

Limitations on Reinforcement for Flange Beams

- Lower Limits
 - Flange in compression

$$\rho_{\min} = \frac{A_s}{b_w d} = \text{ larger of } \left\{ \begin{array}{l} \frac{3\sqrt{f'_c}}{f_y} \\ \frac{200}{f_y} \end{array} \right.$$

Limitations on Reinforcement for Flange Beams

- Lower Limits

- Flange in tension

$$A_{s(\min)} = \text{smaller of } \left\{ \begin{array}{l} \frac{6\sqrt{f'_c}}{f_y} b_w d \\ \text{larger of } \left\{ \begin{array}{l} \frac{3\sqrt{f'_c}}{f_y} b_{\text{eff}} d \\ \frac{200}{f_y} b_{\text{eff}} d \end{array} \right. \end{array} \right.$$

Limitations on Reinforcement for Flange Beams

- **Lower Limits**

- If $A_{s(\text{provided})} \geq 4/3 A_{s(\text{req'd})}$ based on analysis

then $A_{s(\min)}$ is not required (i.e.)

$$\phi M_n \geq 4/3 M_u \text{ for } A_{s(\text{provided})}$$

See ACI 10.5.3

Example - T-Beam

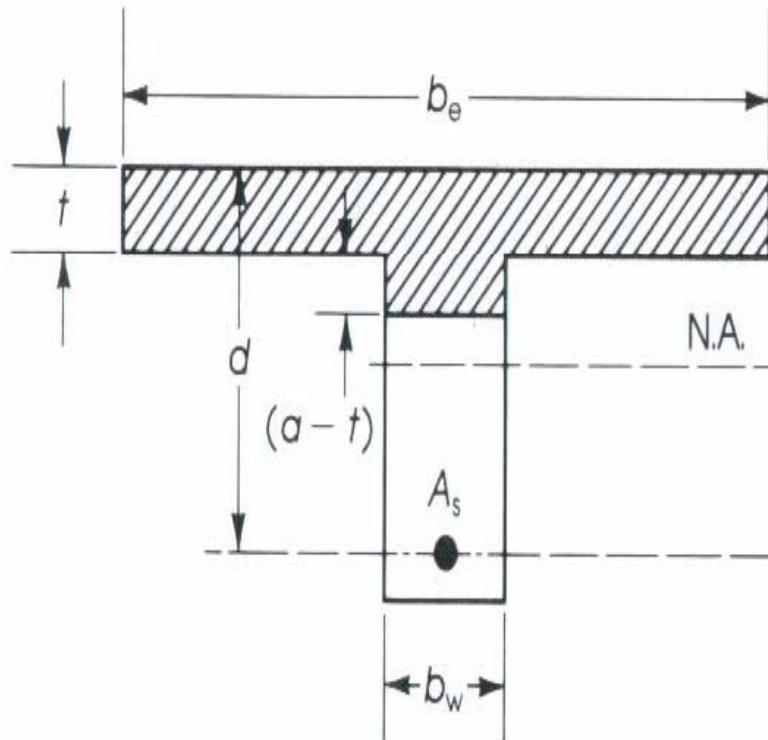
Find M_n and M_u for T-Beam.

$$b_{\text{eff}} = 54 \text{ in. } h_f = 3 \text{ in. } b = 7 \text{ ft.}$$

$$d = 16.5 \text{ in. } A_s = 8.5 \text{ in}^2$$

$$f_y = 50 \text{ ksi } f_c = 3 \text{ ksi}$$

$$b_w = 12 \text{ in. } L = 18 \text{ ft}$$



Example of L-Beam

Confirm b_{eff}

$$b_{\text{eff}} \leq \frac{L}{4} = \frac{18 \text{ ft} \left(\frac{12 \text{ in.}}{\text{ft.}} \right)}{4} = 54 \text{ in.}$$

$$\leq 16h_f + b_w = 16(3 \text{ in.}) + 12 \text{ in.} = 60 \text{ in.}$$

$$\leq b = 7 \text{ ft.} \left(\frac{12 \text{ in.}}{\text{ft.}} \right) = 84 \text{ in.}$$

Example - T-Beam

Compute the equivalent c value and check the strain in the steel, ε_s

$$a = \frac{A_s f_y}{0.85 f'_c b_{\text{eff}}} = \frac{(8.5 \text{ in}^2)(50 \text{ ksi})}{0.85(3 \text{ ksi})(54 \text{ in.})} = 3.09 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{3.09 \text{ in}}{0.85} = 3.63 \text{ in.}$$

$$\varepsilon_s = \left(\frac{d}{c} - 1 \right) 0.003 = \left(\frac{16.5 \text{ in.}}{3.63 \text{ in.}} - 1 \right) 0.003 = 0.0106 > 0.005$$

Steel will yield in the tension zone.

