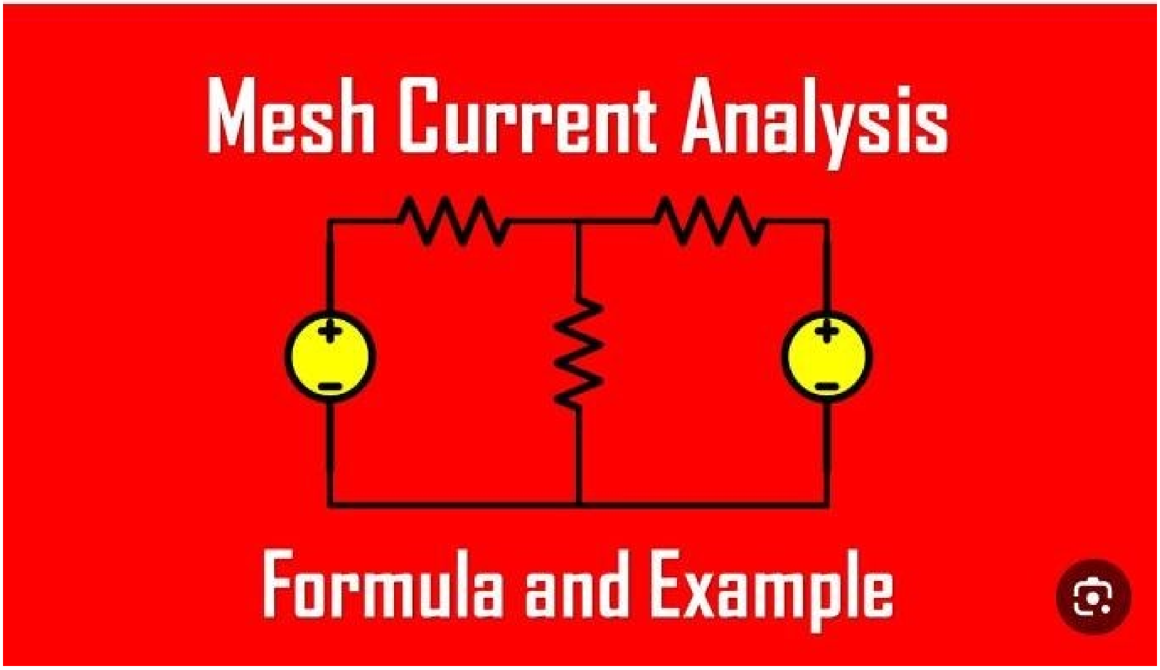

The mesh current method is a network analysis technique where mesh (or loop) current directions are assigned arbitrarily, and then Kirchhoff's voltage law (KVL) and Ohm's law are applied systematically to solve for the unknown currents and voltages.

MAXWELL'S LOOP CURRENT METHOD (MESH ANALYSIS)

Statement:-

This method determines branch currents and voltages across the elements of a network.

Method:-

1. Here, instead of taking branch currents (as in Kickoff's law) loop currents are takenwhich are assumed to flow in the clockwise direction.
2. Branch currents can be found in terms of loop currents
3. Sign conventions for the IR drops and battery emfs are the same as for Kickoff's law.
4. This method is easy if all the sources are given as voltage sluices. If theíe is a currents source present in a network then convert it into equivalent voltage sluice.

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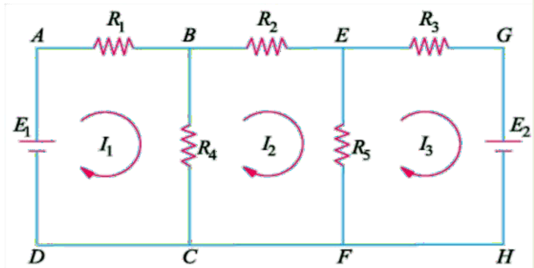
Example:-

Consider a network as shown in Fig. below. It contains three meshes. Let I_1 , I_2 and I_3 are themesh currents of three meshes directed in clockwise



$$E_1 - I_1R_1 - R_4\ (I_1 - I_2)\ = 0 \quad \text{or} \quad I_1\ (R_1 + R_4) - I_2\ R_4 - E_1 = 0 \qquad \text{...loop 1}$$

