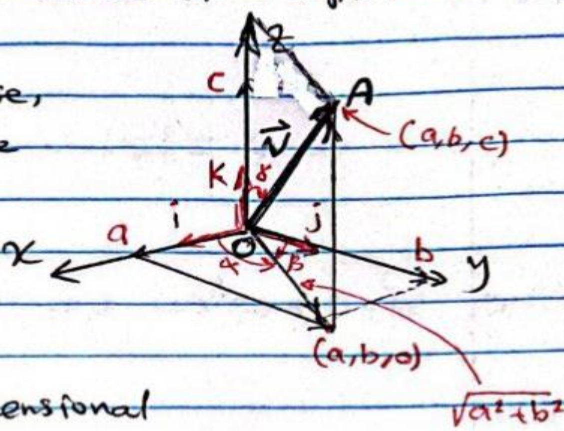
Vectors in a space :

- This could be in three or higher dimensions



- similar to the 2-D case, but we now have three basis vectors i, j & k.
- From these three components unit vectors we can describe any vector in three-dimensional space.

$$\vec{V} = \vec{OA} = ai + bj + ck ,$$

where :

i, j, k = Basis or Fundamental unit vector.
a, b, c = Directional numbers (scalars).
α, β, γ = Directional angles.

$$|V| = |OA| = \sqrt{a^2+b^2+c^2} ,$$

$a = |V| * \cos\alpha$
$b = |V| * \cos\beta$
$c = |V| * \cos\gamma$

$$\frac{\vec{V}}{|V|} = \cos\alpha\, i + \cos\beta\, j + \cos\gamma\, k$$ = unit vector with direction of $\vec{V}$

And,

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

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Ex) Find a vector in space of length (5 units) that makes angles ($70°$) with x-axis, ($85°$) with y-axis?

Solution

$\alpha = 70°$, $\beta = 85°$, $|V| = 5$

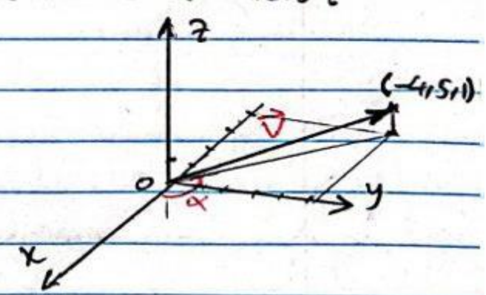
$\gamma = ?$, $\vec{V} = ?$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow \cos^2 70 + \cos^2 85 + \cos^2\gamma = 1$$

$$\therefore \boxed{\cos\gamma = 0.935}$$

$$\therefore \vec{V} = |V|(\cos\alpha\, i + \cos\beta\, j + \cos\gamma\, k)$$
$$= 5(\cos 70\, i + \cos 85\, j + 0.935\, k)$$
$$\boxed{\vec{V} = 1.7\, i + 0.436\, j + 4.675\, k}$$ Ans

Ex) Find the angle between the vector $\vec{V} = -4i + 5j + k$ and the x-axis?



Solution

$a = -4$, $b = 5$, $c = 1$

$$|V| = \sqrt{a^2 + b^2 + c^2}$$

$$|V| = \sqrt{(-4)^2 + (5)^2 + (1)^2} = \sqrt{42}$$

$$\cos\alpha = \frac{a}{|V|} \Rightarrow \alpha = \cos^{-1}\frac{a}{|V|} = \cos^{-1}\frac{-4}{\sqrt{42}}$$

$$\boxed{\alpha = 128°}$$

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Scalar product :- (Dot product)
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Let $\vec{A} = a_1 i + a_2 j + a_3 k$
And $\vec{B} = b_1 i + b_2 j + b_3 k$

Then, $\vec{A}\cdot\vec{B} = |A|\,|B| \cos\theta$

where θ — is the angle between $\vec{A}$ & $\vec{B}$

Properties :-

1- $\vec{A}\cdot\vec{A} = |A|^2$
2- $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A} = a_1 b_1 + a_2 b_2 + a_3 b_3$
3- $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|A|\,|B|} = \dfrac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2+a_2^2+a_3^2} + \sqrt{b_1^2+b_2^2+b_3^2}}$
4- $\vec{A} \perp \vec{B} \Rightarrow \vec{A}\cdot\vec{B} = 0$ [Orthogonal Vectors]
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5- $ai + bj \perp bi - aj$

EX) Find the angle "θ" between $\vec{A} = i - 2j - 2k$ & $\vec{B} = 6i + 3j + 2k$?

