Ministry of Higher Education and Scientific Research
Almustaqbal University, College of Engineering
And Engineering Technologies
Computer Technology Engineering Department

SIX WEEK :

NETWORK THEOREMS (DC)

Course Name : Electrical Engineering Fundamentals
Academic Year : 2024-2025
Stage : One
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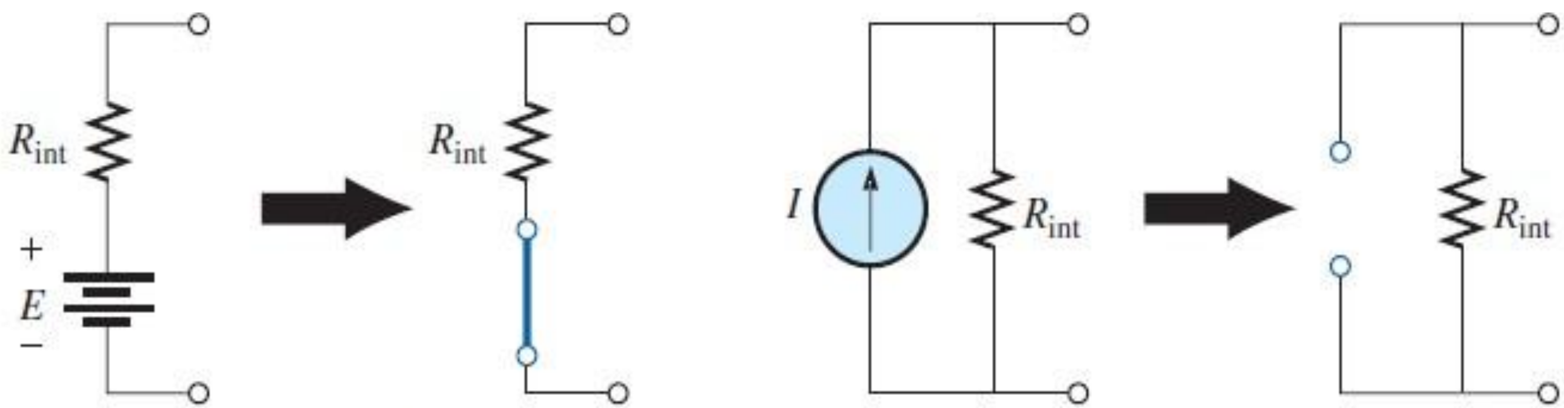
NETWORK THEOREMS (DC)

9.2 SUPERPOSITION THEOREM

The current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source.

When removing a voltage source from a network schematic, replace it with a direct connection (short circuit) of zero ohms. Any internal resistance associated with the source must remain in the network.

When removing a current source from a network schematic, replace it by an open circuit of infinite ohms. Any internal resistance associated with the source must remain in the network.



Removing a voltage source and a current source to permit the application of the superposition theorem.

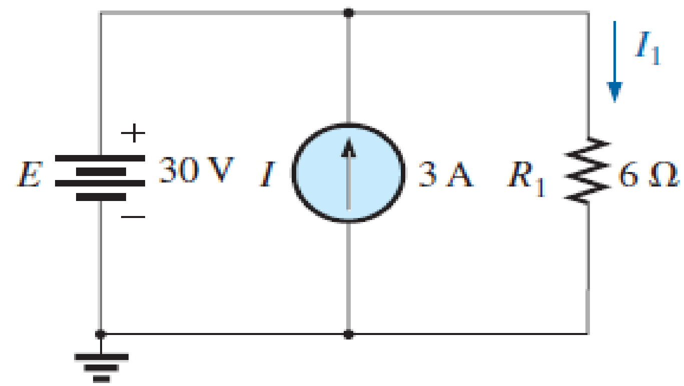
EXAMPLE 9.1 Using the superposition theorem, determine current *I1* for the network in Fig.

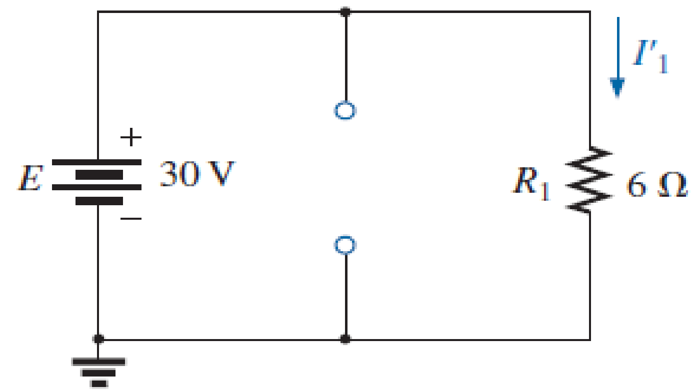
Due to the open circuit, resistor *R1 is in series*

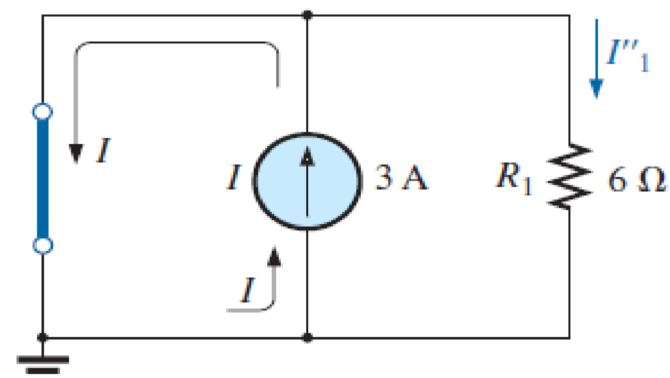
$$I'_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{30\ \text{V}}{6\ \Omega} = 5\ \text{A}$$

$$I''_1 = \frac{R_{sc}I}{R_{sc} + R_1} = \frac{(0\ \Omega)I}{0\ \Omega + 6\ \Omega} = 0\ \text{A}$$

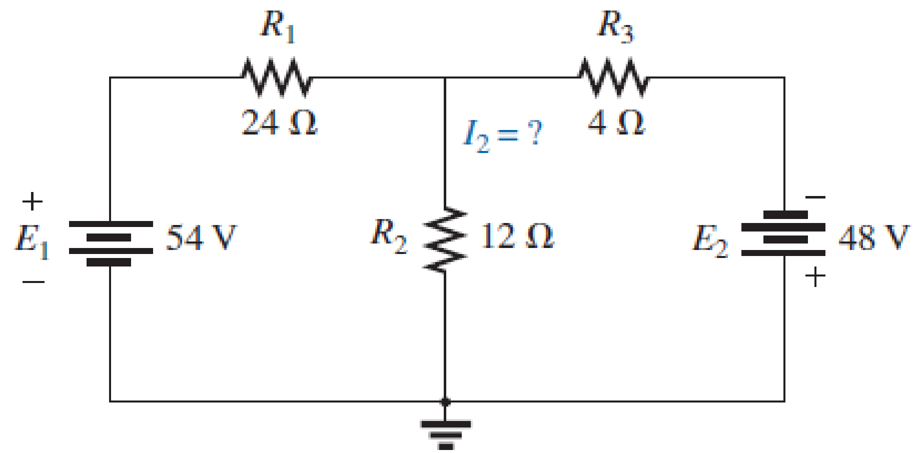
$$I_1 = I'_1 + I''_1 = 5\ \text{A} + 0\ \text{A} = \mathbf{5\ A}$$