Similarly, $-I_2R_2 - R_5\ (I_2 - I_3) - R_4\ (I_2 - I_1) = 0$

or $\quad I_2\ R_4 - I_2\ (R_2 + R_4 + R_5) + I_3R_5\ = 0 \qquad \text{...loop 2}$

Also $\quad -I_3R_3 - E_2 - R_5\ (I_3 - I_2)\ = 0 \quad \text{or} \quad I_2R_5 - I_3\ (R_3 + R_5) - E_2 = 0 \qquad \text{...loop 3}$

Note: When we consider mesh-1, the currents I_1 is gíeateí than I_2. So, cuíient thíough R_4 is I_1-I_2. Similaíly, when we consider mesh-2, the cuíient I_2 is gíeateí than I_1 and I_3 . So, cuíient thíough R_4 is $I_2 - I_1$. and the cuíient thíough R_5 is $I_2 - I_3$.

Mesh Analysis Using Matrix Form

Example *Write the impedance matrix of the network shown in Fig. 2.53 and find the value of current I_3.* **(Network Analysis A.M.I.E. Sec. B.W. 1980)**

Solution. Different items of the mesh-resistance matrix $[R_m]$ are as under :

$$R_{11} = 1 + 3 + 2 = 6\ \Omega;\ R_{22} = 2 + 1 + 4 = 7\ \Omega;\ R_{33} = 3 + 2 + 1 = 6\ \Omega;$$

$$R_{12} = R_{21} = -2\ \Omega;\ R_{23} = R_{32} = -1\ \Omega;\ R_{13} = R_{31} = -3\ \Omega;$$

$$E_1 = +5\ \text{V};\ E_2 = 0;\ E_3 = 0.$$

The mesh equations in the matrix form are

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{bmatrix} = 6(42 - 1) + 2(-12 - 3) - 3\,(2 + 21) = 147$$

$$\Delta_3 = \begin{bmatrix} 6 & -2 & 5 \\ -2 & 7 & 0 \\ -3 & -1 & 0 \end{bmatrix} = 6 + 2(5) - 3(-35) = 121$$

$$I_3 = \Delta_3/\Delta = \frac{121}{147} = 0.823\ \text{A}$$

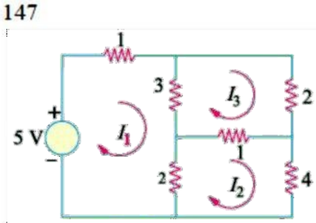


Fig. 2.53

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Example *Determine the current supplied by each battery in the circuit shown in Fig. 2.54.*

(Electrical Engg. Aligarh Univ.)

Solution. Since there are three meshes, let the three loop currents be shown in Fig. 2.51.

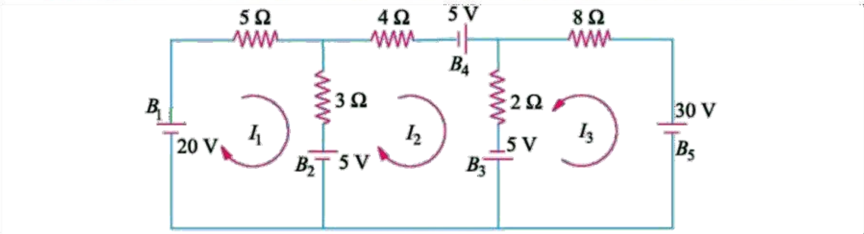

Fig. 2.54

For loop 1 we get

$20 - 5I_1 - 3\ (I_1 - I_2) - 5 = 0$ or $8I_1 - 3I_2 = 15$ ...(*i*)

For loop 2 we have

$-4I_2 + 5 - 2\ (I_2 - I_3) + 5 + 5 - 3\ (I_2 - I_1) = 0$ or $3I_1 - 9I_2 + 2I_3 = -15$ ...(*ii*)

Similarly, for loop 3, we get

$-8I_3 - 30 - 5 - 2(I_3 - I_2) = 0$ or $2I_2 - 10I_3 = 35$ ...(*iii*)

Eliminating I_1 from (*i*) and (*ii*), we get $63I_2 - 16I_3 = 165$ ...(*iv*)

Similarly, for I_2 from (*iii*) and (*iv*), we have $I_2 = 542/299$ A

From (*iv*), $I_3 = -1875/598$ A

Substituting the value of I_2 in (*i*), we get $I_1 = 765/299$ A

Example

Discharge current of B_1 = **765/299A**

Charging current of $B_2 = I_1 - I_2 =$ **220/299 A**

Discharge current of $B_3 = I_2 + I_3 =$ **2965/598 A**

Discharge current of $B_4 = I_2 =$ **545/299 A**; Discharge current of $B_5 =$ **1875/598 A**

Solution by Using Mesh Resistance Matrix.

