



1) Find the domain and range of $y = \frac{2x}{x-1}$

Solution//

$x-1=0 \rightarrow x=1 \rightarrow$ domain function = $\mathbb{R}$ except $\{1\}$

$$y = \frac{2x}{x-1} \rightarrow yx - y = 2x$$

$$x = \frac{y}{y-2}$$

$$y-2=0 \rightarrow y = 2$$

Range function = $\mathbb{R}$ except $\{2\}$

2) Find the domain and range of $y = \sqrt{\frac{x-1}{x+2}}$

Solution/

$$x-1 \geq 0 \rightarrow x \geq 1$$

$$x+2 \geq 0 \rightarrow x > -2$$

Domain function = $\{x: x \geq 1\} \cap \{x: x > -2\}$

$$y = \sqrt{\frac{x-1}{x+2}}$$

$$Y^2 = \frac{x-1}{x+2} \rightarrow xy^2 + 2y^2 = x - 1$$

$$\rightarrow xy^2 - x = -1 - 2y^2$$

$$\rightarrow x = \frac{-1 - 2y^2}{y^2 - 1} \rightarrow y^2 - 1 = 0 \rightarrow y = \pm 1$$

Range function = $\mathbb{R}$ except $\{+1, -1\}$



3) Given $f(x)=(x-2)^3+1$ find $f^{-1}(x)$

Solution//

$$y=(x-2)^3+1$$

$$x=(y-2)^3+1$$

$$x=(y-2)^3+1$$

$$x-1=(y-2)^3$$

$$(x-1)^{1/3}=y-2$$

$$(x-1)^{1/3}+2=y$$

$$f^{-1}(x)=(x-1)^{1/3}+2$$

4) Given $f(x)=5-9x$ find $f^{-1}(x)$

Solution//

$$y=5-9x$$

$$x=5-9y$$

$$9y=5-x$$

$$y=(5-x)/9$$

$$f^{-1}(x)=(5-x)/9$$

Example5: Evaluate the following limits if they exist.

$$1) \lim_{x \rightarrow -1} \frac{\sqrt{2+x}-1}{x+1}, x \neq -1, x \neq -2$$

Sol:

$$\lim_{x \rightarrow -1} \frac{\sqrt{2+x}-1}{x+1} * \frac{\sqrt{2+x}+1}{\sqrt{2+x}+1} = \lim_{x \rightarrow -1} \frac{2+x-1}{x+1(\sqrt{2+x}+1)}$$

$$\lim_{x \rightarrow -1} \frac{x+1}{x+1(\sqrt{2+x}+1)} = \lim_{x \rightarrow -1} \frac{1}{(\sqrt{2+x}+1)} = \frac{1}{(\sqrt{2-1}+1)} = \frac{1}{2}$$