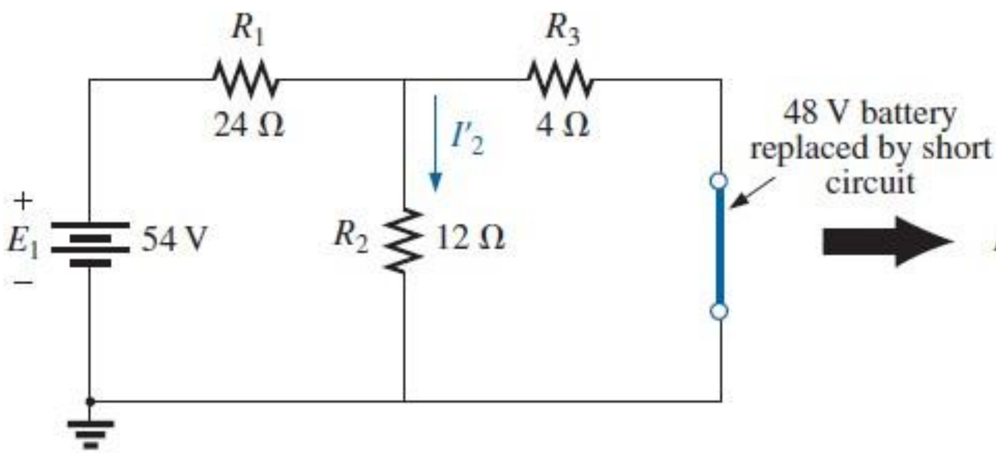


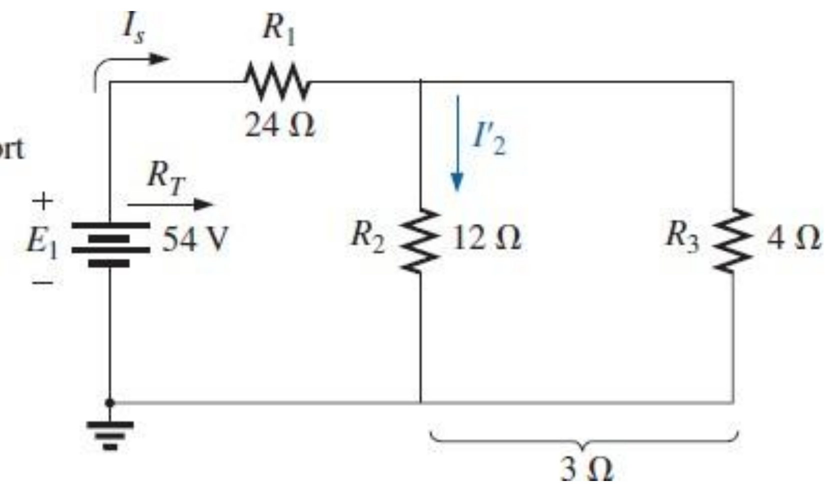




EXAMPLE 9.2 Using the superposition theorem, determine the current through the 12 Ω resistor in Fig. Note that this is a two-source network of the type examined in the previous chapter when we applied branch-current analysis and mesh analysis.







The total resistance seen by the source is therefore

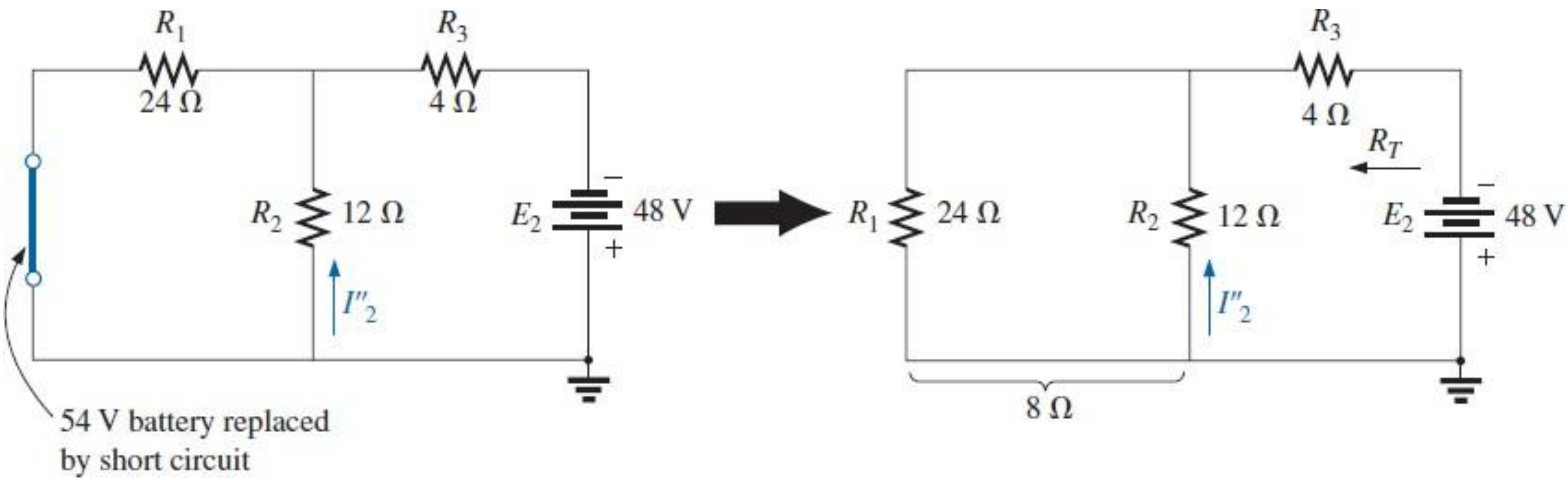
$$R_T = R_1 + R_2 \| R_3 = 24\ \Omega + 12\ \Omega \| 4\ \Omega = 24\ \Omega + 3\ \Omega = 27\ \Omega$$

and the source current is

$$I_s = \frac{E_1}{R_T} = \frac{54\ \text{V}}{27\ \Omega} = 2\ \text{A}$$

Using the current divider rule results in the contribution to I_2 due to the 54 V source:

$$I'_2 = \frac{R_3 I_s}{R_3 + R_2} = \frac{(4\ \Omega)(2\ \text{A})}{4\ \Omega + 12\ \Omega} = 0.5\ \text{A}$$