The different items of the mesh-resistance matrix $[R_m]$ are as under :

$R_{11} = 5 + 3 = 8\ \Omega$; $R_{22} = 4 + 2 + 3 = 9\ \Omega$; $R_{33} = 8 + 2 = 10\ \Omega$

$R_{12} = R_{21} = -3\ \Omega$; $R_{13} = R_{31} = 0$; $R_{23} = R_{32} = -2\ \Omega$

E_1 = algebraic sum of the voltages around mesh (*i*) = 20 – 5 = 15 V

$E_2 = 5 + 5 + 5 = 15$ V ; $E_3 = -30 - 5 = -35$ V

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Example

Hence, the mesh equations in the matrix form are

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix} = 8(90-4) + 3(-30) = 598$$

$$\Delta_1 = \begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{vmatrix} = 15(90-4) - 15(-30) - 35\,(6) = 1530$$

$$\Delta_2 = \begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{vmatrix} = 8(150-70) + 3(150+0) = 1090$$

$$\Delta_3 = \begin{vmatrix} 8 & -3 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{vmatrix} = 8(-315+30) + 3(105+30) = -1875$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{1530}{598} = \frac{765}{299}\text{ A}\,;\ I_2 = \frac{\Delta_2}{\Delta} = \frac{1090}{598} = \frac{545}{299}\text{ A}\,;\ I_3 = \frac{\Delta_3}{\Delta} = \frac{-1875}{598}\text{ A}$$

Home Work

1. Find the ammeter current in Fig. 2.57 by using loop analysis. **[1/7 A]**
2. Using mesh analysis, determine the voltage across the 10 kΩ resistor at terminals *a-b* of the circuit shown in Fig. 2.58. **[2.65 V]**
3. Apply loop current method to find loop currents I_1, I_2 and I_3 in the circuit of Fig. 2.59. **[I_1 = 3.75 A, I_2 = 0, I_3 = 1.25 A]**

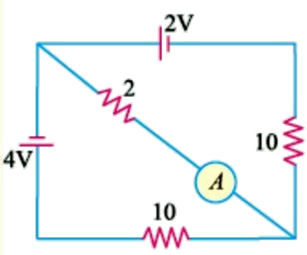

Fig. 2.57

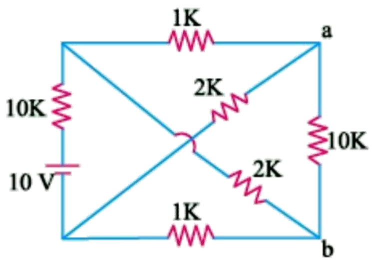

Fig. 2.58

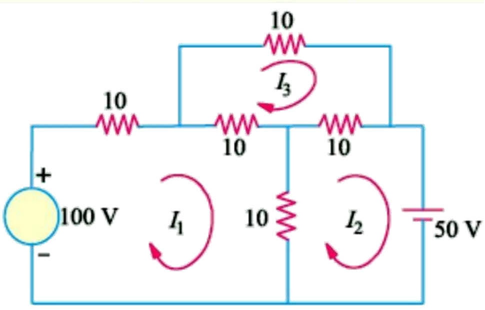

Fig. 2.59

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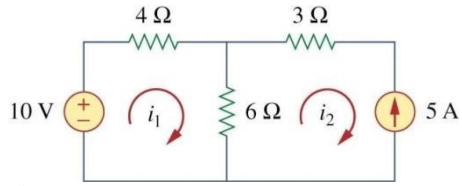
MESH ANALYSIS WITH CURRENT SOURCES

