Department of biology

((GENERAL MATHEMATICS))

1st stage

Week 9- lecture 9

Taylor Series

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Taylor series

The Taylor series is a way to represent a function $f(x)$ as an infinite sum of terms calculated from its derivatives at a single point. The general formula for the Taylor series of $f(x)$ centered at $x = a$ is:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

In summation form:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Where:

- $f^{(n)}(a)$ is the n-th derivative of $f(x)$ evaluated at $x = a$.
- $n!$ is the factorial of n.

Examples:-

1. Maclaurin Series for $f(x) = 1 + x$:

$$f(x) = 1 + x$$

- $f'(x) = 1$, $f''(x) = 0$, $f'''(x) = 0$, etc.
- At $x = 0$:
  $f(0) = 1$, $f'(0) = 1$, $f''(0) = 0$, etc.

Maclaurin Series:

$$f(x) = 1 + x$$

2. Maclaurin Series for $f(x) = x^2$:

$$f(x) = x^2$$

- $f'(x) = 2x$, $f''(x) = 2$, $f'''(x) = 0$, etc.
- At $x = 0$:
  $f(0) = 0$, $f'(0) = 0$, $f''(0) = 2$, etc.

Maclaurin Series:

$$f(x) = \frac{2}{2!}x^2 = x^2$$

3. Maclaurin Series for $f(x) = x^3 + x$:

$$f(x) = x^3 + x$$

- $f'(x) = 3x^2 + 1$, $f''(x) = 6x$, $f'''(x) = 6$, etc.
- At $x = 0$:
  $f(0) = 0$, $f'(0) = 1$, $f''(0) = 0$, $f'''(0) = 6$, etc.

Maclaurin Series:

$$f(x) = x + \frac{6}{3!}x^3 = x + x^3$$

4. Maclaurin Series for $f(x) = x^4$:

$$f(x) = x^4$$

- $f'(x) = 4x^3$, $f''(x) = 12x^2$, $f'''(x) = 24x$, $f^{(4)}(x) = 24$, etc.
- At $x = 0$:
  $f(0) = 0$, $f'(0) = 0$, $f''(0) = 0$, $f'''(0) = 0$, $f^{(4)}(0) = 24$, etc.

Maclaurin Series:

$$f(x) = \frac{24}{4!}x^4 = x^4$$

5. Maclaurin Series for $f(x) = 1 - x$:

$$f(x) = 1 - x$$

- $f'(x) = -1$, $f''(x) = 0$, $f'''(x) = 0$, etc.
- At $x = 0$:
  $f(0) = 1$, $f'(0) = -1$, $f''(0) = 0$, etc.

Maclaurin Series:

$$f(x) = 1 - x$$

6. Maclaurin Series for $f(x) = x^5$:

$$f(x) = x^5$$

- $f'(x) = 5x^4$, $f''(x) = 20x^3$, $f'''(x) = 60x^2$, $f^{(4)}(x) = 120x$, $f^{(5)}(x) = 120$, etc.
- At $x = 0$:
  $f(0) = 0$, $f'(0) = 0$, $f''(0) = 0$, $f'''(0) = 0$, $f^{(5)}(0) = 120$, etc.

Maclaurin Series:

$$f(x) = \frac{120}{5!}x^5 = x^5$$

7. Maclaurin Series for $f(x) = x^2 + 1$:

$$f(x) = x^2 + 1$$

- $f'(x) = 2x$, $f''(x) = 2$, $f'''(x) = 0$, etc.
- At $x = 0$:
  $f(0) = 1$, $f'(0) = 0$, $f''(0) = 2$, $f'''(0) = 0$, etc.

Maclaurin Series:

$$f(x) = 1 + \frac{2}{2!}x^2 = 1 + x^2$$

8. Maclaurin Series for $f(x) = x^3 - x$:

$$f(x) = x^3 - x$$

- $f'(x) = 3x^2 - 1$, $f''(x) = 6x$, $f'''(x) = 6$, etc.
- At $x = 0$:
  $f(0) = 0$, $f'(0) = -1$, $f''(0) = 0$, $f'''(0) = 6$, etc.

Maclaurin Series:

$$f(x) = -x + \frac{6}{3!}x^3 = -x + x^3$$

9. Maclaurin Series for $f(x) = 1 - x^2$:

$$f(x) = 1 - x^2$$

- $f'(x) = -2x$, $f''(x) = -2$, $f'''(x) = 0$, etc.
- At $x = 0$:
  $f(0) = 1$, $f'(0) = 0$, $f''(0) = -2$, $f'''(0) = 0$, etc.

Maclaurin Series:

$$f(x) = 1 - \frac{2}{2!}x^2 = 1 - x^2$$

10. Maclaurin Series for $f(x) = x + x^2$:

$$f(x) = x + x^2$$

- $f'(x) = 1 + 2x$, $f''(x) = 2$, $f'''(x) = 0$, etc.
- At $x = 0$:
  $f(0) = 0$, $f'(0) = 1$, $f''(0) = 2$, etc.

Maclaurin Series:

$$f(x) = x + \frac{2}{2!}x^2 = x + x^2$$