made in the derivation of the members resistance.

Variability in Resistance

Comparison of measured and computed failure moments based on all data for reinforced concrete beams with $f_c > 2000$ psi.