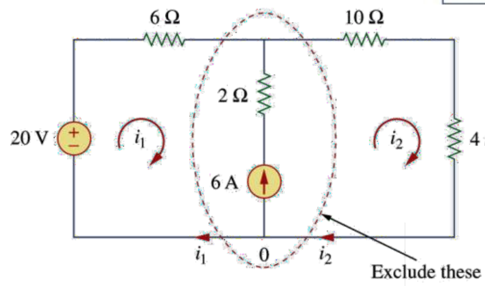


<u>Case 1</u>

- Current source exist only in one mesh
- One mesh variable is reduced

<u>Case 2</u>

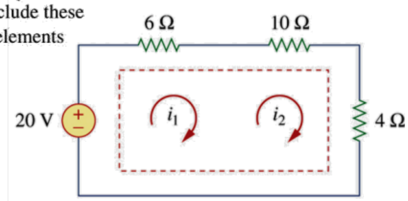
- Current source exists between two meshes, a super-mesh is obtained.



(a)

$$6i_1 + 14i_2 = 20$$

$$i_1 - i_2 = -6$$



(b)

a supermesh results when two meshes have a (dependent , independent) current source in common.

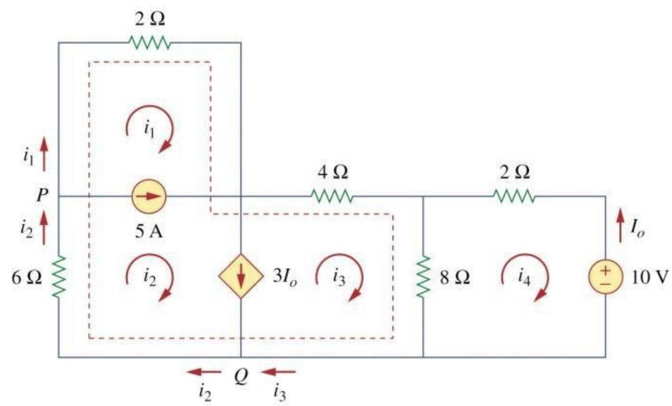
Example For the circuit in Fig. 3.24, find i_1 to i_4 using mesh analysis.

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$i_1 - i_2 = -5$$

$$i_2 - i_3 = 3I_o = -3i_4$$

$$2i_4 + 8(i_4 - i_3) = -10$$



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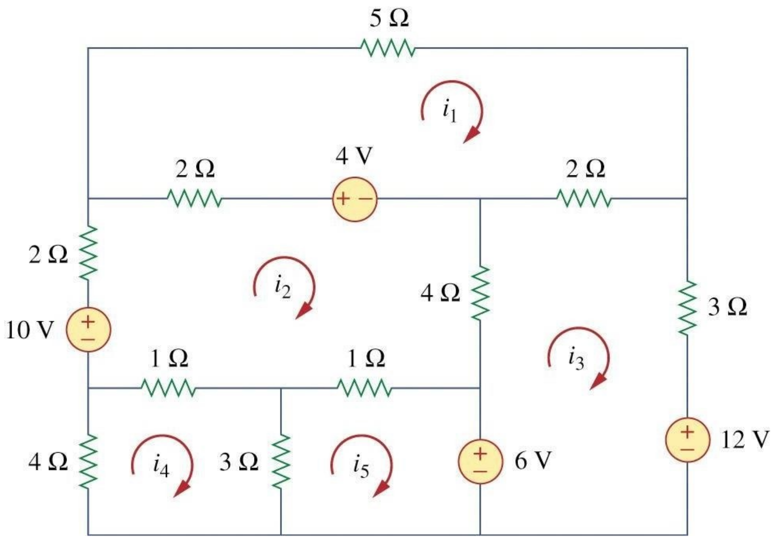
Home Work

Write the mesh current equations in Figure below.



$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

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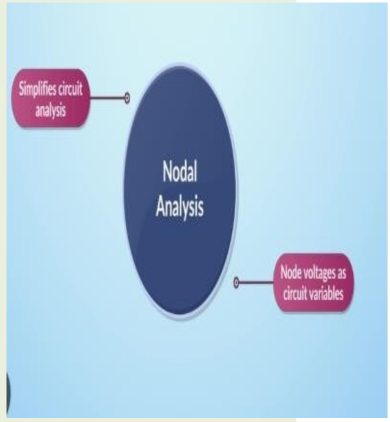
Nodal Analysis



Nodal analysis is used for solving any electrical network, and it is defined as the mathematical method for calculating the voltage distribution between the circuit nodes.

