Module Title: Fundamental of Electrical Engineering (DC)

Module Code:	**UOMU024011**

Chapter 4
Lectures (Week 7, 8, 9,10)
Circuit analysis Theorems:
Nodal Analysis Methods
Kirchhoff's laws
Thevenin's and Norton's Theorems
Examples and solutions
(Week 7)

4.0 Introduction Nodal Analysis Method

Having understood the fundamental laws of circuit theory (Ohm's law and Kirchhoff's laws),

we are now prepared to apply these laws to develop two powerful techniques for nodal circuit analysis:

nodal analysis, which is based on a systematic application of Kirchhoff's current law (KCL), and mesh analysis, which is based on a systematic application of Kirchhoff's voltage law (KVL). The two techniques are so important that this chapter should be regarded as the most important in the lectures.

4.1 Nodal Analysis

Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables. Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously.

To simplify matters, we shall assume in this section that circuits do not contain voltage sources. Circuits that contain voltage sources will be analyzed in the next section.

Steps to Determine Node Voltages:

1. **Select a node as the reference node. Assign voltages v1, v2,. . vn–1 to the remaining n-1 nodes.**
2. **The voltages are referenced with respect to the reference node.**
3. **Apply KCL to each of the n-1 nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.**

4. Solve the resulting simultaneous equations to obtain the unknown node voltages.

We shall now explain and apply these three steps.

The first step in nodal analysis is selecting a node as the reference or datum node. The reference node is commonly called the ground since it is assumed to have zero potential. A reference node is indicated by any of the three symbols in Fig. 4.1. We shall always use the symbol in Fig. 4.1(b). Once we have selected a reference node, we assign voltage designations.

to nonreference nodes. Consider, for example, the circuit in **Fig. 4.2(a)**. Node 0 is the reference node (**v = 0**), while nodes 1 and 2 are assigned voltages **v1** and **v2**, respectively. Keep in mind that the node voltages are defined with respect to the reference node. As illustrated in **Fig. 4.2(a)**, each node voltage is the voltage with respect to the reference node.

The number of nonreference nodes is equal to the number of independent equations that we will derive.

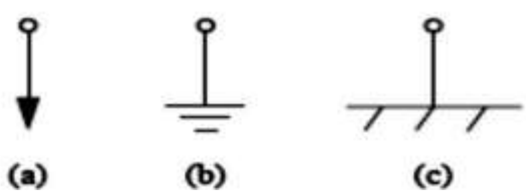

Figure 4.1 Common symbols for indicating a reference node

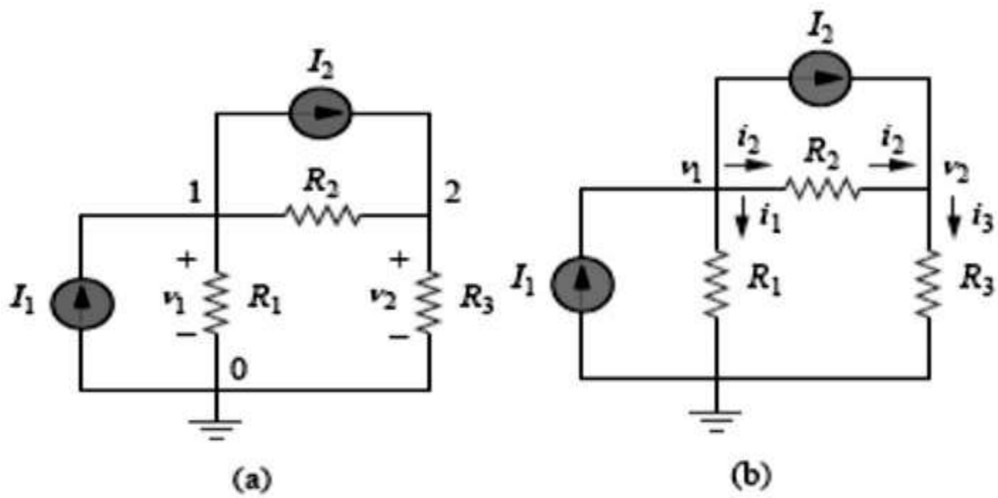

Figure 4.2 Typical circuits for nodal analysis.

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Chapter 4
Lectures (Week 8)
Kirchhoff's laws

4.1 Kirchhoff's Rules

Learning Objectives

By the end of the section, you will be able to:

• State Kirchhoff's junction rule.

• State Kirchhoff's loop rule.

• Analyze complex circuits using Kirchhoff's rules.

Introduction:

We have just seen that some circuits may be analyzed by reducing a circuit to a single voltage source and an equivalent resistance. Many complex circuits cannot be analyzed with the series-parallel techniques developed in the preceding sections. In this section, we elaborate on the use of Kirchhoff's rules to analyze more complex circuits. For example, the circuit in Figure 4.3 is known as a multi-loop circuit, which consists of junctions. A junction, also known as a node, is a connection of three or more wires.

In this circuit, the previous methods cannot be used, because not all the resistors are in clear series or parallel configurations that can be reduced. Give it a try.

The resistors **R1** and **R2, R3** are in series and can be reduced to an equivalent resistance.

The same is true of resistors **R4** and **R5** . But what do you do then?

Even though this circuit cannot be analyzed using the methods already learned, two circuit analysis rules can be used to analyze any circuit, simple or complex. The rules are known as Kirchhoff's rules, after their **inventor Gustav Kirchhoff (1824–1887)**.