Therefore, the total resistance seen by the 48 V source is

$$R_T = R_3 + R_2 \,\|\, R_1 = 4\ \Omega + 12\ \Omega \,\|\, 24\ \Omega = 4\ \Omega + 8\ \Omega = 12\ \Omega$$

and the source current is

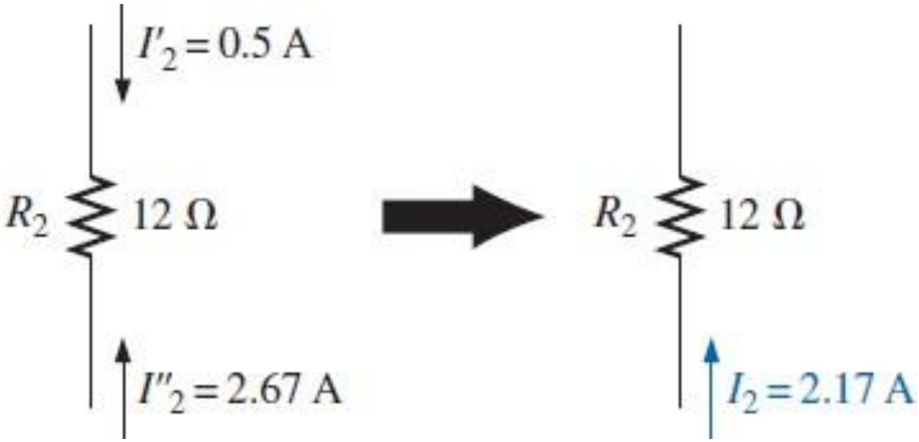
$$I_s = \frac{E_2}{R_T} = \frac{48\ \text{V}}{12\ \Omega} = 4\ \text{A}$$

Applying the current divider rule results in

$$I''_2 = \frac{R_1(I_s)}{R_1 + R_2} = \frac{(24\ \Omega)(4\ \text{A})}{24\ \Omega + 12\ \Omega} = 2.67\ \text{A}$$

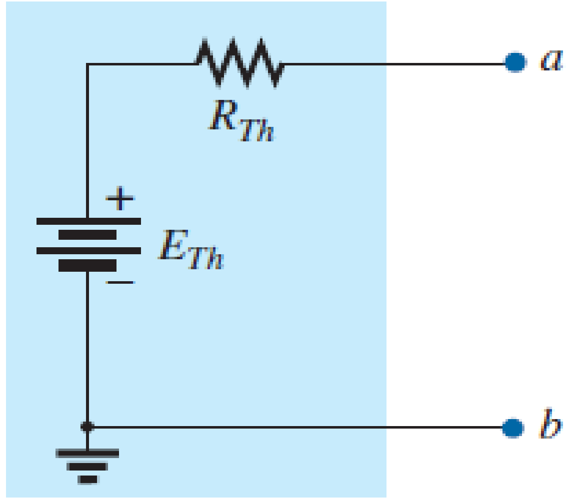
It is now important to realize that current *I2 due to each source has a* different direction, as shown in Fig. The net current therefore is the difference of the two and the direction of the larger as follows:

$$I_2 = I''_2 - I'_2 = 2.67\ \text{A} - 0.5\ \text{A} = \mathbf{2.17\ A}$$



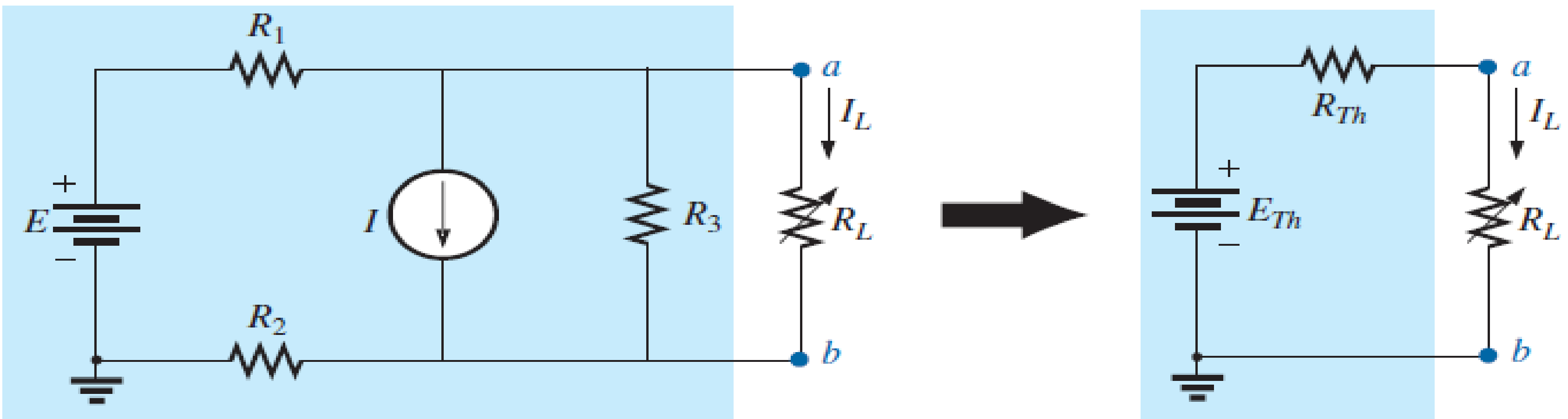
9.3 THÉVENIN’S THEOREM

Any two-terminal dc network can be replaced by an equivalent circuit consisting solely of a voltage source and a series resistor as shown in Fig.



Thévenin’s Theorem Procedure

1. Remove that portion of the network where the Thévenin equivalent circuit is found. In Fig., this requires that the load resistor RL be temporarily removed from the network.
2. Mark the terminals of the remaining two-terminal network.



R_{Th}:

3.Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

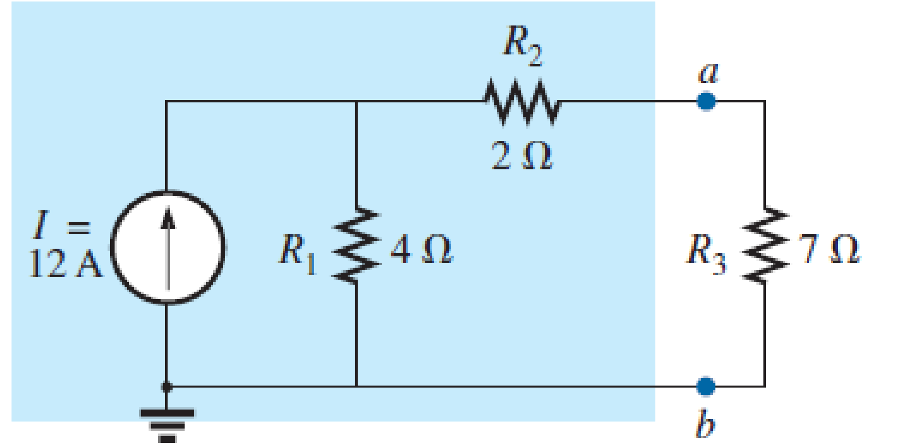
E_{Th}:

4.Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that causes most confusion and errors. In all cases, keep in mind that it is the open circuit potential between the two terminals marked in step 2.)

