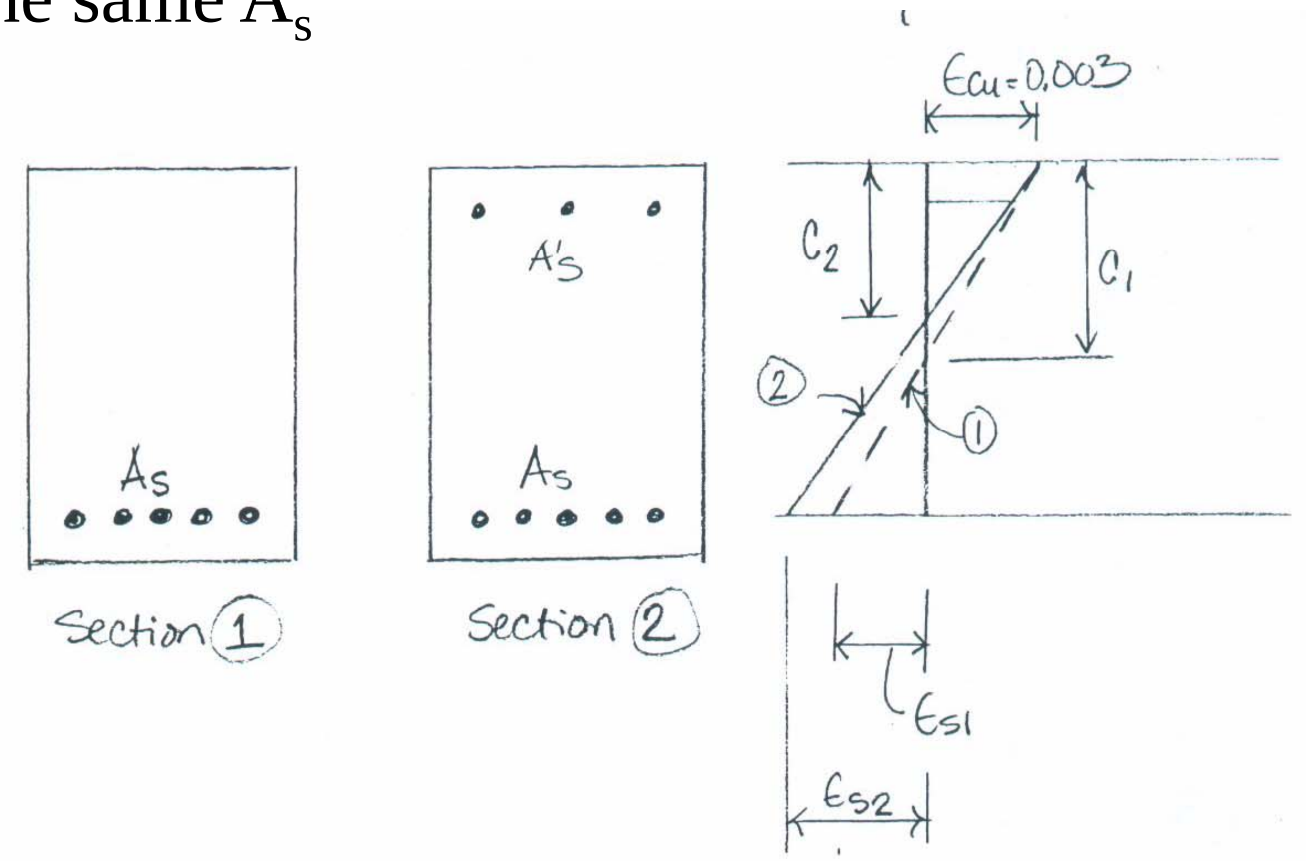
Effect of Compression Reinforcement

Compare the strain distribution in two beams with the same A_s



Effect of Compression Reinforcement

Section 1:

$$T = A_s f_s$$

$$T = C_{c1} = 0.85 f_c' ba = 0.85 f_c' b \beta_1 c_1$$

$$c_1 = \frac{A_s f_s}{0.85 f_c' b \beta_1}$$

Section 2:

$$T = A_s f_s$$

$$T = C_s' + C_{c1}$$

$$= A_s' f_s' + 0.85 f_c' b a_2$$

$$= A_s' f_s' + 0.85 f_c' b \beta_1 c_2$$

$$c_2 = \frac{A_s f_s - A_s' f_s'}{0.85 f_c' b \beta_1}$$

Addition of A'_s strengthens compression zone so that less concrete is needed to resist a given value of T. ⟶ NA goes up ($c_2 < c_1$) and ε_s increases ($\varepsilon_{s2} > \varepsilon_{s1}$).