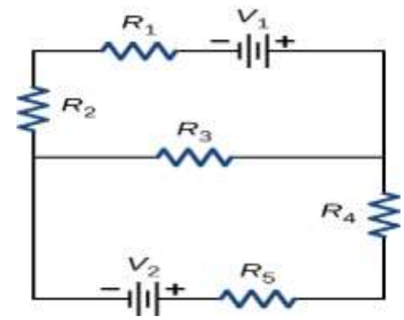

Figure 4.3 This circuit cannot be reduced to a combination of series and parallel connections.

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4.1.1 Kirchhoff's Rules

- **<u>Kirchhoff's first rule</u>**: the junction rule. The sum of all currents entering a junction must equal the sum of all currents leaving the junction:

  $\sum I_{in} = \sum I_{out}$. (4.1)

- **<u>Kirchhoff's second rule</u>** : the loop rule. The algebraic sum of changes in potential around any closed-circuit path (loop) must be zero:

  $\sum \mathbf{V} = \mathbf{0}$. (4.2)

We now provide explanations of these two rules, followed by problem-solving hints for applying them and a worked example that uses them.

- **<u>Kirchhoff's First Rule:</u>**

Kirchhoff's first rule (the junction rule) applies to the charge entering and leaving a junction (Figure 4.2).

As stated earlier, a junction, or node, is a connection of three or more wires. Current is the flow of charge, and charge is conserved; thus, whatever charge flows into the junction must flow out.

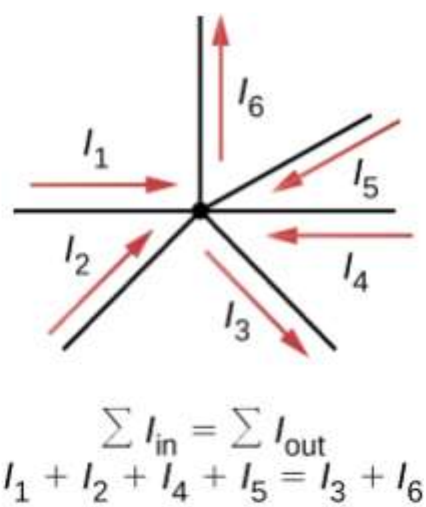

Figure 4.2 Charge must be conserved, so the sum of currents into a junction must be equal to the sum of currents out of the junction.

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Although it is an over-simplification, an analogy can be made with water pipes connected in a plumbing junction. If the wires in Figure 4.2 were replaced by water pipes, and the water was assumed to be incompressible, the volume of water flowing into the junction must equal the volume of water flowing out of the junction.

- **<u>Kirchhoff's Second Rule:</u>**

Kirchhoff's second rule (the loop rule) applies to potential differences. The loop rule is stated in terms of potential V rather than potential energy, but the two are related since U = qV.

In a closed loop, whatever energy is supplied by a voltage source, the energy must be transferred into other forms by the devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. Kirchhoff's loop rule states that the algebraic sum of potential differences,

including voltage supplied by the voltage sources and resistive elements, in any loop must be equal to zero. For example, consider a simple loop with no junctions, as in Figure 4.3.

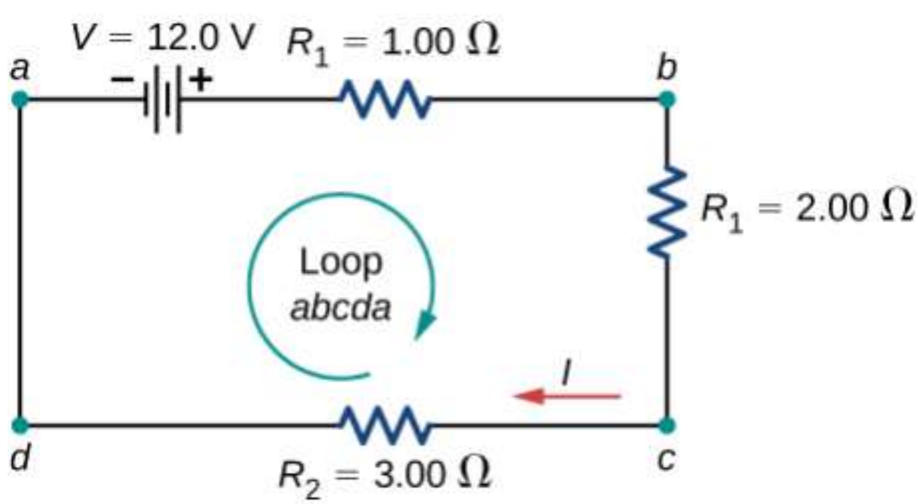

Figure 4.3 A simple loop with no junctions. Kirchhoff's loop rule states that the algebraic sum of the voltage differences is equal to zero.

Kirchhoff’s laws worksheet with example

4.2Brief Worksheet

- Kirchhoff’s Current Law - states that the current entering a point in a circuit is equal to the summation of the currents exiting.
- Kirchhoff’s Voltage Law - states that the summation of all voltage drops in a closed loop must equal to zero which is a result of the electrostatic field being conservative.
- (Conventional) current flowing through the cell has a positive voltage (gains energy). Current going through a resistor has a negative voltage (loses energy).

Example worksheet Problem (for students)

In figure 4.4 below the circuit has two loops and two source of supply voltage

1. Begin by labelling the junctions in our circuit, J1 and J2. Then we label the currents as I, I1 and I2 in an arbitrary direction as shown in the figure below. (Direction of currents will be confirmed once we complete the problem).