Solution

$\vec{A}\cdot\vec{B} = (1*6) + (-2*3) + (-2*2) = \boxed{-4}$

$|A| = \sqrt{1^2 + (-2)^2 + (-2)^2} = \sqrt{9} = \boxed{3}$

$|B| = \sqrt{6^2 + 3^2 + 2^2} = \sqrt{49} = \boxed{7}$

$\Rightarrow |A|\,|B| = \boxed{21}$

$\therefore \cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|A|\,|B|} \Rightarrow \theta = \cos^{-1}\dfrac{-4}{21} \approx \boxed{101^\circ}$ Ans

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Vector Product :- (Cross-product)

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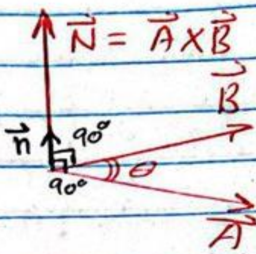
Normal vector is what yields from vector product or cross product.

$$\vec{N} = \vec{A} \times \vec{B} = \vec{n}\,|A|\,|B|\,\sin\theta$$



where $\vec{n}$ — is a normal unit vector

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$; where, $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3 k$, $\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3 k$

Properties :-

1- $\vec{A} \times \vec{A} = 0 \longrightarrow$ "$\sin 0 = 0$"

2- $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

3- $\vec{A} \parallel \vec{B} \Longrightarrow \vec{A} \times \vec{B} = 0 \longrightarrow \sin 0 = 0$

4- Area of $\Delta ABC = \frac{1}{2}|\vec{A} \times \vec{B}|$

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Ex) Find $\vec{A}\times\vec{B}$ & $\vec{B}\times\vec{A}$ if

$$\vec{A} = 2i + j + k$$
$$\vec{B} = -4i + 3j + k$$

Solution)

$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} i - \begin{vmatrix} 2 & 1 \\ -4 & 1 \end{vmatrix} j + \begin{vmatrix} 2 & 1 \\ -4 & 3 \end{vmatrix} k$$

$$= (1\times1 - (3\times1))\,i - (2\times1 - (-4\times1))\,j + (2\times3 - (-4\times1))\,k$$

$$\boxed{\vec{A}\times\vec{B} = -2i - 6j + 10k}$$

but,

$$\vec{B}\times\vec{A} = -\vec{A}\times\vec{B} = \boxed{2i + 6j - 10k}$$

Triple Product :- الضرب الثلاثي المحدود

A- Scalar triple product :-

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C}$$

Note)

1- Box Volume is $\Longrightarrow$ $V_{box} = |\vec{A}\cdot\vec{B}\times\vec{C}|$

2- Pyramid Volume is $\longrightarrow$ $V_{Py} = \frac{1}{6}|\vec{A}\cdot\vec{B}\times\vec{C}|$

B- Vector triple Product :-

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\,B - (\vec{A}\cdot\vec{B})\,\vec{C}$$

Note)

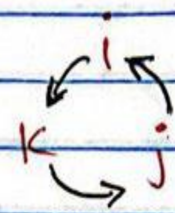
$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = k\cdot k = 1$; $i\times i = j\times j = k\times k = 0$

$i \cdot j = j \cdot k = k \cdot i = 0$

$i \times j = k$

$j \times k = i$

$k \times i = j$



HW#2

1- Find the Length & direction of these vectors & the angles make with the x-axis?

a- $5i + 12j$

b- $\sqrt{3}\, i + j$

2- Find a vector 6 units long in the direction of $\vec{A} = 2i + 2j - k$

3- Find the area of the triangle whose vertices are $A(1,-1,0)$, $B(2,1,-1)$, & $C(-1,1,2)$?

4- If $\vec{A} = 2i - j$ & $B = i + 3j - 2k$, find $A \times B$, then calculate $(A \times B) \cdot A$?

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