Example - T-Beam

Compute the reinforcement ρ and check to make sure it is greater than ρ_{\min}

$$\rho = \frac{A_s}{b_w d} = \frac{(8.5 \text{ in}^2)}{(12 \text{ in.})(16.5 \text{ in.})} = 0.0429$$

$$\rho_{\min} = \begin{cases} \frac{200}{f_y} = \frac{200}{50000} = 0.004 \\ \frac{3\sqrt{f_c}}{f_y} = \frac{3\sqrt{3000}}{50000} = 0.00329 \end{cases} \Rightarrow \rho_{\min} = 0.004$$

0.0429 > 0.004

Section works for minimum reinforcement.

Example - T-Beam

Compute ω and check that the c value is greater than h_f

$$\varpi = \rho \frac{f_y}{f'_c} = 0.0429 \left(\frac{50 \text{ ksi}}{3 \text{ ksi}} \right) = 0.7155$$

$$h_f \leq \frac{1.18\varpi d}{\beta_1} \Rightarrow 3 \text{ in.} \leq \frac{1.18(0.7155)(16.5 \text{ in.})}{0.85} = 16.388$$

$$h_f \leq a \Rightarrow 3 \text{ in.} \leq 3.09 \text{ in.}$$

Analysis the beam as a T-beam.

Example - T-Beam

Compute ω and check that the c value is greater than h_f

$$A_{sf} = \frac{0.85 f'_c (b_{\text{eff}} - b_w) h_f}{f_y} = \frac{0.85 (3 \text{ ksi}) (54 \text{ in.} - 12 \text{ in.}) (3 \text{ in.})}{(50 \text{ ksi})}$$
$$= 6.426 \text{ in}^2$$

Compute a

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c b_w} = \frac{(8.5 \text{ in}^2 - 6.426 \text{ in}^2)(50 \text{ ksi})}{0.85 (3 \text{ ksi})(12 \text{ in.})}$$
$$= 3.889 \text{ in.}$$

Example - T-Beam

Compute nominal moment components

$$\begin{aligned}M_{n1} &= (A_s - A_{sf}) f_y \left(d - \frac{a}{2} \right) \\&= (8.5 \text{ in}^2 - 6.426 \text{ in}^2)(50 \text{ ksi}) \left(16.5 \text{ in.} - \frac{3.889 \text{ in.}}{2} \right) \\&= 1535.34 \text{ k-in.}\end{aligned}$$

$$\begin{aligned}M_{n2} &= A_{sf} f_y \left(d - \frac{h_f}{2} \right) = (6.426 \text{ in}^2)(50 \text{ ksi}) \left(16.5 \text{ in.} - \frac{3 \text{ in.}}{2} \right) \\&= 4819.5 \text{ k-in.}\end{aligned}$$

Example - T-Beam

Compute nominal moment

$$\begin{aligned}M_n &= M_{n1} + M_{n2} \\&= 1535.34 \text{ k-in.} + 4819.5 \text{ k-in.} \\&= 6354.84 \text{ k-in.} \Rightarrow 529.57 \text{ k-ft.}\end{aligned}$$