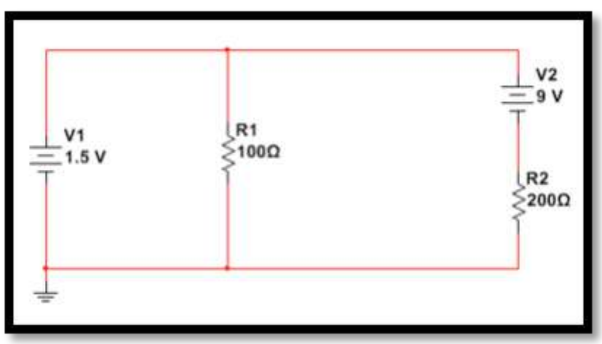

Figure 4.4

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Lectures (Week 9, 10)
Thevenin's and Norton's Theorems
Examples and solutions

Problem 1: Find the Thevenin's equivalent circuit for the following circuit shown in Figure 5.

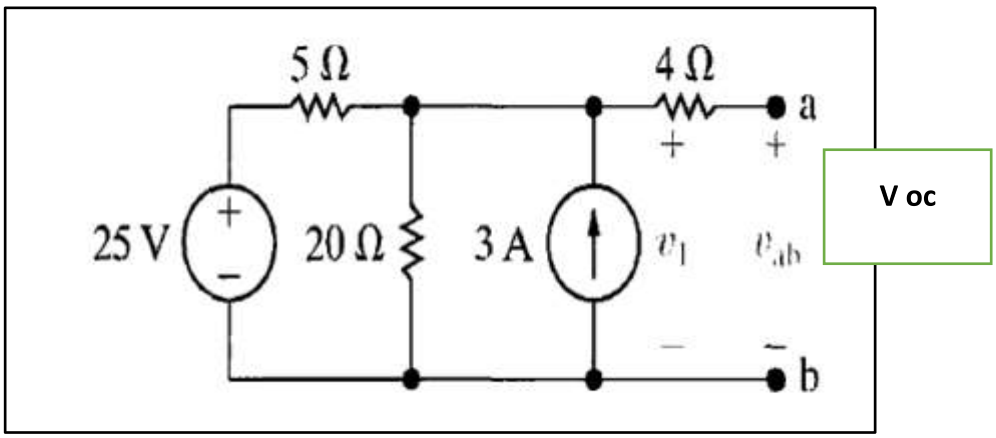


Figure 5

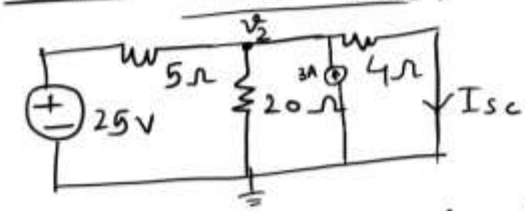
V_{th}:

* $V_{oc} = V_{ab} = v_1$

* $\frac{v_1 - 25}{5} + \frac{v_1}{20} = 3$

$\frac{v_1}{5} + \frac{v_1}{20} = 8 \Rightarrow \boxed{v_1 = 32V}$

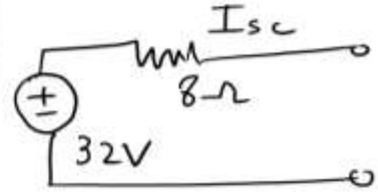
R_{th}: Find I_{sc}:



$\frac{v_2 - 25}{5} + \frac{v_2}{20} + \frac{v_2}{4} = 3$

$v_2 = 16V \Rightarrow \boxed{I_{sc} = 4A}$

$R_{th} = \frac{v_{th}}{I_{sc}} = 8\Omega$



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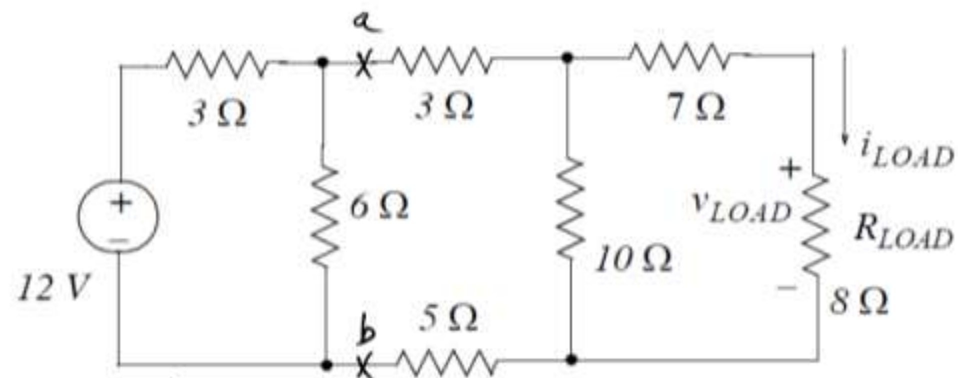
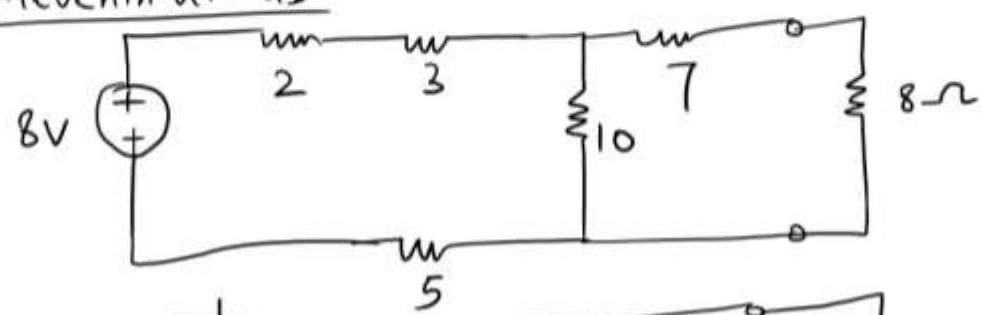
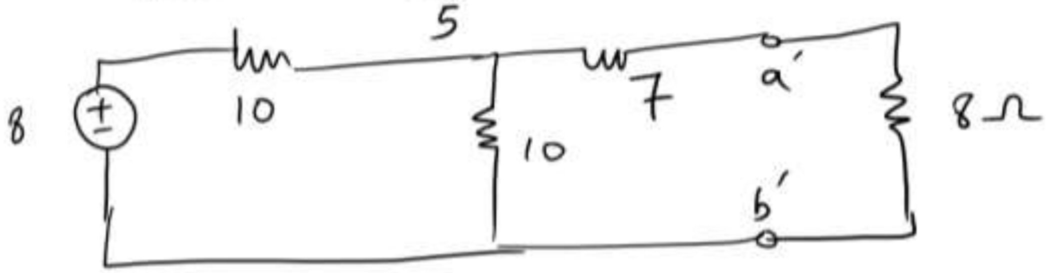
Problem 2: Find i_{LOAD} through R_{LOAD} using Thevenin's theorem