This method is also known as the node-voltage method since the node voltages are with respect to the ground. The following are the three laws that define the equation related to the voltage that is measured between each circuit node:

Ohm's law
Kirchhoff's voltage law
Kirchhoff's current law

NODAL ANALYSIS

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltage v_1, v_2, ...v_{n-1} to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the n-1 nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Common symbols for indicating a reference node,
(a) common ground,

(b) ground,

(c) chassis.

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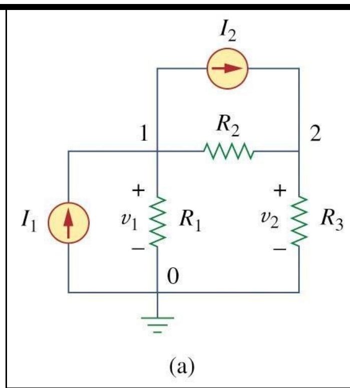
NODAL ANALYSIS

Typical circuit for nodal analysis



(a)

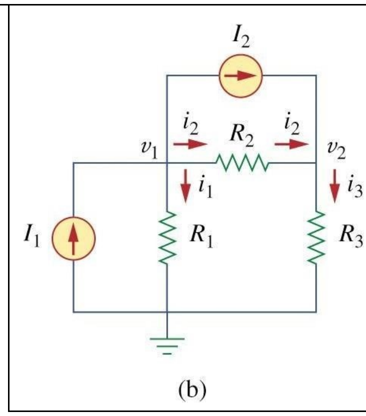
$$I_1 = I_2 + i_1 + I$$

$$i_1 = \frac{v_1 - 0}{R_1} \quad \text{or} \quad i_1 = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2} \quad \text{or} \quad i$$

$$i_3 = \frac{v_2 -}{}$$

$$\boxed{i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}}$$



(b)

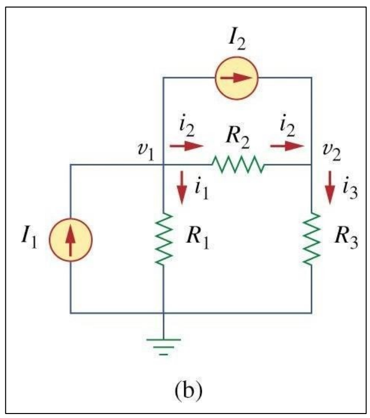
NODAL ANALYSIS

$$\Rightarrow I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

$$\boxed{\begin{aligned} \Rightarrow I_1 - I_2 &= G_1 v_1 + G_2 (v_1 - v_2) \\ I_2 &= -G_2 (v_1 - v_2) + G_3 v_2 \end{aligned}}$$

$$\Rightarrow \boxed{\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}}$$



(b)

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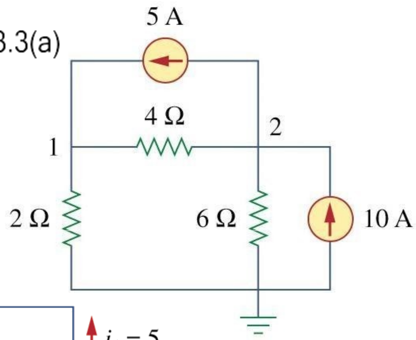
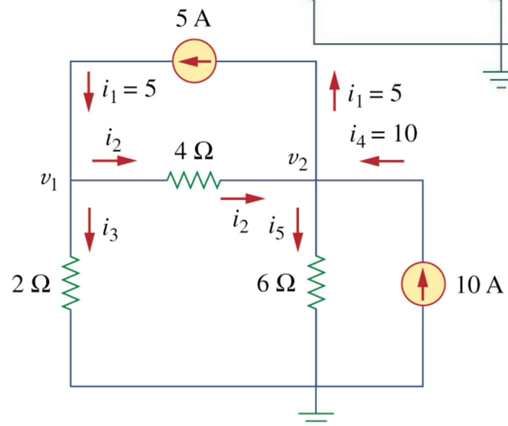
Example Calculus the node voltage in the circuit shown in Fig. 3.3(a)





