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كلية العلوم قسم الكيمياء الحياتية

المحاضرة السابعة

Differentiation

المادة : الرياضيات

المرحلة : الاولى

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The Definition of the derivative:

The derivative of the function f is the function f' whose value at x is defined by the eq.

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \quad \text{whenever the limit exists}$$

Example 1: Use the definition of the derivative to find the derivative of the function

$$f(x) = 3x - 4$$

$$\begin{aligned} \text{Sol: } f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{3(x+\Delta x) - 4 - (3x - 4)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{3x + 3\Delta x - 4 - 3x + 4}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{3\Delta x}{\Delta x} = \lim_{\Delta x \rightarrow 0} 3 = 3 \end{aligned}$$

Example 2: Use the definition of the derivative to find the derivative of the function

$$f(x) = x^2$$

$$\begin{aligned} \text{Sol: } f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x^2 - 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} = 2x \end{aligned}$$

H.W: Use the definition of the derivative to find the derivative of the following functions:

$$1- f(x) = 2x - 1 \quad 2- f(x) = (x + 1)^2 \quad 3- f(x) = x^2 - 1$$

$$4- f(x) = \sqrt{x} \quad 5- f(x) = \frac{x}{x-9} \quad 6- f(x) = x^3$$

Derivative Formulas

Powers of x rule:

$$\text{If } f(x) = x^n, \text{ then } f'(x) = n(x)^{n-1}$$

Constant rule:

$$\text{If } f(x) = C, \text{ then } f'(x) = 0$$

Coefficient rules:

$$\text{If } f(x) = c \cdot u(x), \text{ then } f'(x) = c \cdot u'(x)$$

$$\text{If } f(x) = k \cdot x^n, \text{ then } f'(x) = kn(x)^{n-1}$$

$$\text{If } f(x) = kx, \text{ then } f'(x) = k$$

Sum rule:

$$\text{If } f(x) = u(x) + v(x), \text{ then } f'(x) = u'(x) + v'(x)$$

Difference rule:

$$\text{If } f(x) = u(x) - v(x), \text{ then } f'(x) = u'(x) - v'(x)$$

Product rule:

$$\text{If } f(x) = u(x) \cdot v(x), \text{ then } f'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

Quotient rule:

$$\text{If } f(x) = \frac{u(x)}{v(x)}, \text{ then } f'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{(v(x))^2}$$

Power (Chain) rule:

$$\text{If } f(x) = (u(x))^n, \text{ then } f'(x) = n(u(x))^{n-1} \cdot u'(x)$$

Examples: Find the derivative of the following functions:

$$1- f(x) = 5 \Rightarrow \frac{d}{dx} f(x) = \frac{d}{dx} (5) = 0$$

$$2- f(x) = x^6 \Rightarrow \frac{d}{dx} f(x) = \frac{d}{dx} (x^6) = 6x^5$$

$$3- f(x) = x^6 + x^3 \Rightarrow \frac{d}{dx} (x^6 + x^3) = \frac{d}{dx} (x^6) + \frac{d}{dx} (x^3) = 6x^5 + 3x^2$$

$$4- f(x) = 3x^5 \Rightarrow \frac{d}{dx} f(x) = \frac{d}{dx} (3x^5) = 15x^4$$

$$5- f(x) = (x^2 + 2)(x^3 + 3x + 1) \Rightarrow$$

$$\frac{d}{dx} f(x) = (x^2 + 2)(3x^2 + 3) + (x^3 + 3x + 1) \cdot 2x$$

$$6- f(x) = (x^3 - \frac{x}{2})^6 \Rightarrow \frac{d}{dx} f(x) = 6 (x^3 - \frac{x}{2})^5 (3x^2 - \frac{1}{2})$$

$$7- f(x) = \frac{x^2-1}{x^4+1} \Rightarrow \frac{d}{dx} \left(\frac{x^2-1}{x^4+1} \right) = \frac{(x^4+1) \cdot \frac{d}{dx}(x^2-1) - (x^2-1) \cdot \frac{d}{dx}(x^4+1)}{(x^4+1)^2}$$