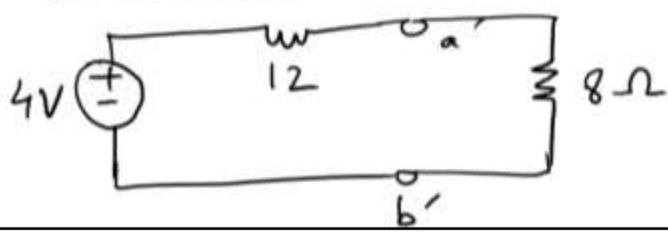


Thevenin at ab





Thevenin at a′ – b′



$$i_{LOAD} = \frac{4}{20} = 0.2A$$

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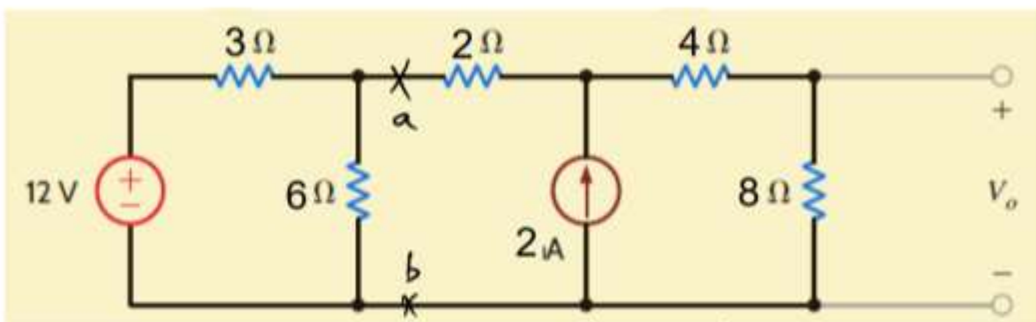
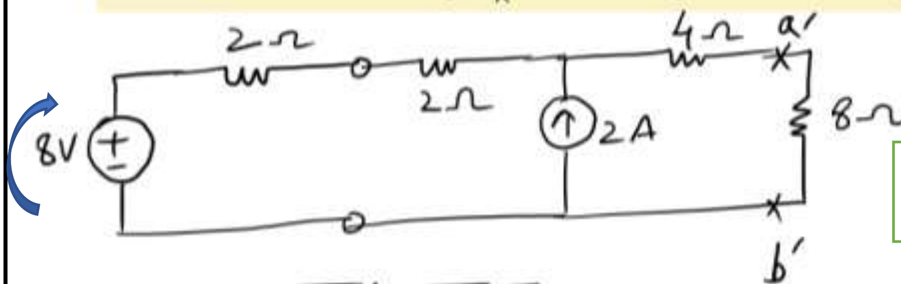
Problem 3: Find V_o using Thevenin's theorem