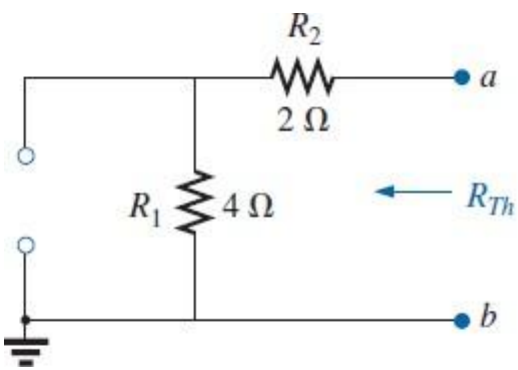
Conclusion:

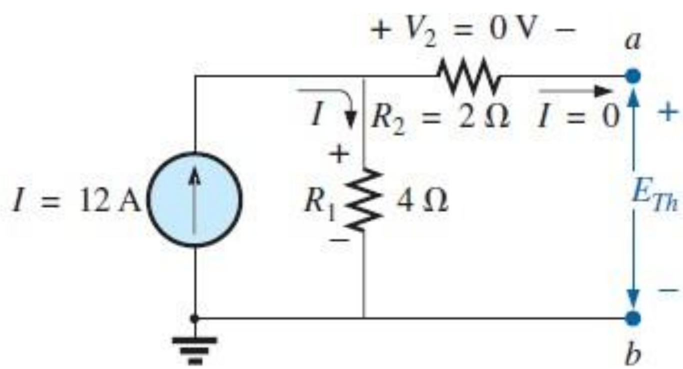
5.Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor RL between the terminals of the Thévenin equivalent circuit as shown in Fig. 9.25(b).

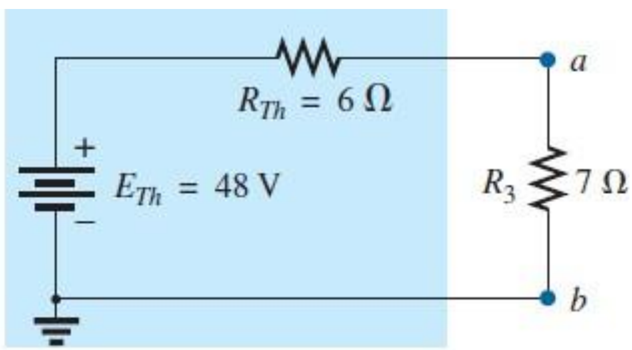
EXAMPLE 9.7 Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig.



$$R_{Th} = R_1 + R_2 = 4\ \Omega + 2\ \Omega = \mathbf{6\ \Omega}$$







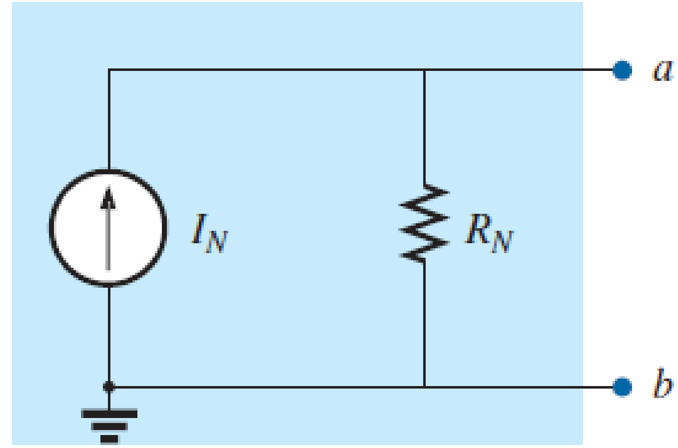
In this case, since an open circuit exists between the two marked terminals, the current is zero between these terminals and through the 2 Ω resistor. The voltage drop across R_2 *is, therefore,*

$$V_2 = I_2R_2 = (0)R_2 = 0\text{ V}$$

and

$$E_{Th} = V_1 = I_1R_1 = IR_1 = (12\text{ A})(4\ \Omega) = \mathbf{48\ V}$$

9.4 NORTON'S THEOREM

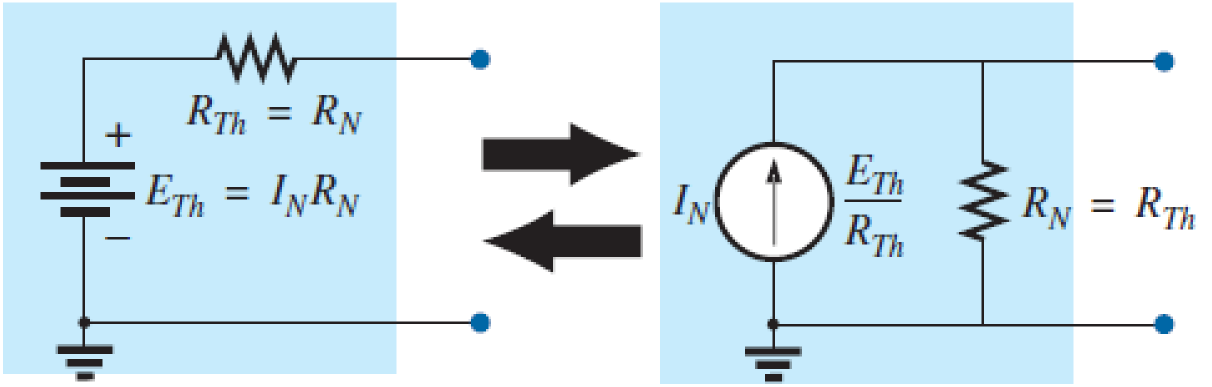


Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor, as shown in Fig.

