EXAMPLE 4 Suppose that the two towropes in Fig. 5.11 pull with horizontal forces of 2.5×10^5 N and 1.0×10^5 N, respectively, and that these forces make angles of 30° and 15° with the long axis of the barge (see Fig. 5.12). Suppose that the friction force is zero. What are the magnitude and direction of the net horizontal force the towropes exert on the barge?

SOLUTION: The net force is the vector sum

$$\mathbf{F}_{\text{net}} = \mathbf{F}_1 + \mathbf{F}_2 \tag{5.10}$$

where $\mathbf{F}_1$ is the force of the first towrope and $\mathbf{F}_2$ that of the second. The net force is shown in Fig. 5.12a. With the x and y axes arranged as in Fig. 5.12a, the forces can be resolved into x and y components. The x component of the net force is the sum of the x components of the individual forces (see Fig. 5.12b),

$$\begin{aligned} F_{\text{net},x} &= F_{1,x} + F_{2,x} \\ &= 2.5 \times 10^5 \text{ N} \times \cos 30° + 1.0 \times 10^5 \text{ N} \times \cos 15° \\ &= 2.5 \times 10^5 \text{ N} \times 0.866 + 1.0 \times 10^5 \text{ N} \times 0.966 \\ &= 3.1 \times 10^5 \text{ N} \end{aligned} \tag{5.11}$$

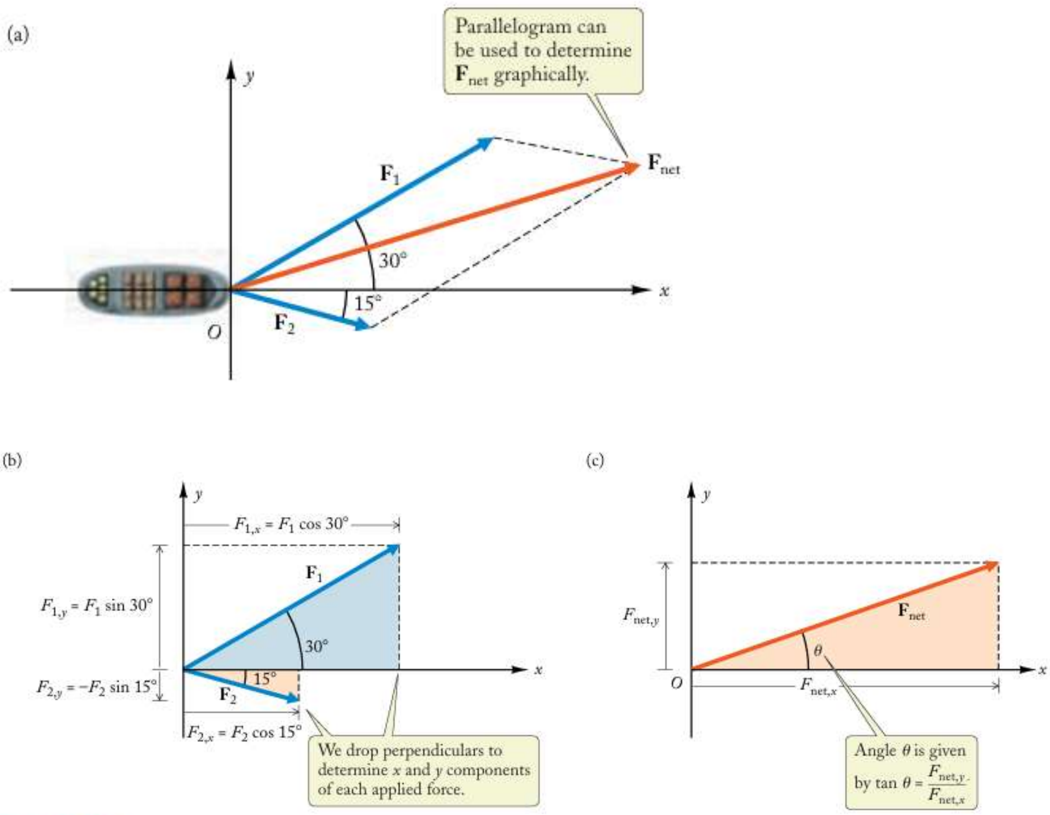


FIGURE 5.12 (a) One tugboat pulls with a force $\mathbf{F}_1$, and the other pulls with a force $\mathbf{F}_2$. The magnitudes of these forces are $F_1 = 2.5 \times 10^5$ N and $F_2 = 1.0 \times 10^5$ N, respectively. The net force $\mathbf{F}_{\text{net}}$ is the vector sum of the two forces $\mathbf{F}_1$ and $\mathbf{F}_2$. (b) The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ and their x and y components. (c) The net force $\mathbf{F}_{\text{net}}$ and its x and y components.