8 Volt= (12x6)/ 9

$$V_o = 16 \times \frac{8}{16} = 8V$$

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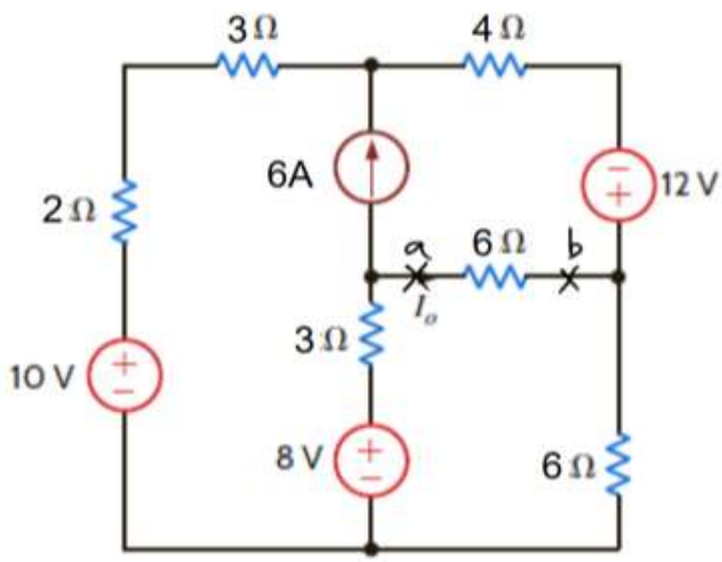
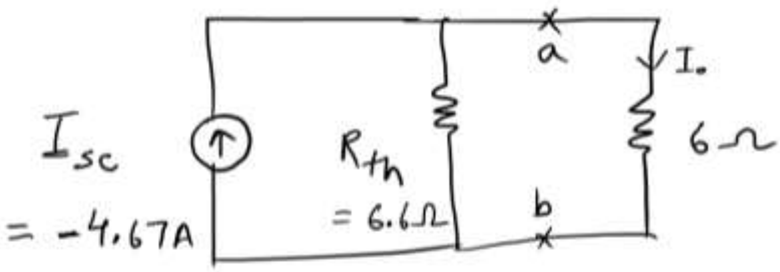
Problem 4: Find I_o using Thevenin's or Norton's theorem



Equivalent : (computed on next page)



$$I_o = \frac{6.6}{12.6} \times (-4.67) = -2.44A$$

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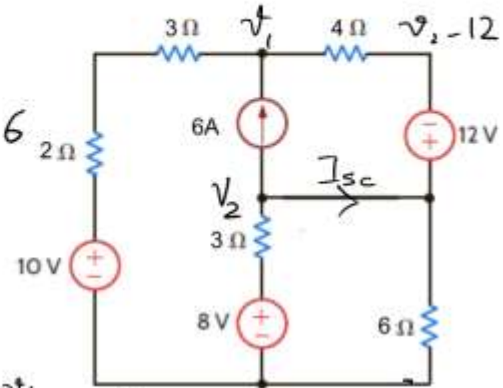
Thevenin's and Norton's Theorems

Problems – In class



$$\frac{v_1 - 10}{5} + \frac{v_1 - (v_2 - 12)}{4} = 6$$

$$\boxed{9v_1 - 5v_2 = 100}$$

$$\frac{v_2 - 8}{3} + \frac{v_2}{6} + \frac{v_2 - 12 - v_1}{4} = -6$$

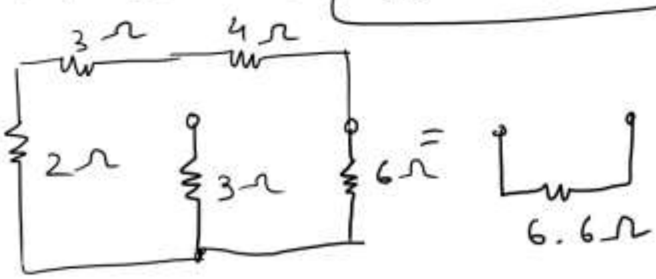
$$\Rightarrow \frac{4v_2 - 24 + 2v_2 + 3v_2 - 36 - 3v_1}{12} = -6$$

$$\boxed{-3v_1 + 9v_2 = -4} \Rightarrow -9v_1 + 27v_2 = -12$$

$$\Rightarrow 22v_2 = 88 \Rightarrow \boxed{v_2 = 4V}$$

$$\frac{v_2 - 8}{3} + 6 + I_{sc} = 0 \Rightarrow \boxed{I_{sc} = -4.667 A}$$

R_{th} :



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(Week 10)
Extra solved problems
Norton's Theorems

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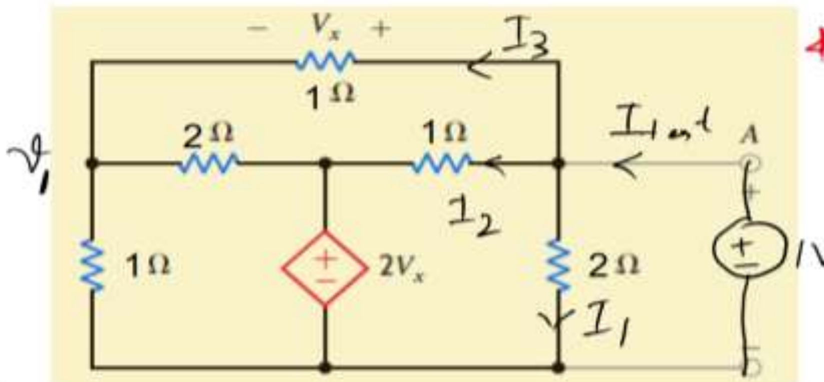
Problem 5: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals AB (Irwin – Example 5.8)