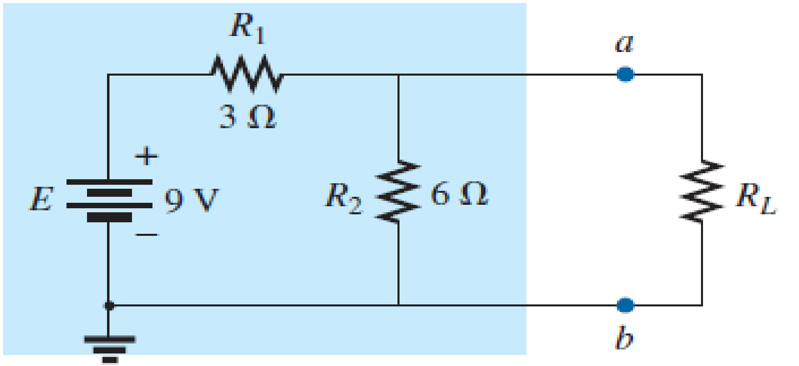
Norton's Theorem Procedure

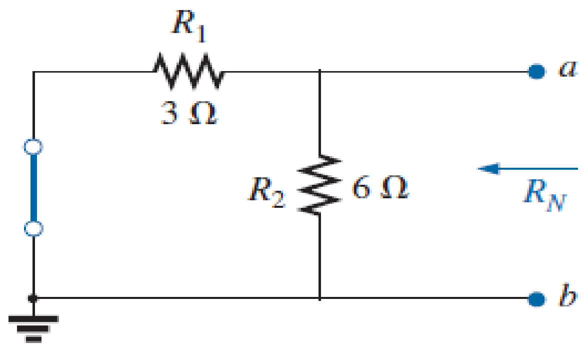
1. Remove that portion of the network across which the Norton equivalent circuit is found.
2.Mark the terminals of the remaining two-terminal network. R_N:
3.Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since $R_N = R_{Th}$, the procedure and value obtained using the approach described for Thévenin's theorem will determine the proper value of R_N.
I_N:
4.Calculate I_N by first returning all sources to their original position and then finding the short- circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.
Conclusion:
5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

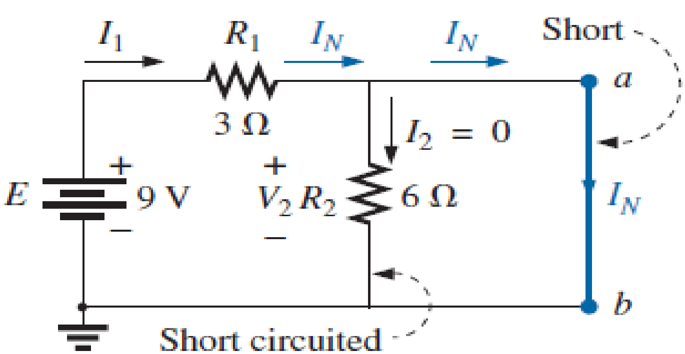
The Norton and Thévenin equivalent circuits can also be found from each other by using the source transformation discussed earlier in this chapter and reproduced in Fig.



EXAMPLE 9.11 Find the Norton equivalent circuit for the network in the shaded area in Fig.



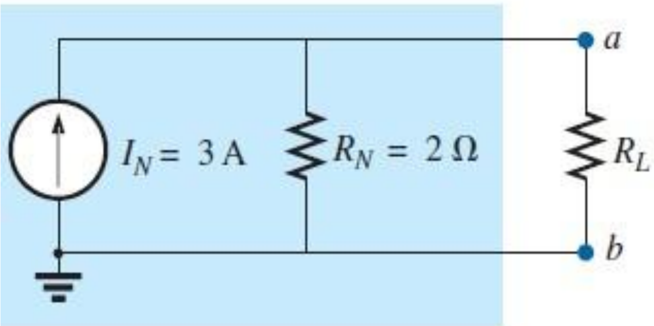


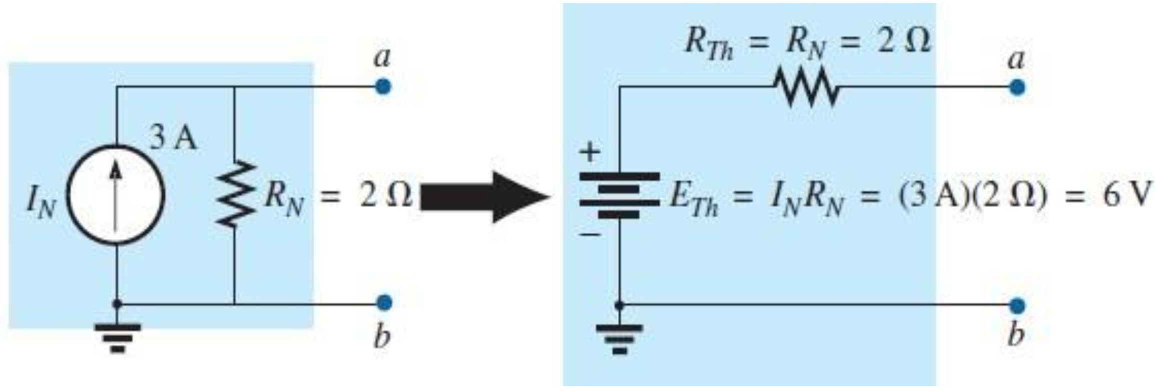


$$R_N = R_1 \| R_2 = 3\ \Omega \| 6\ \Omega = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = 2\ \Omega$$

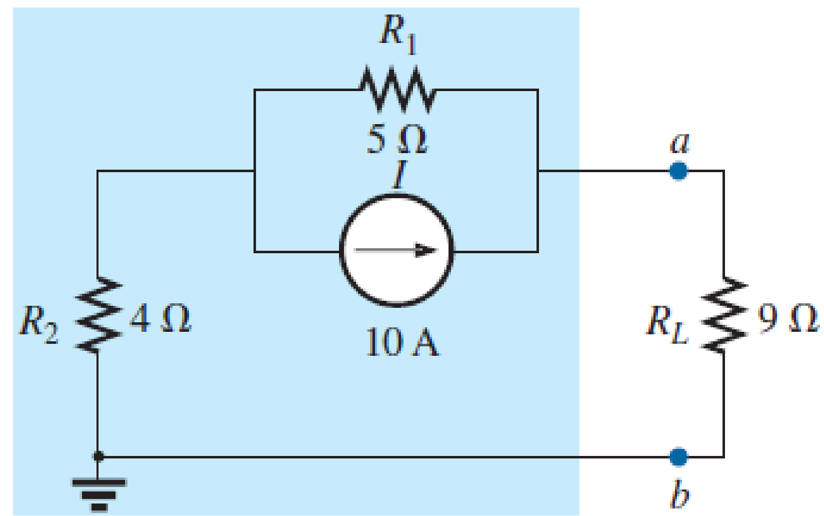
$$V_2 = I_2 R_2 = (0)6\ \Omega = 0\ \text{V}$$