At node 1

$$i_1 = i_2 + i_3$$

$$\Rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

At node 2

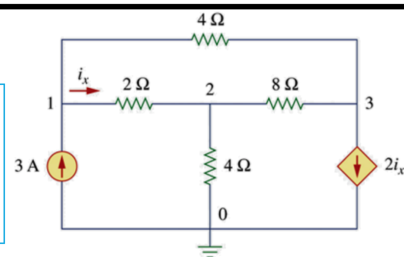
$$i_2 + i_4 = i_1 + i_5$$

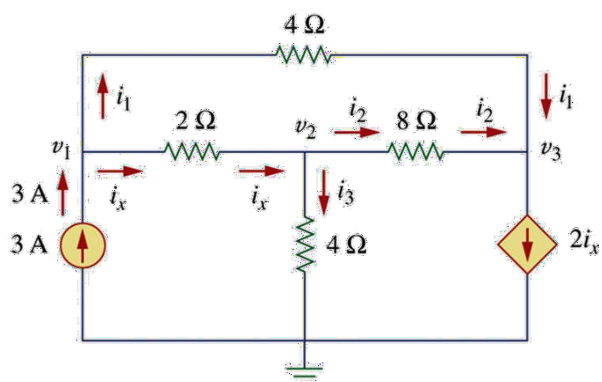
$$\Rightarrow 5 = \frac{v_2 - v_1}{4} + \frac{v_2 - 0}{6}$$

In matrix form:

$$\begin{bmatrix} \frac{1}{2} + \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{6} + \frac{1}{4} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

Example Determine the voltage at the nodes in Fig. 3.5(a)





At node 1, $3 = i_1 + i_x$

$$\Rightarrow 3 = v_1 -$$

At node 2 $i_x = i_2 + i_3$

$$\Rightarrow \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

At node 3 $i_1 + i_2 = 2i_x$

$$\Rightarrow \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

In matrix form:

$$\begin{bmatrix} \frac{3}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{7}{8} & -\frac{1}{8} \\ \frac{3}{4} & -\frac{9}{8} & \frac{3}{8} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

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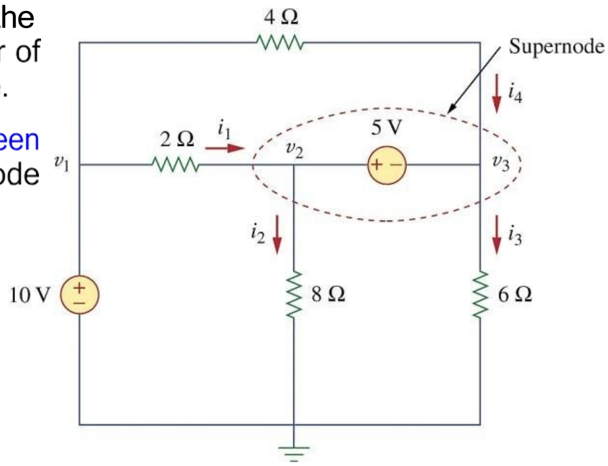
NODAL ANALYSIS WITH VOLTAGE SOURCES

Case 1: The voltage source is connected between a nonreference node and the reference node: The nonreference node voltage is equal to the magnitude of voltage source and the number of unknown nonreference nodes is reduced by one.

Case 2: The voltage source is connected between two nonreferenced nodes: a generalized node (supernode) is formed.

Fig. 3.7 A circuit with a supernode.



Example For the circuit shown, find the node voltages.

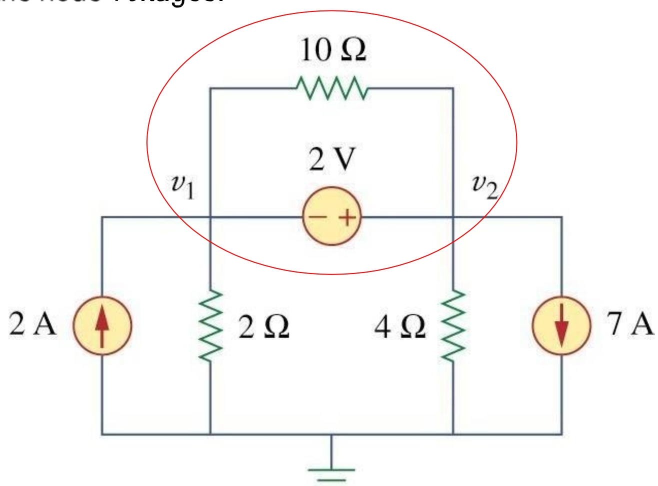
$$2 - 7 - i_{2\Omega} - i_{4\Omega} = 0$$