$\Rightarrow V_{th} = 0V$

We have

$$v_x + v_1 = 1V \quad \Rightarrow \quad v_1 = 1 - v_x$$

KCL ;
$$\frac{v_1}{1} + \frac{v_1 - 2v_x}{2} + \frac{v_1 - 1}{} = 0$$

$$\Rightarrow \quad 1 - v_x + \frac{1 - 3v_x}{2} - v_x = 0$$

$$\Rightarrow \quad 3 = 7v_x \quad \Rightarrow \quad \boxed{v_x = \frac{3}{7}\ V}$$

$$I_1 = \frac{1}{2} A, \quad I_2 = \frac{1 - 2(3/7)}{1} = 1/7\ A$$

$$I_3 = 3/7\ A$$

$$I_{test} = I_1 + I_2 + I_3 = \boxed{\frac{15}{14} A}$$

$$\Rightarrow R_{th} = \frac{1}{I_{test}} = \frac{14}{15}\ \Omega$$

EE240 Circuits I 6

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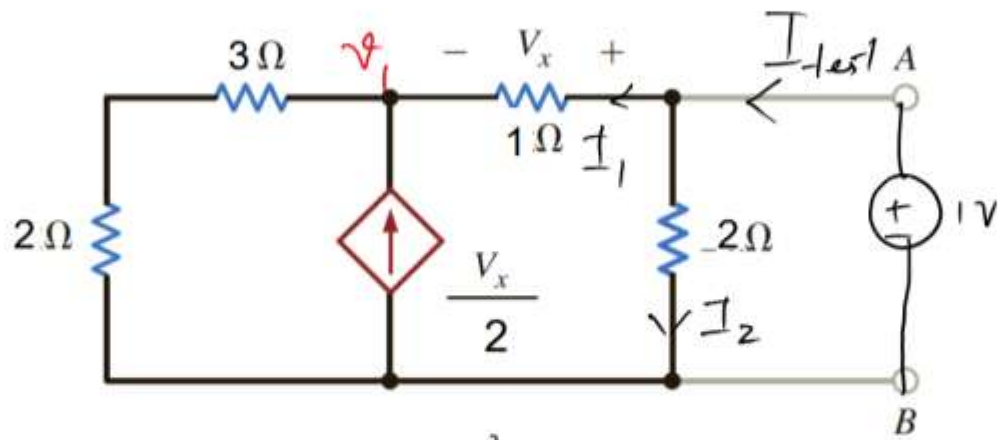
Problem 6: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals AB (Irwin – E 5.13)



$$\frac{v_1}{5} + \frac{v_1 - 1}{1} = \frac{V_x}{2}$$

$$V_x + v_1 = 1 \Rightarrow V_x = 1 - v_1$$

$$\Rightarrow \boxed{v_1 = \frac{15}{17}\ V}$$

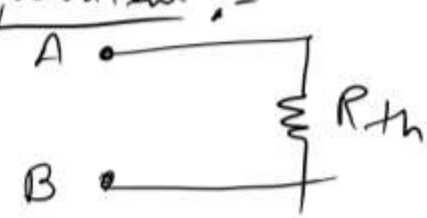
$$I_{test} = I_1 + I_2 = \frac{1}{2} + \frac{2}{17} = \frac{21}{34}\ A$$

$$I_1 = \frac{1 - v_1}{1} = \frac{2}{17}\ A$$

$$I_2 = \frac{1}{2}$$

$$\Rightarrow R_{th} = \frac{1}{I_{test}} = \frac{34}{21}\ \Omega$$

Equivalent :-



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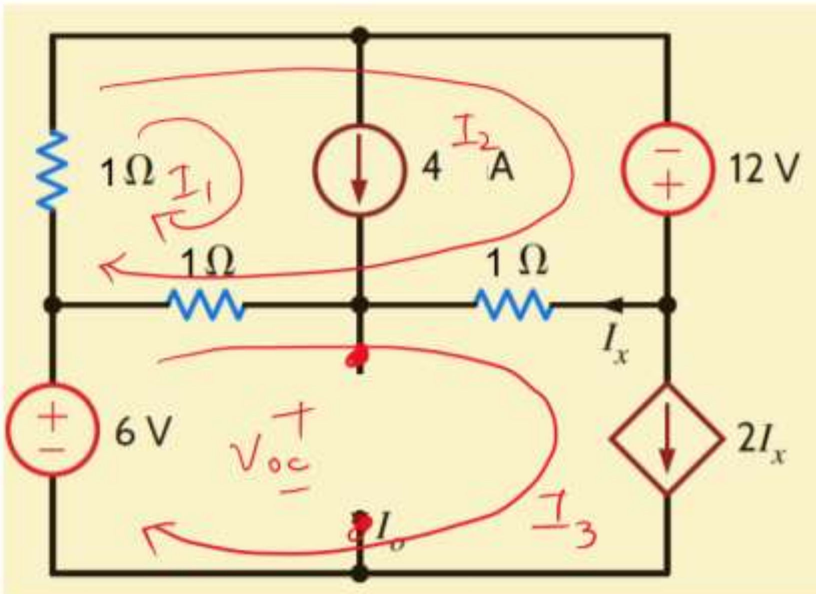
Problem 7: Find I_o using Thevenin's theorem (See problem sheet for problems)