Likewise, the y component of the net force is the sum of the y components of the individual forces,

$$\begin{aligned} F_{\text{net},y} &= F_{1,y} + F_{2,y} \\ &= 2.5 \times 10^5\ \text{N} \times \sin 30° - 1.0 \times 10^5\ \text{N} \times \sin 15° \\ &= 2.5 \times 10^5\ \text{N} \times 0.500 - 1.0 \times 10^5\ \text{N} \times 0.259 \\ &= 1.0 \times 10^5\ \text{N} \end{aligned} \tag{5.12}$$

The y components of the individual forces are of opposite sign because one tugboat pulls the barge to the left (up in Fig. 5.12) and the other to the right (down in Fig. 5.12).

The components $F_{\text{net},x}$ and $F_{\text{net},y}$ uniquely specify the net force, and we could end our calculation of the net force with these components. However, the problem asks for the magnitude and the direction of the net force, and we therefore have to take our calculation a step further. According to Eq. (3.15), the magnitude of the net force is the square root of the sum of squares of the components:

$$\begin{aligned} F_{\text{net}} &= \sqrt{(F_{\text{net},x})^2 + (F_{\text{net},y})^2} \\ &= \sqrt{(3.1 \times 10^5\ \text{N})^2 + (1.0 \times 10^5\ \text{N})^2} = 3.3 \times 10^5\ \text{N} \end{aligned} \tag{5.13}$$

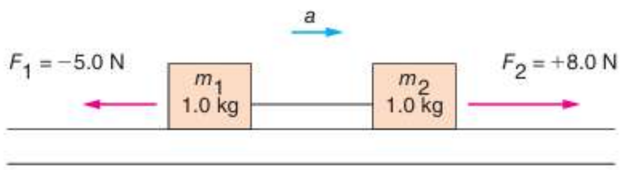
The direction of the net force makes an angle θ with the x axis (see Fig. 5.12c). According to Eq. (3.16), this angle is given by

$$\tan\theta = \frac{F_{\text{net},y}}{F_{\text{net},x}} = \frac{1.0 \times 10^5\ \text{N}}{3.1 \times 10^5\ \text{N}} = 0.32 \tag{5.14}$$

With our calculator, we find that the angle with this tangent is 18°.

EXAMPLE 3.1 Finding Acceleration with Two Applied Forces

Forces are applied to blocks connected by a string and resting on a frictionless surface, as illustrated in ● Fig. 3.8. If the mass of each block is 1.0 kg and the mass of the string is negligible, then what is the acceleration of the system?



Thinking It Through

Note in the figure that the forces act in opposite directions. Hence in Newton's second law, $a = F/m$, the force F would be the vector sum or net force, $F_{net} = F_2 + F_1$, where force directions are indicated by + and − signs as given in the figure. Effectively, F_1 cancels part of F_2. The total mass of the system being accelerated is $m_1 + m_2$.

F_1 = negative 5 N pulls m_1 to the left. F_2 = 8 N pulls m_2 to the right.

Solution

Step 1 *Given:* $m_1 = 1.0$ kg, $F_1 = -5.0$ N (left, negative direction)
$m_2 = 1.0$ kg, $F_2 = +8.0$ N (right, positive direction)

Step 2 *Wanted:* a (acceleration)
(The units are standard in the metric system.)