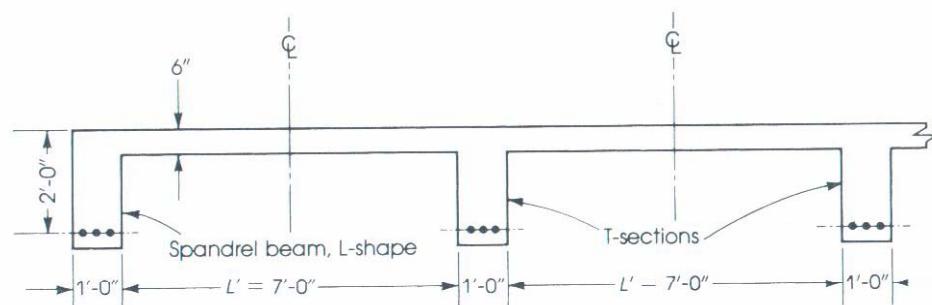
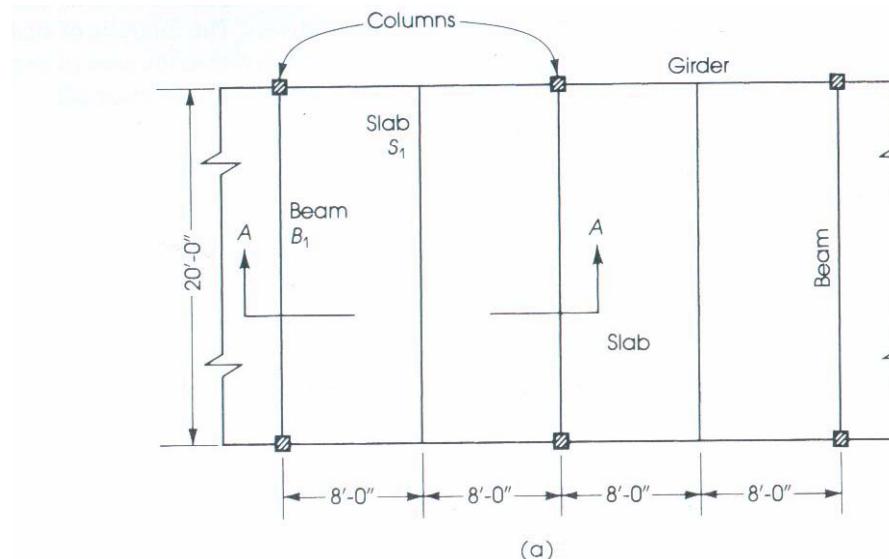
Compute ultimate moment

$$\begin{aligned}M_u &= \phi M_n = 0.9(529.57 \text{ k-ft.}) \\&= 416.6 \text{ k-ft.}\end{aligned}$$

Example of L-Beam

Determine the effective b for the spandrel beam and do the analysis.

Use 4 #9 bars and find the ultimate moment capacity.
 $f_y = 50 \text{ ksi}$, $f_c = 3 \text{ ksi}$



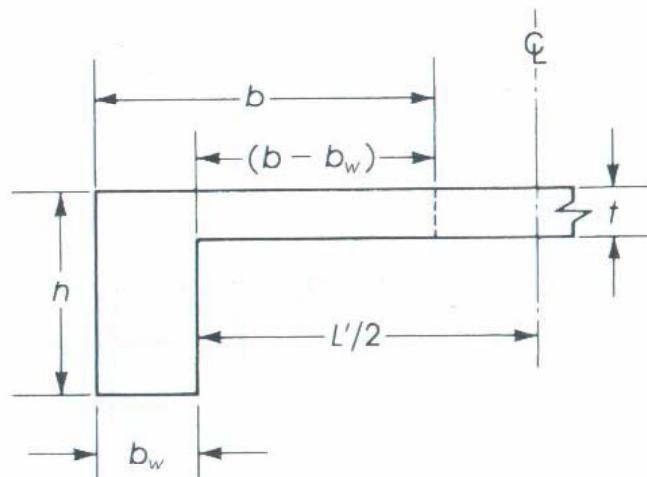
Example of L-Beam

Compute b_{eff}

$$b_{\text{eff}} \leq \frac{L}{12} + b_w$$

$$\leq 6h_f + b_w$$

$$\leq b_{\text{actual}} = b_w + 0.5 * (\text{clear distance to next web})$$



Example of L-Beam

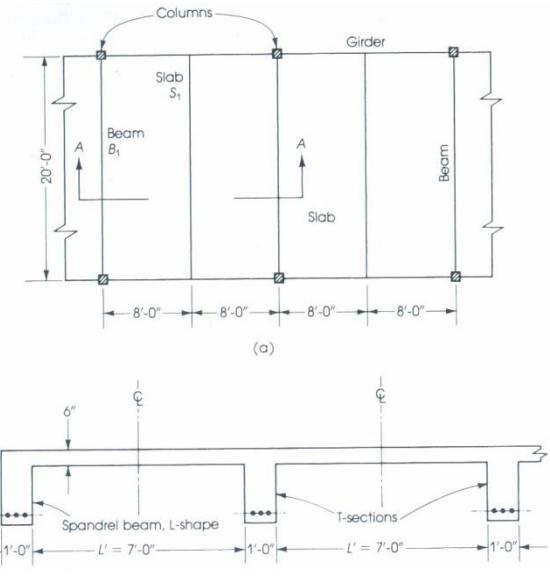
Compute b_{eff}

$$b_{\text{eff}} \leq \frac{L}{12} + b_w = \frac{20 \text{ ft} \left(\frac{12 \text{ in.}}{\text{ft}} \right)}{12} + 12 \text{ in.} = 32 \text{ in.}$$