$$2 - 7 - \frac{v_1}{2} - \frac{v_2}{4} = 0$$

$$\therefore \quad 2v_1 + v_2 = -20 \cdots (1)$$

and

$$v_1 - v_2 = -2 \cdots (2)$$



BASIC

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Example Find the node voltages in the circuit of Fig. 3.12.

At suopernode 1-2,

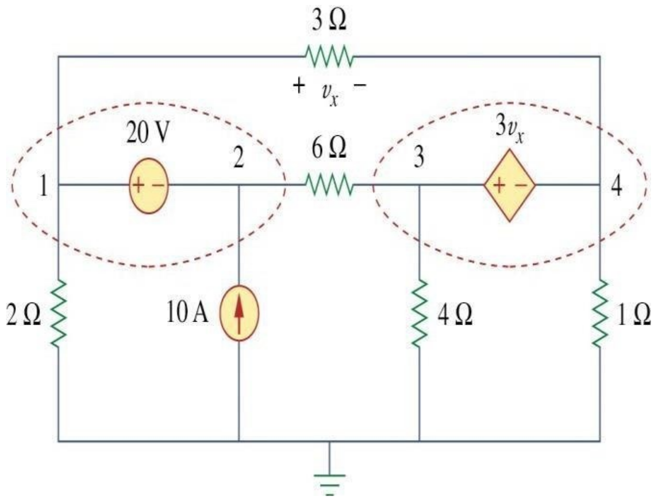
$$\frac{v_1}{2} + \frac{v_1 - v_4}{3} + \frac{v_2 - v_3}{6} = 10$$

$$v_1 - v_2 = 20$$

At supernode 3-4,

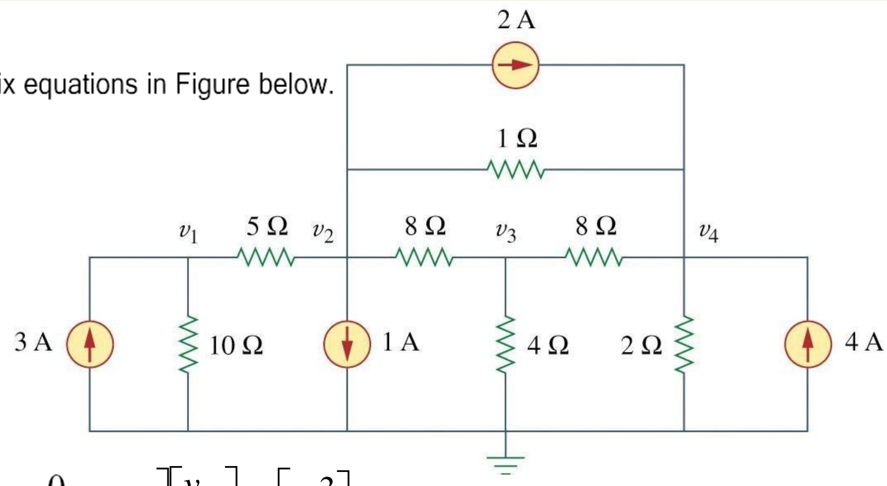
$$\frac{v_3}{4} + \frac{v_4}{1} + \frac{v_3 - v_2}{6} + \frac{v_4 - v_1}{3} = 0$$

$$v_3 - v_4 = 3v_x = 3(v_1 - v_4)$$



Home Work

Write the node voltage matrix equations in Figure below.



$$\begin{bmatrix} 0.3 & -0.2 & 0 & 0 \\ -0.2 & 1.325 & -0.125 & -1 \\ 0 & -0.125 & 0.5 & -0.125 \\ 0 & -1 & -0.125 & 1.625 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \\ 6 \end{bmatrix}$$