$$= \frac{(x^4+1) \cdot (2x) - [(x^2-1) \cdot 4x^3]}{(x^4+1)^2} = \frac{2x^5 + 2x - [4x^5 - 4x^3]}{(x^4+1)^2}$$

$$= \frac{2x^5 + 2x - 4x^5 + 4x^3}{(x^4+1)^2} = \frac{-2x^5 + 4x^3 + 2x}{(x^4+1)^2}$$

$$8- f(x) = (2x^2 - 5x^{-2})^{-5} \Rightarrow$$

$$\frac{d}{dx} f(x) = -5(2x^2 - 5x^{-2})^{-6} \cdot (4x + 10x^{-3})$$

Derivative of trigonometric function

$$1\text{-if } f(x) = \sin x \Rightarrow f'(x) = \cos x$$

$$2\text{-if } f(x) = \cos x \Rightarrow f'(x) = -\sin x$$

$$3\text{-if } f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$$

$$4\text{-if } f(x) = \cot x \Rightarrow f'(x) = -\csc^2 x$$

$$5\text{-if } f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$$

$$6\text{-if } f(x) = \csc x \Rightarrow f'(x) = -\csc x \cot x$$

$$7\text{-if } f(x) = e^x \Rightarrow f'(x) = e^x$$

$$8\text{-if } f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x} \cdot f'(x)$$

Note: we use the following rules when $u(x)$ is a function of x :

$$1- \frac{d}{dx} (\sin u(x)) = \cos u(x) \cdot u'(x)$$

$$2- \frac{d}{dx} (\cos u(x)) = -\sin u(x) \cdot u'(x)$$

$$3- \frac{d}{dx} (\tan u(x)) = \sec^2 u(x) \cdot u'(x)$$

$$4- \frac{d}{dx} (\cot u(x)) = -\csc^2 u(x) \cdot u'(x)$$

$$5- \frac{d}{dx} (\sec u(x)) = \sec u(x) \tan u(x) \cdot u'(x)$$

$$6- \frac{d}{dx} (\csc u(x)) = -\csc u(x) \cot(u(x)) \cdot u'(x)$$

Examples: Find the derivative of the following functions:

$$1- f(x) = \sin x^3 \Rightarrow f'(x) = \cos x^3 \cdot 3x^2 = 3x^2 \cos x^3$$

$$2- f(x) = \cos(5x^2 + 2)^3 + \cot(1 - x^2) \Rightarrow f'(x) = -\sin(5x^2 + 2)^3 \cdot 3(5x^2 + 2)^2 \cdot 10x - \csc(1 - x^2) \cot(1 - x^2) \cdot -2x$$

$$3- f(x) = \tan(x^2 + 1) \Rightarrow f'(x) = \sec^2(x^2 + 1) \cdot 2x$$

$$4- f(x) = (3 - \sec(5x))^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2}(3 - \sec(5x))^{-\frac{1}{2}} \cdot \sec(5x) \tan(5x) \cdot 5$$

$$5- f(x) = \csc^2(2x) + 4x^2 \sin x$$

$$f'(x) = 2 \csc(2x) \cdot -\csc(2x) \cot(2x) \cdot 2 + 4(x^2 \cos x + \sin x \cdot 2x)$$

$$f'(x) = -2 \csc^2(2x) \cot(2x) \cdot 2 + 4(x^2 \cos x + 2x \sin x)$$

$$1- = \ln(x^2 - 8) \Rightarrow f'(x) = \frac{1}{x^2 - 8} \cdot 2x = \frac{2x}{x^2 - 8}$$

$$2- f(x) = e^{-2x} \Rightarrow f'(x) = -2e^{-2x}$$

H.W: Find the derivative of the following functions:

$$1- f(x) = \tan^3(\cos 5x^3)^{\frac{1}{2}} \quad 2- f(x) = \cos(x^4 + 3x^3 + x^2)^7 + \csc(\tan x^3)$$

$$3- f(x) = \frac{(\sin \sqrt{x})^3}{\sqrt{x}} \quad 4- f(x) = x^3 \sin(2x^2 + 3)$$