Doubly Reinforced Beams

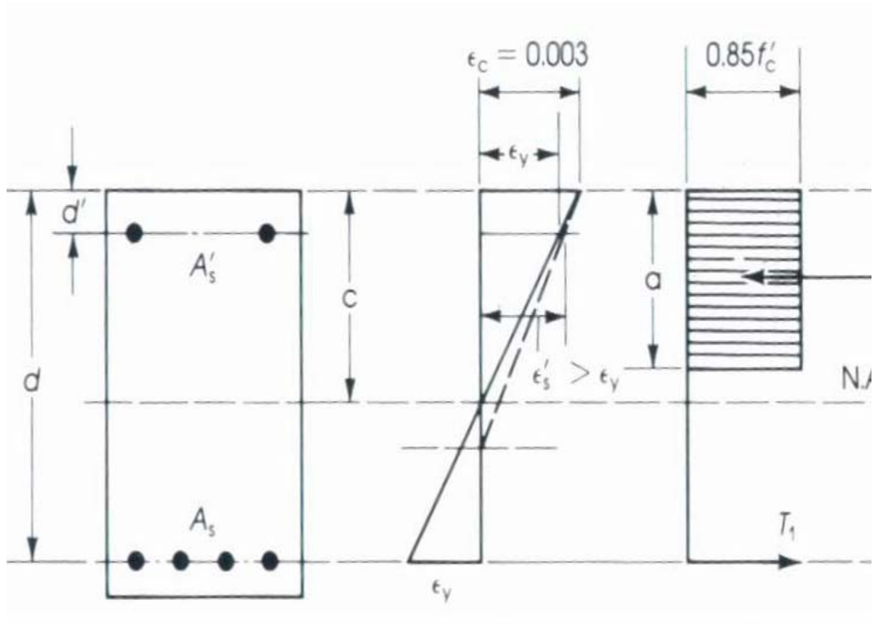
Four Possible Modes of Failure

- Under reinforced Failure
  - (Case 1) Compression and tension steel yields
  - (Case 2) Only tension steel yields
- Over reinforced Failure
  - (Case 3) Only compression steel yields
  - (Case 4) No yielding Concrete crushes

Analysis of Doubly Reinforced Rectangular Sections

<u>Strain Compatibility Check</u>

Assume ε_s’ using similar triangles



$$\frac{\varepsilon_s}{(c-d')} = \frac{0.003}{c} \Rightarrow$$

$$\varepsilon_s = \frac{(c-d')*0.003}{c}$$

Analysis of Doubly Reinforced Rectangular Sections

<u>Strain Compatibility</u>

Using equilibrium and find a

$$T = C_{\mathrm{c}}' + C_{\mathrm{s}}' \Rightarrow a = \frac{\left(A_{\mathrm{s}} - A_{\mathrm{s}}'\right) f_{\mathrm{y}}}{0.85 f_{\mathrm{c}}' b}$$

$$c = \frac{a}{\beta_1} = \frac{\left(A_{\mathrm{s}} - A_{\mathrm{s}}'\right) f_{\mathrm{y}}}{\beta_1 \left(0.85 f_{\mathrm{c}}' b\right)} = \frac{\left(\rho - \rho'\right) d \; f_{\mathrm{y}}}{\beta_1 \left(0.85 f_{\mathrm{c}}'\right)}$$

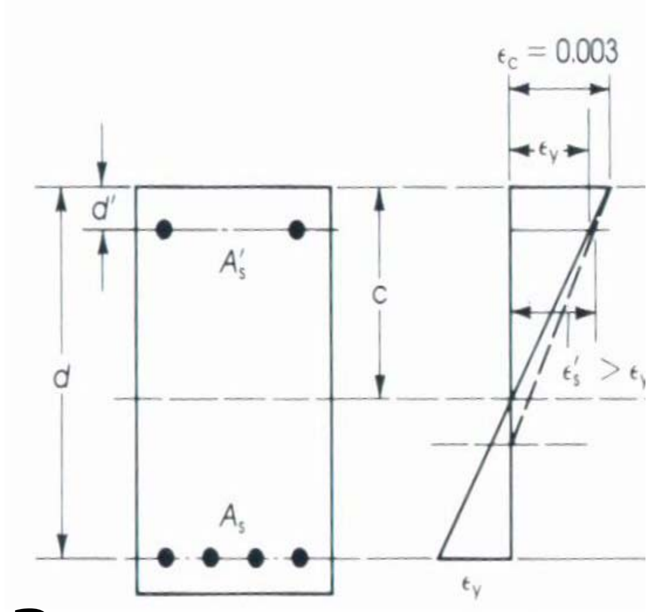
Analysis of Doubly Reinforced Rectangular Sections