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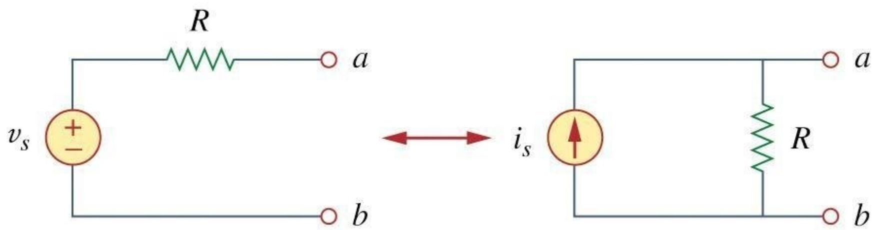
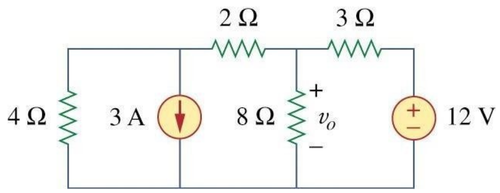
SOURCE CONVERSION

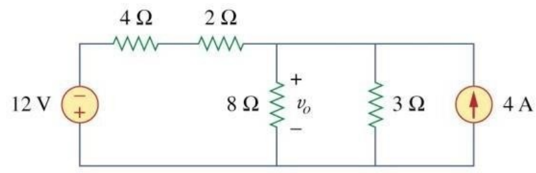
A given voltage source with a series resistance can be converted into (or replaced by) and equivalent current source with a parallel resistance.
This current is I = V/R

Conversely, a current source with a parallel resistance can be converted into a voltage source with a series resistance.
The voltage source of voltage V = IR



Example Use source transformation to find v_o in the circuit in Figure





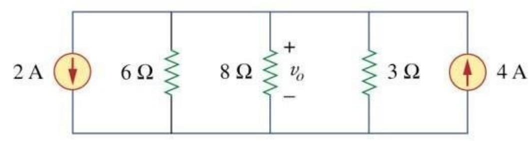
(a)

we use current division in Figure (c) to get

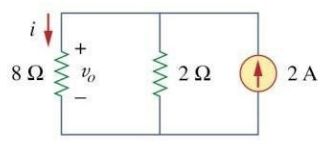
$$i = \frac{2}{2+8}(2) = 0.4\text{A}$$

and

$$v_o = 8i = 8(0.4) = 3.2\text{V}$$



(b)



(c)

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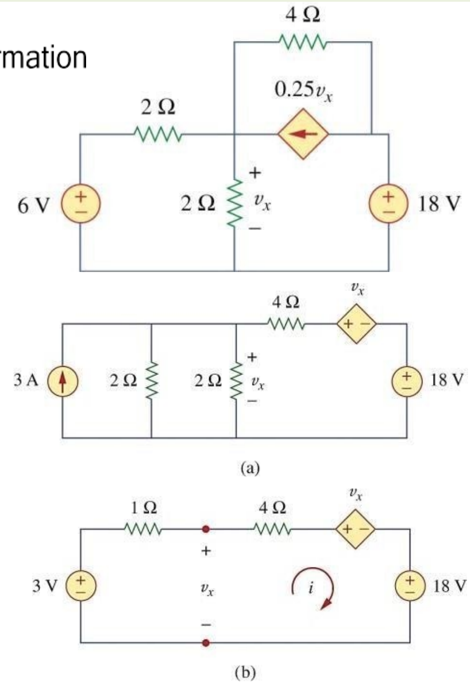
Example Find v_x in Figure below using source transformation



Applying KVL around the loop in Figure (b) gives

$-3 + 5i + v_x + 18 = 0$ or $15 + 5i + v_x = 0$

Appling KVL to the loop containing only the 3V voltage source, the 1Ω resistor, and v_x yields

$-3 + 1i + v_x = 0 \Rightarrow v_x = 3 - i$

Substituting this into first Equation, we obtain

$15 + 5i + 3 - i = 0$ *or* $4i = -18 \Rightarrow i = -4.5\text{A}$

Alternatively

$-v_x + 4i + v_x + 18 = 0 \Rightarrow i = -4.5\text{A}$

thus

$v_x = 3 - i = 7.5\text{V}$

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