$I_1 = 4A, \quad I_3 = 2I_x$

Loop 2
$$-12 + 1(I_2 - I_3) + 1(I_1 + I_2 - I_3) + (I_1 + I_2)1 = 0$$

$I_x = I_2 - I_3 \Rightarrow \boxed{3I_x = I_2}$

Solving

$I_3 = \frac{8}{5}A, \quad I_2 = \frac{12}{5}A, \quad I_x = \frac{4}{5}A$

$V_{oc} = 6 + (I_1 + I_2 - I_3)1 = \frac{54}{5}V$

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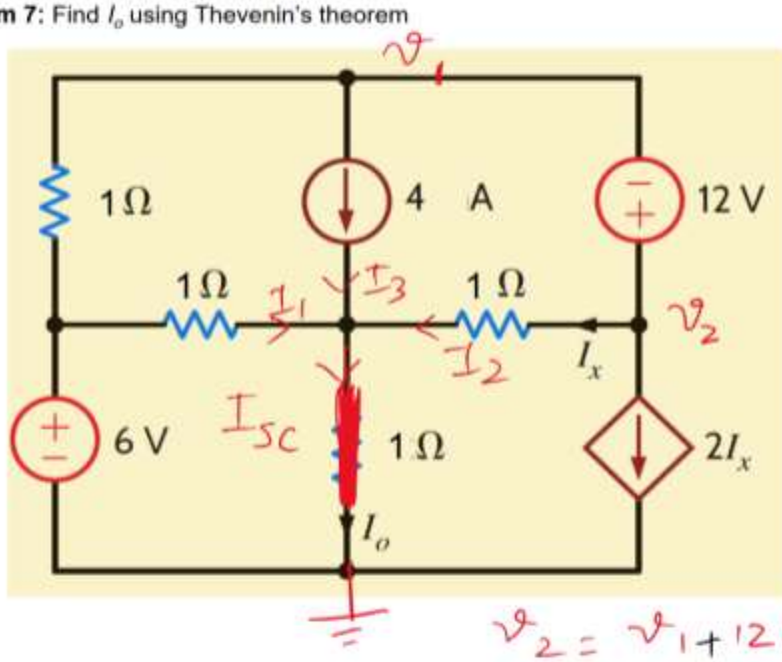
Thevenin's and Norton's Theorems

Problems – In class

Problem 7: Find I_o using Thevenin's theorem



$$v_2 = v_1 + 12 \text{ (Super Node)}$$

$$\frac{v_1 - 6}{1} + 4 + \frac{v_2}{1} + 2I_x = 0 \quad \Big\} \; I_x = \frac{v_2}{1}$$

$$\Rightarrow v_1 - 6 + 4 + 3v_1 + 36 = 0$$

$$\Rightarrow v_1 = -\frac{17}{2}\text{ V}, \quad v_2 = 7/2$$

$$I_{sc} = \frac{6}{1} + \frac{7}{2} + 4 = \frac{27}{2}\text{ A}$$

$$\Rightarrow R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{54/5}{27/2} = \frac{4}{5}\ \Omega$$

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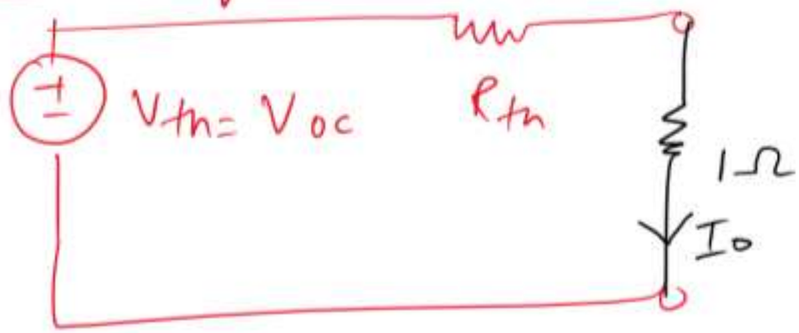
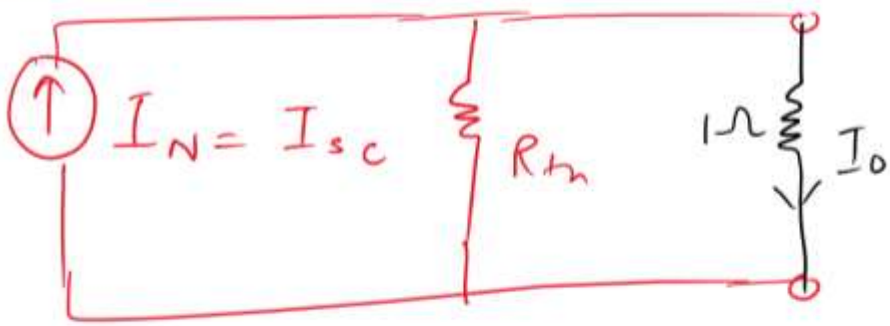
Problem 7: Find I_o using Thevenin's theorem

Thevenin Equivalent



Norton Equivalent



$$I_o = \frac{54/5}{1+\frac{4}{5}} = 6\ A$$

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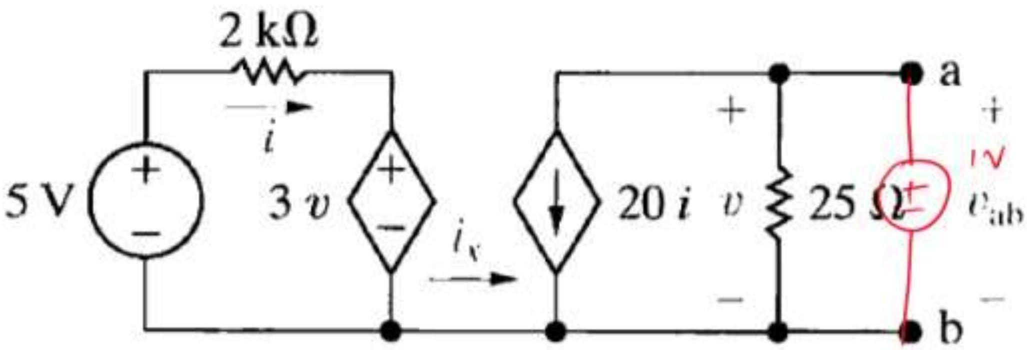
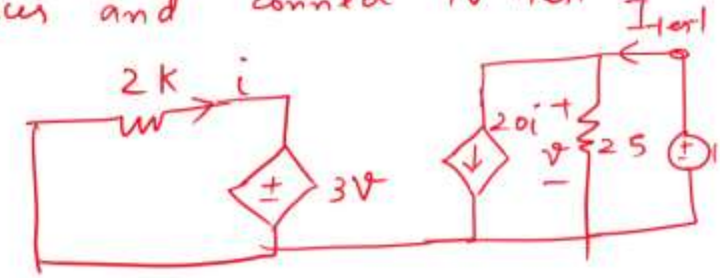
Problem 8: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals a,b