Strain Compatibility

The strain in the compression steel is

$$\varepsilon_s' = \left(1 - \frac{d'}{c}\right)\varepsilon_{cu}$$

$$= \left(1 - \frac{\beta_1 \left(0.85 f_c'\right) d'}{\left(\rho - \rho'\right) d\ f_y}\right) 0.003$$

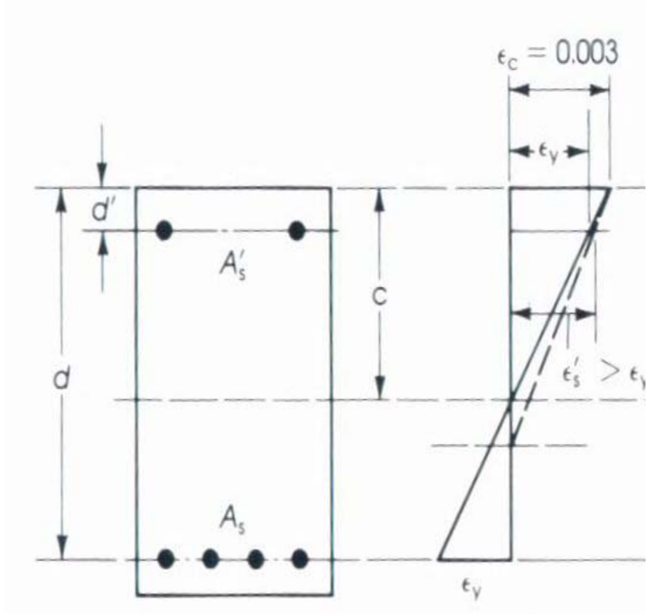


Analysis of Doubly Reinforced Rectangular Sections

Strain Compatibility

Confirm

$$\varepsilon_s' \geq \varepsilon_y = \frac{f_y}{E_s}; \qquad \varepsilon_s \geq \varepsilon_y$$



$$\varepsilon_s' = \left(1 - \frac{\beta_1 \left(0.85 f_c'\right) d'}{\left(\rho - \rho'\right) d\ f_y}\right) 0.003 \geq \frac{f_y}{E_s} = \frac{f_y}{29 \text{ x } 10^6}$$

Analysis of Doubly Reinforced Rectangular Sections

<u>Strain Compatibility</u>

Confirm

$$-\frac{\beta_1\left(0.85 f_c'\right)d'}{\left(\rho-\rho'\right)d\ f_y} \geq \frac{f_y - 87000}{87000}$$

$$\left(\rho-\rho'\right) \geq \left(\frac{\beta_1\left(0.85 f_c'\right)d'}{d\ f_y}\right)\left(\frac{87000}{87000 - f_y}\right)$$

Analysis of Doubly Reinforced Rectangular Sections

If the statement is true than

$$M_{\text{n}} = \left(A_{\text{s}} - A'_{\text{s}} \right) f_{\text{y}} \left(d - \frac{a}{2} \right) + A'_{\text{s}} f_{\text{y}} \left(d - d' \right)$$

else the strain in the compression steel

$$f_{\text{s}} = E\varepsilon_{\text{s}}$$

Analysis of Doubly Reinforced Rectangular Sections

Strain

Compute the stress in the compression steel.

$$f_s' = 29 \text{ x } 10^6 \left(1 - \frac{\beta_1 \left(0.85 f_c' \right) d'}{\left(\rho - \rho' \right) d \ f_y} \right) 0.003$$

Analysis of Doubly Reinforced Rectangular Sections

Go back and calculate the equilibrium with f_s'

$$T = C_c' + C_s' \Rightarrow a = \frac{\left(A_s f_y - A_s' f_s\right)}{0.85 f_c' b}$$

$$c = \frac{a}{\beta_1}$$

Iterate until the c value is adjusted for the f_s'

$$f_s' = \left(1 - \frac{d'}{c}\right) 87000$$

Analysis of Doubly Reinforced Rectangular Sections

Go back and calculate the moment capacity of the beam

$$M_n = \left(A_s f_y - A_s' f_s'\right)\left(d - \frac{a}{2}\right) + A_s' f_s'\left(d - d'\right)$$

Limitations on Reinforcement Ratio for Doubly Reinforced beams

Lower limit on ρ (ACI 10.5)
same as for single reinforce beams.

$$\rho_{\min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y}$$