Matrices :- المصفوفات

A rectangular array of numbers or symbols with each element being distinct and seperate (m rows, n columns) is called an $m \times n$ matrix when certain laws of combination, yet to be specified, are laid down.

Example :-

$$y_i = \sum_{j=1}^{n} a_{ij} x_j \qquad \text{where } i = 1, 2, \ldots, m$$

Solution

$$y_1 = a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n$$

$$\vdots$$

$$y_m = a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n$$

This can be written as a matrix

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}}_{[Y]} = \underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}}_{[A]} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{[X]}$$

$$[Y] = [A][X]$$

Email (hussein.kadhim@uomus.edu.iq)

Special Matrices :-

① **Square matrix :-**

$$[A] = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{nn} \end{bmatrix} \Longrightarrow (m = n)$$

② **Diagonal matrix :-**

$$[A] = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & a_{33} & \\ 0 & 0 & 0 & a_{nn} \end{bmatrix}$$

③ **Unit matrix :-** It may called by <u>Identity matrix</u>

$$[I] = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

④ **Upper triangular matrix :-**

$$[U] = \begin{bmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ 0 & u_{22} & u_{23} & \cdots & u_{2n} \\ 0 & 0 & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & u_{nn} \end{bmatrix}$$

⑤ **Upper unit triangular matrix :-**

$[I] = [U]$ with diagonal elements equal to 1.

(6) Lower triangular matrix :-

[L] = all elements above the diagonal are zero

(7) Inverse of Square matrix [A] :-

Defined by:-

(Identity or unit matrix)

$$[A]^{-1}[A] = [I] \qquad a^{-1}a = \frac{\cancel{a}}{\cancel{a}} = 1$$

Where,

$[A]^{-1}$ is called A inverse

(8) Tridiagonal matrix :- مصفوفة قطرية

$$\begin{bmatrix} b_1 & c_1 & & & \text{Zero} \\ a_2 & b_2 & c_2 & & \\ & a_3 & b_3 & c_3 & \\ & & a_{n-1} & b_{n-1} & c_{n-1} \\ \text{Zero} & & & a_n & b_n \end{bmatrix}$$

(9) Transpose of a matrix :-

A matrix form when columns & rows are interchange, designated as A^T or $[A]^T$.

$$[A] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \rightarrow [A]^T = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

Email (hussein.kadhim@uomus.edu.iq)

(10) Elemantry matrix operations :-

a- Two matrices are equal

$[A] = [B]$ if $a_{ij} = b_{ij}$, $\begin{cases} i = 1,2,\dots,m \\ j = 1,2,\dots,n \end{cases}$

b- Matrix addition :-

$[A] + [B] = [C]$ if $c_{ij} = a_{ij} + b_{ij}$

c- Matrix multiplication :-

$[C] = [A][B]$ if $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$

$\begin{cases} i = 1,2,\dots,m \\ k = 1,2,\dots,q \end{cases}$

Example :-

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} \underbrace{(a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31})}_{c_{11}} & \underbrace{(a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32})}_{c_{12}} \\ \underbrace{(a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31})}_{c_{21}} & \underbrace{(a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32})}_{c_{22}} \end{bmatrix}$$

$$= \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

Matrix properties :-

1- Matrix multiplication is associative

$[A]\{[B][C]\} = \{[A][B]\}[C]$

2- Not Commutative

$[A][B] \neq [B][A]$

But there is exception $\Rightarrow [I][A] = [A][I] = [A]$

3- Is distributive $\Rightarrow [A]\{[B]+[C]\} = [A][B] + [A][C]$