First find R_{th} :-

Switch off sources and connect 1V test source



$v = 1V$

$3v = 3V$

$i = -3/2k$

$\Rightarrow I_{test} = \frac{1}{25} - (20)(i) = 0.04 - 0.03 = 0.01A$

$\Rightarrow$ $R_{th} = 100\Omega$ $\leftarrow \frac{1}{I_{sc}}$

V_{th} :- $V_{ab} = v = v_{th}$

$V_{th} = -20i \times 25$ — ①

$i = \frac{25 - v_{th}}{2k}$ — ②

①+② $\Rightarrow$ $V_{th} = -5V$

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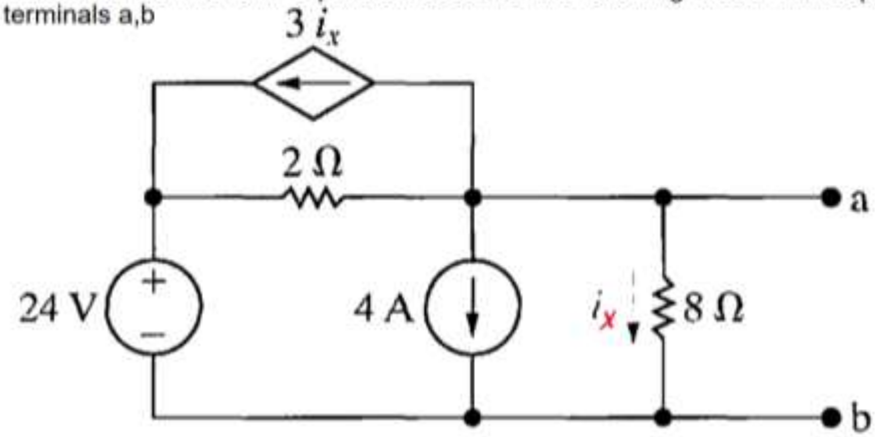
Problem 9: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals a,b



* Circuit contains both dependent and Independt Sources; we can use either of the following techniques

1) Determine V_{ab} and I_{sc}
2) Determine V_{ab} ; Determine R_{th} by switching off independent sources and applying test current (or voltage) source at a-b.

Let's apply 2)

V_{ab} : Using KCL;

$$\frac{V_{ab}}{8} + \frac{V_{ab}-24}{2} + 3i_x + 4 = 0$$

where $i_x = \frac{V_{ab}}{8} \Rightarrow \boxed{V_{ab} = 8i_x}$

Solving $\boxed{V_{ab} = 8V}$

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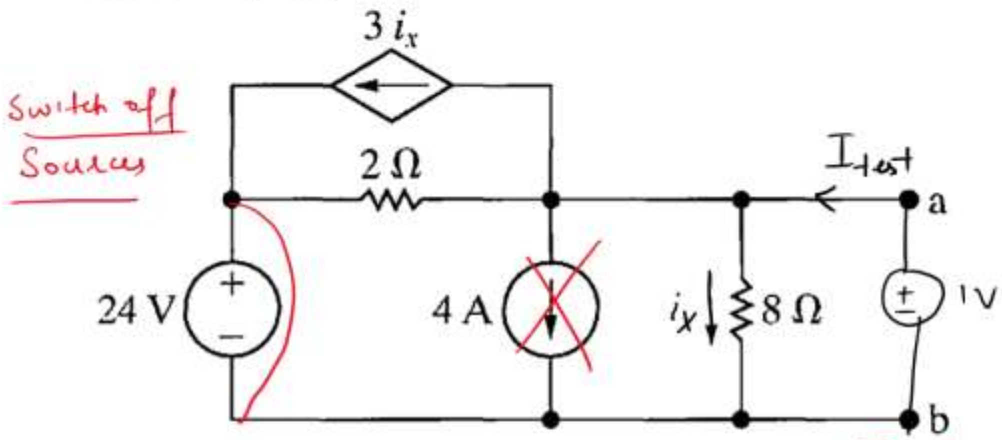
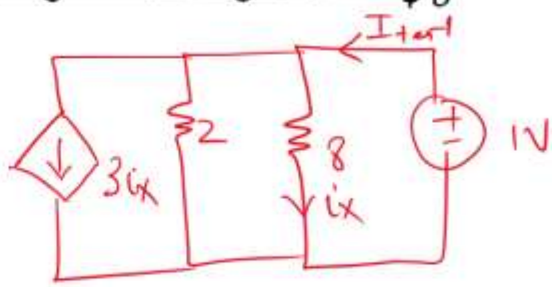
R_{th} : Apply 1V voltage source :





$$i_x = \frac{1}{8}\ A$$

$$I_{test} = \frac{1}{8} + \frac{1}{2} + \frac{3}{8}$$

$$= 8/8 = 1A$$

$$\Rightarrow R_{th} = \frac{1V}{I_{test}} = \boxed{1\ \Omega}$$

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