Step 3 The acceleration may be calculated using Eq. 3.1, $F = ma$, or $a = F/m$. Note, however, that F is the unbalanced (net) force, and $F_{net} = F_2 + F_1 = 8.0\,N - 5.0\,N$ in the direction of F_2. The total mass of the system being accelerated is $m = m_1 + m_2$. Hence, we have an acceleration in the direction of the net force (to the right).

$$a = \frac{F}{m} = \frac{F_{net}}{m_1 + m_2} = \frac{8.0\text{ N} - 5.0\text{ N}}{1.0\text{ kg} + 1.0\text{ kg}} = \frac{3.0\text{ N}}{2.0\text{ kg}} = 1.5\text{ m/s}^2$$

Question: What would be the case if the surface were not frictionless?
Answer: There would be another (frictional) force in the direction of F_1 opposing the motion.

Example 5.1 An Accelerating Hockey Puck AM

A hockey puck having a mass of 0.30 kg slides on the frictionless, horizontal surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck shown in Figure 5.4. The force $\vec{F}_1$ has a magnitude of 5.0 N, and is directed at $\theta = 20°$ below the x axis. The force $\vec{F}_2$ has a magnitude of 8.0 N and its direction is $\phi = 60°$ above the x axis. Determine both the magnitude and the direction of the puck's acceleration.

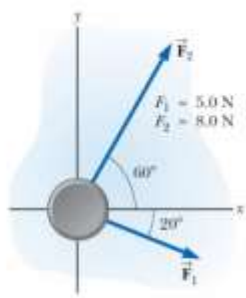


Figure 5.4 (Example 5.1) A hockey puck moving on a frictionless surface is subject to two forces $\vec{F}_1$ and $\vec{F}_2$.

SOLUTION

Conceptualize Study Figure 5.4. Using your expertise in vector addition from Chapter 3, predict the approximate direction of the net force vector on the puck. The acceleration of the puck will be in the same direction.

Categorize Because we can determine a net force and we want an acceleration, this problem is categorized as one that may be solved using Newton's second law. In Section 5.7, we will formally introduce the *particle under a net force* analysis model to describe a situation such as this one.

Analyze Find the component of the net force acting on the puck in the x direction:

$$\sum F_x = F_{1x} + F_{2x} = F_1 \cos\theta + F_2 \cos\phi$$

Find the component of the net force acting on the puck in the y direction:

$$\sum F_y = F_{1y} + F_{2y} = F_1 \sin\theta + F_2 \sin\phi$$

Use Newton's second law in component form (Eq. 5.3) to find the x and y components of the puck's acceleration:

$$a_x = \frac{\sum F_x}{m} = \frac{F_1 \cos\theta + F_2 \cos\phi}{m}$$

$$a_y = \frac{\sum F_y}{m} = \frac{F_1 \sin\theta + F_2 \sin\phi}{m}$$

Substitute numerical values:

$$a_x = \frac{(5.0\text{ N})\cos(-20°) + (8.0\text{ N})\cos(60°)}{0.30\text{ kg}} = 29\text{ m/s}^2$$

$$a_y = \frac{(5.0\text{ N})\sin(-20°) + (8.0\text{ N})\sin(60°)}{0.30\text{ kg}} = 17\text{ m/s}^2$$

Find the magnitude of the acceleration:

$$a = \sqrt{(29\text{ m/s}^2)^2 + (17\text{ m/s}^2)^2} = \boxed{34\text{ m/s}^2}$$

Find the direction of the acceleration relative to the positive x axis:

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = \boxed{31°}$$

Finalize The vectors in Figure 5.4 can be added graphically to check the reasonableness of our answer. Because the acceleration vector is along the direction of the resultant force, a drawing showing the resultant force vector helps us check the validity of the answer. (Try it!)

WHAT IF? Suppose three hockey sticks strike the puck simultaneously, with two of them exerting the forces shown in Figure 5.4. The result of the three forces is that the hockey puck shows *no* acceleration. What must be the components of the third force?

Answer If there is zero acceleration, the net force acting on the puck must be zero. Therefore, the three forces must cancel. The components of the third force must be of equal magnitude and opposite sign compared to the components of the net force applied by the first two forces so that all the components add to zero. Therefore, $F_{3x} = -\sum F_x = -(0.30\text{ kg})(29\text{ m/s}^2) = -8.7\text{ N}$ and $F_{3y} = -\sum F_y = -(0.30\text{ kg})(17\text{ m/s}^2) = -5.2\text{ N}$.