$$\leq 6h_f + b_w = 6(6 \text{ in.}) + 12 \text{ in.} = 48 \text{ in.}$$

$$\leq b_{\text{actual}} = b_w + 0.5 * (\text{clear distance to next web})$$

$$= 12 \text{ in.} + 0.5 * \left(7 \text{ ft} \left(\frac{12 \text{ in.}}{\text{ft}} \right) \right) = 54 \text{ in.}$$

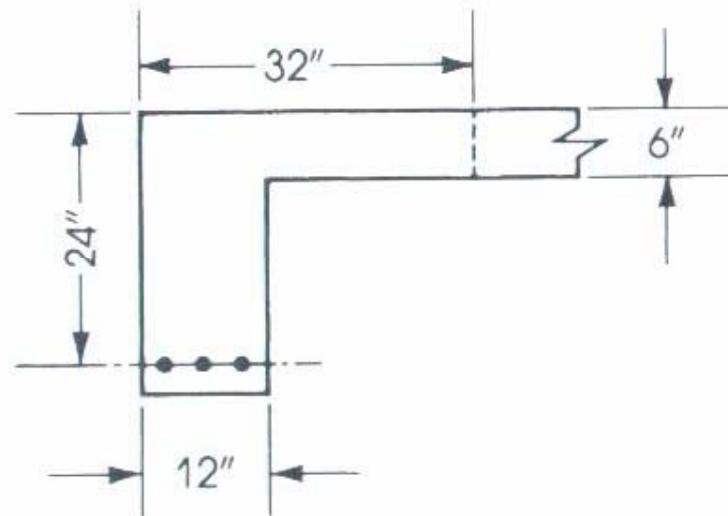


Example of L-Beam

The value b_{eff} and A_s

$$b_{\text{eff}} = 32 \text{ in.}$$

$$A_s = 4(1.0 \text{ in}^2) = 4.0 \text{ in}^2$$



Example - L-Beam

Compute the equivalent c value and check the strain in the steel, ε_s

$$a = \frac{A_s f_y}{0.85 f'_c b_{\text{eff}}} = \frac{(4.0 \text{ in}^2)(50 \text{ ksi})}{0.85(3 \text{ ksi})(32 \text{ in.})} = 2.45 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{2.45 \text{ in}}{0.85} = 2.88 \text{ in.}$$

$$\varepsilon_s = \left(\frac{d}{c} - 1 \right) 0.003 = \left(\frac{24 \text{ in.}}{2.88 \text{ in.}} - 1 \right) 0.003 = 0.0220 > 0.005$$

Steel will yield in the tension zone.

Example - L-Beam

Compute the reinforcement ρ and check to make sure it is greater than ρ_{\min}

$$\rho = \frac{A_s}{b_w d} = \frac{(4.0 \text{ in}^2)}{(12 \text{ in.})(24 \text{ in.})} = 0.0139$$

$$\rho_{\min} = \begin{cases} \frac{200}{f_y} = \frac{200}{50000} = 0.004 \\ \frac{3\sqrt{f_c}}{f_y} = \frac{3\sqrt{3000}}{50000} = 0.00329 \end{cases} \Rightarrow \rho_{\min} = 0.004$$

0.0139 > 0.004

Section works for minimum reinforcement.

Example - T-Beam

Compute ω and check that the c value is greater than h_f

$$\varpi = \rho \frac{f_y}{f'_c} = 0.0139 \left(\frac{50 \text{ ksi}}{3 \text{ ksi}} \right) = 0.2315$$

$$h_f \leq \frac{1.18\varpi d}{\beta_1} \Rightarrow 6 \text{ in.} \leq \frac{1.18(0.2315)(24 \text{ in.})}{0.85} = 7.71 \text{ in.}$$

$$h_f \leq a \Rightarrow 6 \text{ in.} \leq 2.45 \text{ in.}$$

False!

Analysis the beam as a Singly reinforced beam.