$$I_N = \frac{E}{R_1} = \frac{9\ \text{V}}{3\ \Omega} = \mathbf{3\ A}$$

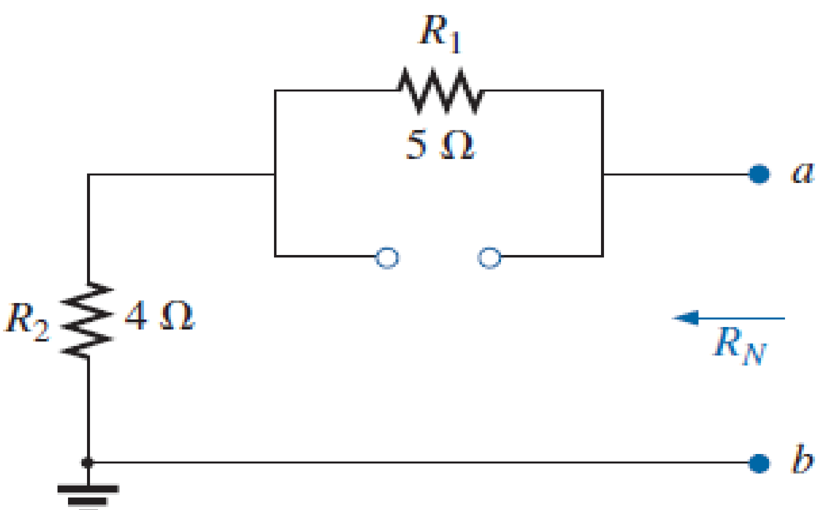


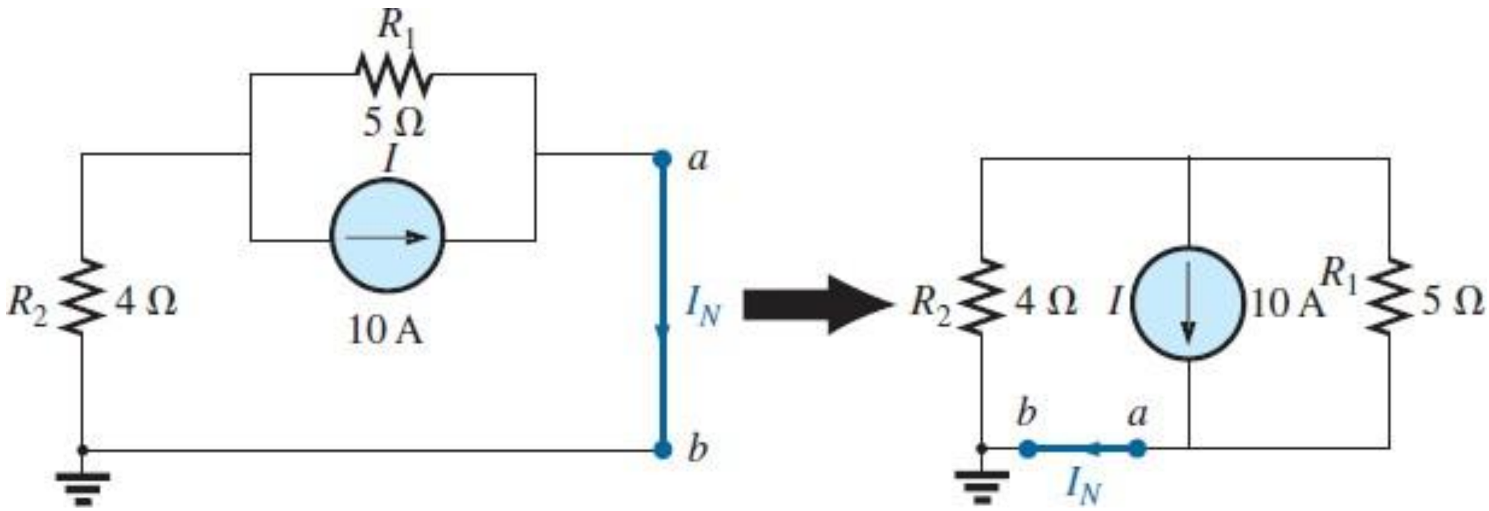


EXAMPLE 9.12 Find the Norton equivalent circuit for the network external to the 9 Ω resistor in Fig.

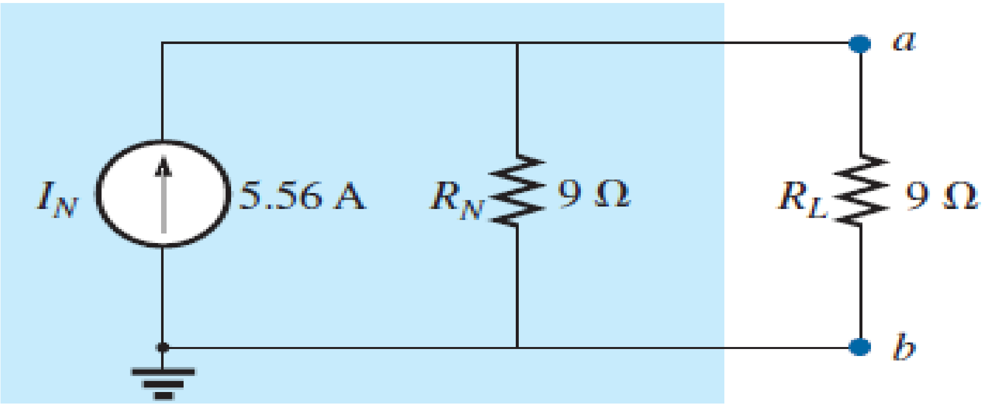


$$R_N = R_1 + R_2 = 5\ \Omega + 4\ \Omega = 9\ \Omega$$





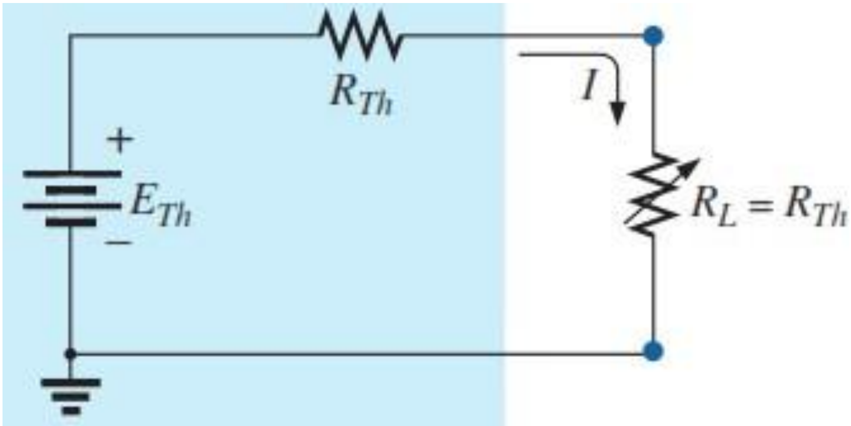
$$I_N = \frac{R_1 I}{R_1 + R_2} = \frac{(5\ \Omega)(10\ \text{A})}{5\ \Omega + 4\ \Omega} = \frac{50\ \text{A}}{9} = \mathbf{5.56\ A}$$



9.5 MAXIMUM POWER TRANSFER THEOREM

A load will receive maximum power from a network when its resistance is exactly equal to the Thévenin resistance of the network applied to the load. That is,

$$R_L = R_{Th}$$



The maximum power delivered to the load can be determined by first finding the current:

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{E_{Th}}{R_{Th} + R_{Th}} = \frac{E_{Th}}{2R_{Th}}$$

Then substitute into the power equation:

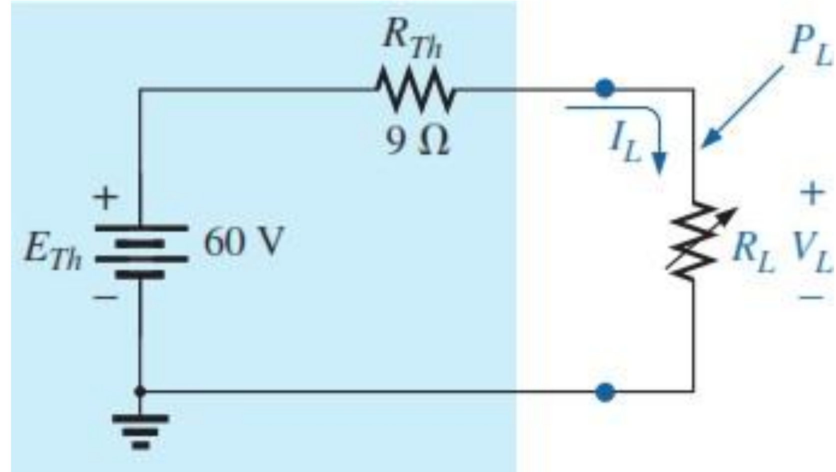
$$P_L = I_L^2 R_L = \left(\frac{E_{Th}}{2R_{Th}}\right)^2 (R_{Th}) = \frac{E_{Th}^2 R_{Th}}{4R_{Th}^2}$$

and

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}}$$

Maximum power transfer occurs when the load voltage and current are one-half of their maximum possible values.

For the circuit in Fig., the current through the load is determined by



$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{60\ \text{V}}{9\ \Omega + R_L}$$

The voltage is determined by

$$V_L = \frac{R_L E_{Th}}{R_L + R_{Th}} = \frac{R_L(60\ \text{V})}{R_L + R_{Th}}$$

and the power by $$P_L = I_L^2 R_L = \left(\frac{60\ \text{V}}{9\ \Omega + R_L}\right)^2 (R_L) = \frac{3600 R_L}{(9\ \Omega + R_L)^2}$$

R_L (Ω)	P_L (W)		I_L (A)		V_L (V)	
5	91.84	↓	4.29	↓	21.43	↓
6	96.00		4.00		24.00	
7	98.44	Increase	3.75	Decrease	26.25	Increase
8	99.65		3.53		28.23	
9 (R_{Th})	**100.00 (Maximum)**		**3.33 ($I_{max}/2$)**		**30.00 ($E_{Th}/2$)**	
10	99.72		3.16		31.58	
11	99.00		3.00		33.00	
12	97.96		2.86		34.29	
13	96.69		2.73		35.46	

Under maximum power conditions, only half the power delivered by the source gets to the load. On an efficiency basis, we are working at only a **50% level**, but we are content because we are getting **maximum power** out of our system.

The dc operating efficiency is defined as the ratio of the power delivered to the load (P_L) to the power delivered by the source (P_s). That is:

$$\eta\% = \frac{P_L}{P_s} \times 100\%$$

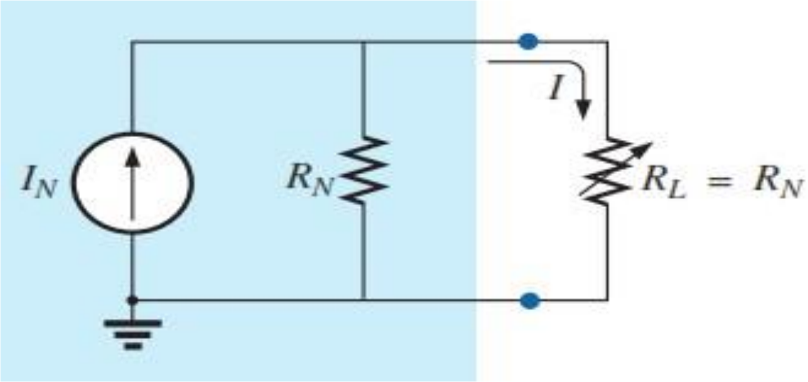
For the situation where $R_L = R_{Th}$,

$$\eta\% = \frac{I_L^2 R_L}{I_L^2 R_T} \times 100\% = \frac{R_L}{R_T} \times 100\% = \frac{R_{Th}}{R_{Th} + R_{Th}} \times 100\%$$

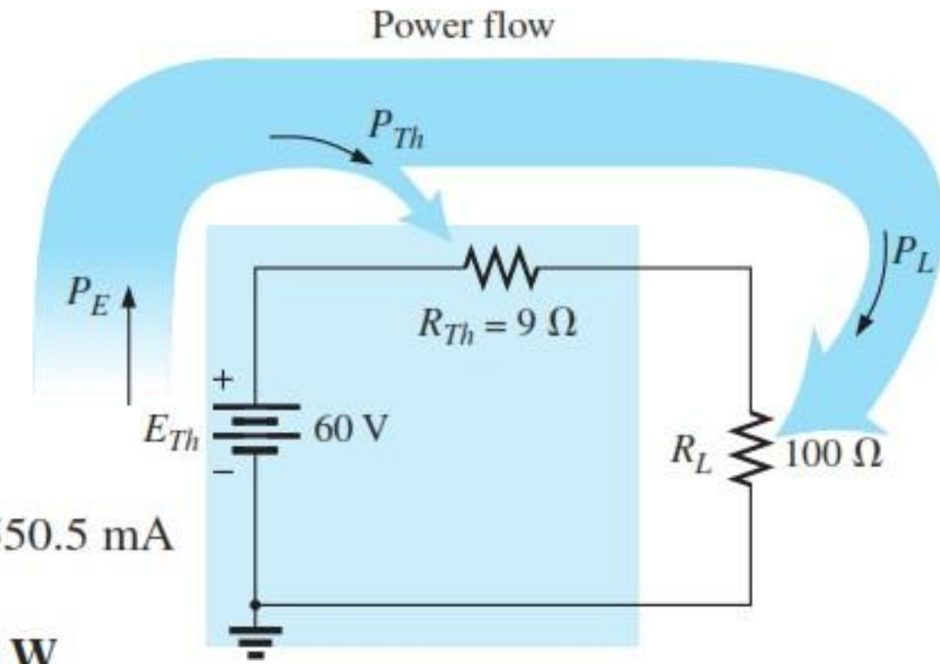
$$= \frac{R_{Th}}{2R_{Th}} \times 100\% = \frac{1}{2} \times 100\% = \mathbf{50\%}$$

$$R_L = R_N$$

$$P_{L_{max}} = \frac{I_N^2 R_N}{4} \quad \text{(W)}$$



Consider the circuit in Fig. where the load resistance is set at 100 Ω and the power to the Thévenin resistance and to the load are calculated as follows:



$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{60\text{ V}}{9\ \Omega + 100\ \Omega} = \frac{60\text{ V}}{109\ \Omega} = 550.5\text{ mA}$$

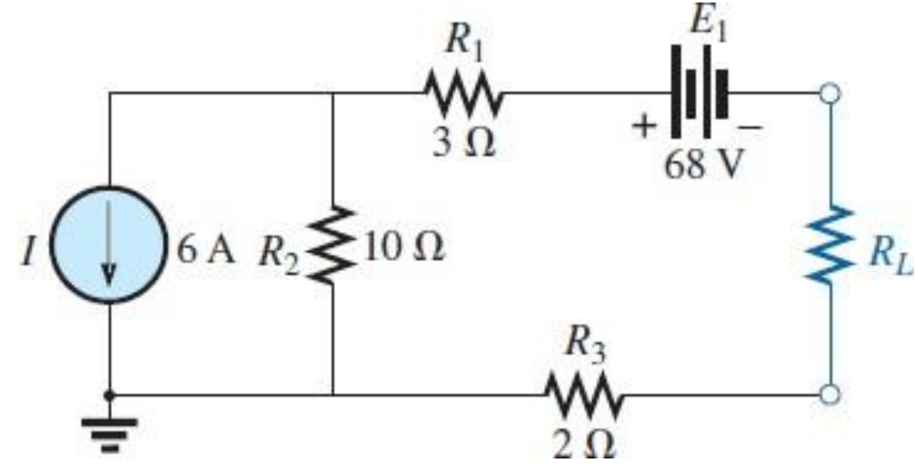
with $P_{R_{Th}} = I_L^2 R_{Th} = (550.5\text{ mA})^2(9\ \Omega) \cong \mathbf{2.73\ W}$

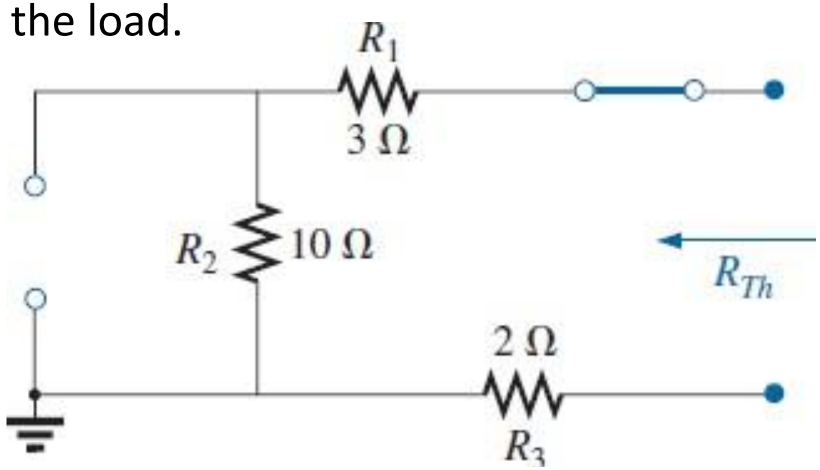
and $P_L = I_L^2 R_L = (550.5\text{ mA})^2(100\ \Omega) \cong \mathbf{30.3\ W}$

The results clearly show that most of the power supplied by the battery is getting to the load—a desirable attribute on an efficiency basis. However, the power getting to the load is only 30.3 W compared to the 100 W obtained under maximum power conditions.

If efficiency is the overriding factor, then the load should be much larger than the internal resistance of the supply. If maximum power transfer is desired and efficiency less of a concern, then the conditions dictated by the maximum power transfer theorem should be applied.

EXAMPLE 9.17 Given the network in Fig., find the value of R_L for maximum power to the load, and find the maximum power to the load.





$$R_{Th} = R_1 + R_2 + R_3 = 3\,\Omega + 10\,\Omega + 2\,\Omega = 15\,\Omega$$

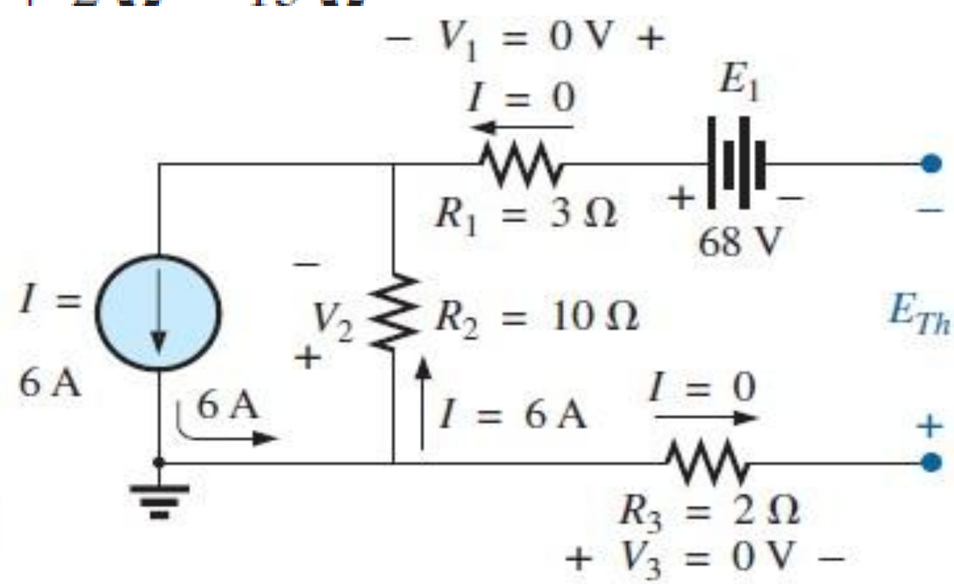
$$R_L = R_{Th} = \mathbf{15\,\Omega}$$

$$V_1 = V_3 = 0\text{ V}$$

$$V_2 = I_2R_2 = IR_2 = (6\text{ A})(10\,\Omega) = 60\text{ V}$$

$$E_{Th} = V_2 + E = 60\text{ V} + 68\text{ V} = \mathbf{128\text{ V}}$$

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(128\text{ V})^2}{4(15\,\Omega)} = \mathbf{273.07\text{ W}}$$



PROBLEMS

SECTION 9.2 Superposition Theorem: 1, 2, 3, 5

SECTION 9.3 Thévenin's Theorem: 8, 9, 10, 11, 12, 16

SECTION 9.4 Norton's Theorem: 20, 21, 23

SECTION 9.5 Maximum Power Transfer Theorem: 24, 26, 27, 28, 30

Thank you very much